Long-Distance Contributions to the Rare Kaon Decay $K^+ \to \pi^+ \ell^+ \ell^-$

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LATTICE 2015

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Outline

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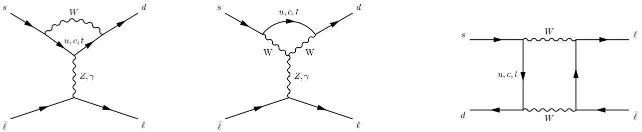
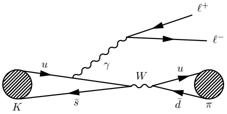
2. Lattice Methodology (arXiv:1507.03094)
   - Operators
   - Wick Contractions
   - Renormalisation
3. Analysis
   - Correlators
   - Integration
   - Divergence Subtraction
4. Results

Phenomenology



Example short distance contributions to the transition $s \to d\ell\bar{\ell}$.

- The processes $K \to \pi\ell\bar{\ell}$ proