Exploring possibly existing $qq\bar{b}\bar{b}$ tetraquark states with $qq = ud, ss, cc$

Antje Peters
peters@th.physik.uni-frankfurt.de

Goethe-Universität Frankfurt am Main, Germany

in collaboration with Pedro Bicudo, Krzysztof Cichy,
Björn Wagenbach and Marc Wagner
arXiv:1505.00613 [hep-lat]

The 33rd International Symposium on Lattice Field Theory
Kobe, Japan

July 16, 2015

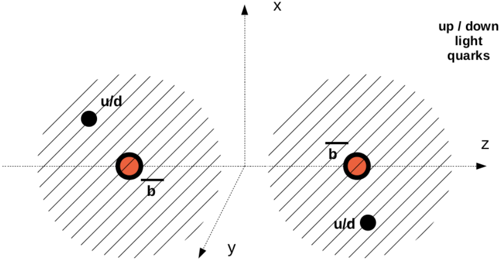
Study of tetraquark states

Motivation

- a number of mesons found in particle detectors (LHCb, Belle) not well understood
- e.g. charged charmonium and bottomonium states (${Z_c}^{\pm}$ and ${Z_b}^{\pm}$)
- mesons (integer spin), but no two-quark structure

Born-Oppenheimer approximation

- Lattice computation of the potential of two static antiquarks in the presence of two light quarks



[Bicudo, Wagner, arXiv:1209.6274 [hep-ph]]

- Solve the Schrödinger equation to check whether potentials are sufficiently attractive to form a bound state.

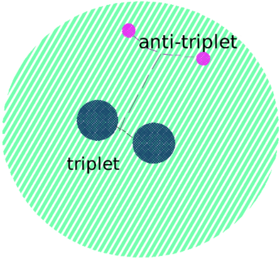
The tetraquark system - Expectations

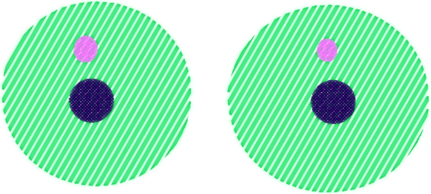
small separations of the static antiquarks:

- interaction due to 1-gluon exchange
- bound state: static $\bar{Q}\bar{Q}$ pair in a color triplet (attractive) $\longrightarrow$ antidiquark

large separations of the static antiquarks:

- screening of the antiquark-antiquark interaction due to light quarks (stronger, the more massive the light quarks)
- basically 2 static-light mesons





Composition of the operator

Approximate the two-meson state:

- operator $\mathcal{O}(t)$ generates quantum numbers (spin, parity, ...)

Composition of the operator

Approximate the two-meson state:

- operator $\mathcal{O}(t)$ generates quantum numbers (spin, parity, ...)
- obtain correlation function in time for each separation r of the static quarks:

$$C(t)\Big|_r = \langle\Omega|\,\mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle \underset{t\to\infty}{\propto} \exp(-V(r)t)$$

Composition of the operator

Approximate the two-meson state:

- operator $\mathcal{O}(t)$ generates quantum numbers (spin, parity, ...)
- obtain correlation function in time for each separation r of the static quarks:

$$C(t)\Big|_r = \langle\Omega|\, \mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle \underset{t\to\infty}{\propto} \exp(-V(r)t)$$

- for large t one finds the potential (effective mass) of the hadronic state

Composition of the operator

Approximate the two-meson state:

- operator $\mathcal{O}(t)$ generates quantum numbers (spin, parity, ...)
- obtain correlation function in time for each separation r of the static quarks:

$$C(t)\Big|_r = \langle\Omega|\, \mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle \underset{t\to\infty}{\propto} \exp(-V(r)t)$$

- for large t one finds the potential (effective mass) of the hadronic state
- use two heavy quarks and two quarks of finite mass
- heavy quarks treated in the static approximation, their spins are irrelevant $\longrightarrow$ possible to label mesons by the spin of the light quarks
- here: $\overline{QQ}qq$ with $\overline{QQ} = \overline{bb}$ and $q \in \{u, d, s, c\}$

Composition of the operator

Approximate the two-meson state:

- operator $\mathcal{O}(t)$ generates quantum numbers (spin, parity, ...)
- obtain correlation function in time for each separation r of the static quarks:

$$C(t)\big|_r = \langle\Omega|\,\mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle \underset{t\to\infty}{\propto} \exp(-V(r)t)$$

- for large t one finds the potential (effective mass) of the hadronic state
- use two heavy quarks and two quarks of finite mass
- heavy quarks treated in the static approximation, their spins are irrelevant $\longrightarrow$ possible to label mesons by the spin of the light quarks
- here: $\overline{QQ}qq$ with $\overline{QQ} = \overline{bb}$ and $q \in \{u, d, s, c\}$
- compute the temporal correlation function of the trial state:

$$\mathcal{O}(t) = (\mathcal{C}\Gamma)_{AB}\left(\bar{Q}_C\left(-\frac{r}{2}\right)q_A^{(\alpha)}\left(-\frac{r}{2}\right)\right)\left(\bar{Q}_D\left(+\frac{r}{2}\right)q_B^{(\beta)}\left(+\frac{r}{2}\right)\right)$$

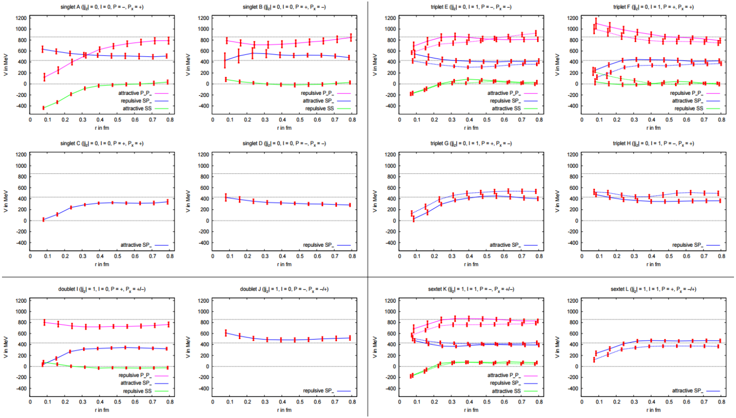
- Γ: arbitrary combinations of γ matrices (account for spin and parity)

Lattice setup

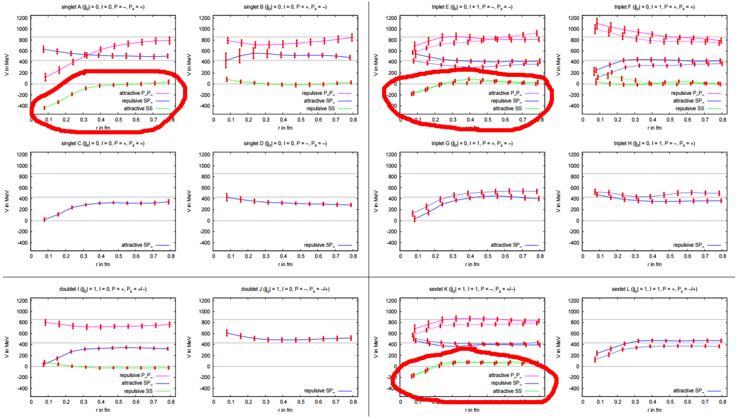
- 2 ensembles generated by ETMC
- $n_f = 2$
- quark action: Wilson twisted mass at maximal twist
- gluon action: tree-level Symanzik improved

β	size	μ_l	a in fm	m_π in MeV	number of configurations
3.90	$24^3 \times 48$	0.00400	0.079	340	480
4.35	$32^3 \times 64$	0.00175	0.042	352	100

Many different quantum numbers, many different potentials...



Many different quantum numbers, many different potentials...



Attractive channels I

- at large separations, we find a pair B mesons in spacially symmetric s-wave $\longrightarrow$ total system has $P = +$
- interaction of heavy quarks due to 1-gluon exchange; in case of attractive potential quarks in an antisymmetric color triplet
- since flavour is symmetric and color is antisymmetric, heavy quark spin must be symmetric, too, i.e. $j_{heavy} = 1$

$qq = ud$

- scalar isosinglet, i.e. $j_{light} = 0$

$$qq = \frac{ud - du}{\sqrt{2}}$$

$$\Longrightarrow I(J^P) = 0(1^+)$$

- vector isotriplet, i.e. $j_{light} = 1$

$$qq \in \left\{uu, \frac{ud + du}{\sqrt{2}}, dd\right\}$$

$$\Longrightarrow I(J^P) \in \{1(0^+), 1(1^+), 1(2^+)\}$$

Attractive channels II

$qq = ss, cc$

- consider vector isotriplet

$$qq \in \{ss, cc\}$$

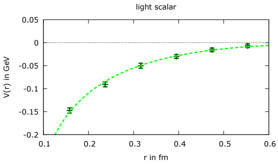
- in case of scalar isosinglet consider two hypothetical degenerate flavours $s^{(1)}, s^{(2)}$ and $c^{(1)}, c^{(2)}$

$$qq = \frac{s^{(1)}s^{(2)} - s^{(2)}s^{(1)}}{\sqrt{2}} \quad \text{and} \quad qq = \frac{c^{(1)}c^{(2)} - c^{(2)}c^{(1)}}{\sqrt{2}}$$

$\Longrightarrow I(J^P) \in \{0(0^+), 0(1^+), 0(2^+)\}$

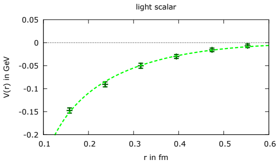
fit an ansatz to the potentials:

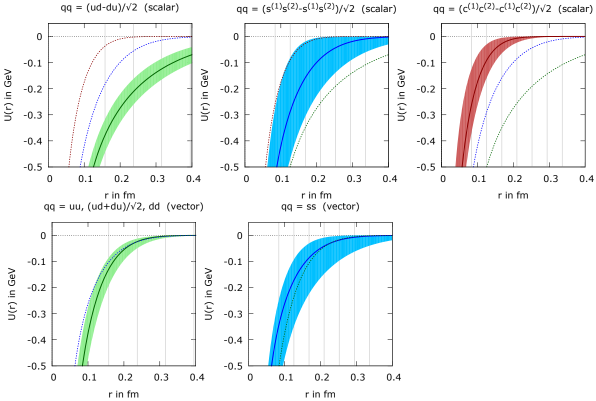
$$V(r) = \underbrace{-\frac{\alpha}{r}}_{Coulomb-like} \times \overbrace{e^{-\left(\frac{r}{d}\right)^2}}^{colour\ screening}$$



fit an ansatz to the potentials:

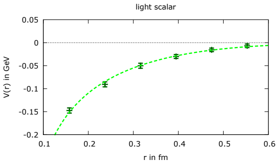
$$V(r) = \underbrace{-\frac{\alpha}{r}}_{Coulomb-like} \times \overbrace{e^{-\left(\frac{r}{d}\right)^2}}^{colour\ screening}$$

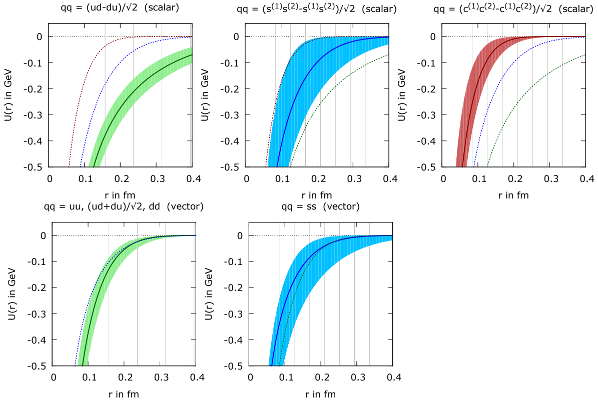




fit an ansatz to the potentials:

$$V(r) = \underbrace{-\frac{\alpha}{r}}_{Coulomb-like} \times \overbrace{e^{-\left(\frac{r}{d}\right)^2}}^{colour\ screening}$$





Potentials show more promise for binding the less massive the light quarks are!

Solve Schrödinger's equation

- notice: Born-Oppenheimer approximation is valid for $m_q \ll m_Q$
- solve Schrödinger's equation:

$$\left(-\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V(r)\right) R(r) = E_B R(r) \quad , \qquad \psi(r) = R(r)/r$$

- lowest eigenvalue $E_B < 0$ binding (4-quark bound state), $E_B > 0$ no binding (2 mesons)

Systematic investigation

For scalar isosinglet and vector isotriplet and for $q = u, d, s, c$:

- vary fit ranges $[t_{min}, t_{max}]$ for effective mass plateaus
- vary fit range $[r_{min}, r_{max}]$ for $V(r)$

Systematic investigation

For scalar isosinglet and vector isotriplet and for $q = u, d, s, c$:

- vary fit ranges $[t_{min}, t_{max}]$ for effective mass plateaus
- vary fit range $[r_{min}, r_{max}]$ for $V(r)$
- perform a fit $V(r) = -\frac{\alpha}{r} \mathrm{e}^{-\left(\frac{r}{d}\right)^2}$ for every set of input parameters
- determine binding energy E_B for every set of input parameters

Systematic investigation

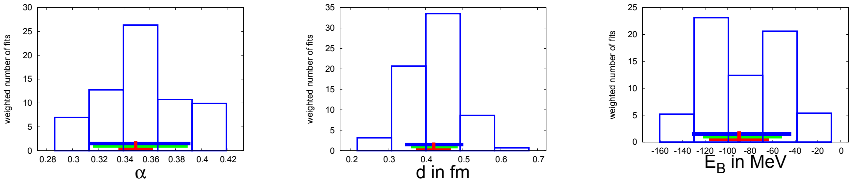
For scalar isosinglet and vector isotriplet and for $q = u, d, s, c$:

- vary fit ranges $[t_{min}, t_{max}]$ for effective mass plateaus
- vary fit range $[r_{min}, r_{max}]$ for $V(r)$
- perform a fit $V(r) = -\frac{\alpha}{r} \mathrm{e}^{-\left(\frac{r}{d}\right)^2}$ for every set of input parameters
- determine binding energy E_B for every set of input parameters
- weight α, d and E_B according to the respective χ^2/dof of the fit $\longrightarrow$ determine one histogram for α, d and E_B
- determine systematic errors on α, d and binding energy E_B according to the width of the histograms

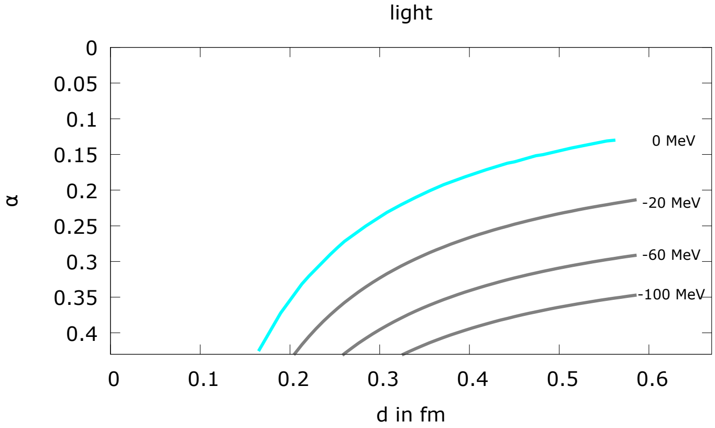
Systematic investigation

For scalar isosinglet and vector isotriplet and for $q = u, d, s, c$:

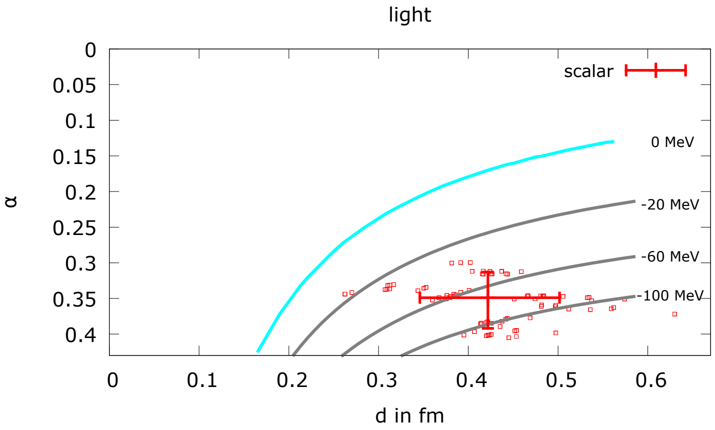
- vary fit ranges $[t_{min}, t_{max}]$ for effective mass plateaus
- vary fit range $[r_{min}, r_{max}]$ for $V(r)$
- perform a fit $V(r) = -\frac{\alpha}{r} e^{-\left(\frac{r}{d}\right)^2}$ for every set of input parameters
- determine binding energy E_B for every set of input parameters
- weight α, d and E_B according to the respective χ^2/dof of the fit $\longrightarrow$ determine one histogram for α, d and E_B
- determine systematic errors on α, d and binding energy E_B according to the width of the histograms



Visualize errors on the fit parameters in the $\alpha - d$ plane:

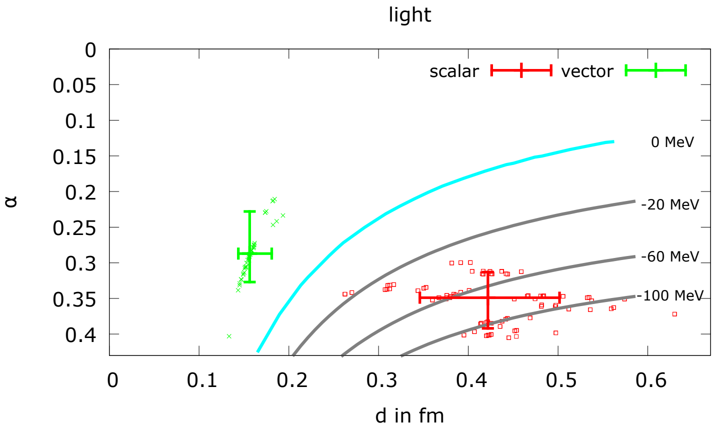


Visualize errors on the fit parameters in the $\alpha - d$ plane:



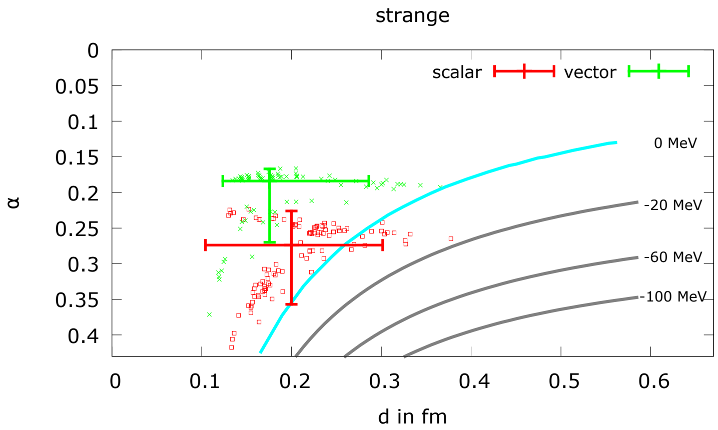
scalar light: bound state!!!

Visualize errors on the fit parameters in the $\alpha - d$ plane:

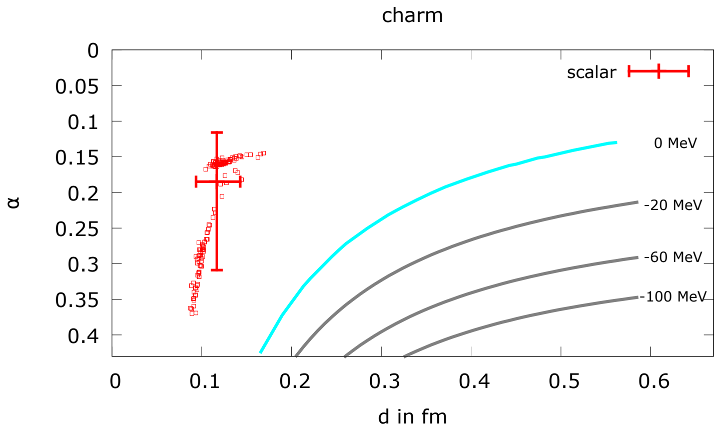


scalar light: bound state!!!

Visualize errors on the fit parameters in the $\alpha - d$ plane:



Visualize errors on the fit parameters in the $\alpha - d$ plane:



Summary

- for scalar light channel $qq = ud$ with quantum numbers $I(J^P) = 0(1^+)$ we find $E_B = -93^{+47}_{-43} \longrightarrow$ binding with 2σ confidence level!
- $qq = ss, cc$: no binding
- scalar channels in general more attractive than respective vector channels

Outlook

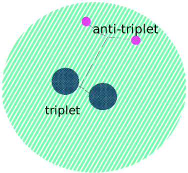
- Computations done for a pion mass of 340 MeV: Repeat analysis for another pion mass and extrapolate to physical point.
  - Binding in vector light channel?
- investigate structure: 2-meson or diquark-antidiquark?
- investigate $Q\overline{Q}$ systems

Appendix

The tetraquark system - Expectations I

At small separations of the static antiquarks:

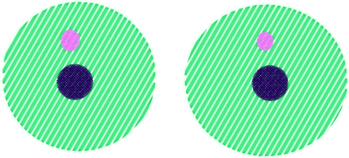
- interaction between quarks due to 1-gluon exchange
- in case of a bound state: static antiquark-antiquark pair is in a color triplet (attractive) $\longrightarrow$ formation of an antidiquark
- tetraquark system must be a color singlet, so light quarks necessarily form a color antitriplet (antisymmetric)
- total two-quark wave function must be antisymmetric (Pauli principle): implies symmetric structure in light flavour and spin
- bound states in (spin) scalar isosinglet (i.e. spin singlet with antisymmetric flavour combination) and (spin) vector isotriplet (spin triplet with symmetric flavour combination) are expected



The tetraquark system - Expectations II

At large separations of the static antiquarks:

- screening of the antiquark-antiquark interaction due to light quarks
- stronger screening in case of more massive light quarks (wave function more compact)
- essentially no overlap between the wave functions of the light quarks
- basically two static-light mesons
- if light quarks are sufficiently heavy, screening should prevent formation of a bound state



Systematic investigation I

For scalar isosinglet and vector isotriplet and for $q = u, d, s, c$:

- vary fit ranges $[t_{min}, t_{max}]$ for effective mass
  - $t_{max} - t_{min} \geq 1$
  - do not include too small t_{min}: contamination by excited states
  - large t_{min} and t_{max} increase statistical errors
- vary fit range $[r_{min}, r_{max}]$ for $V(r)$
  - do not include lattice results for $r_{min} < 2a$: sizeable discretization errors
  - fits with $r_{min} = 2$ or $r_{min} = 3$: large difference $\longrightarrow$ spreading of results
  - choice of r_{max}: basically no effect on fit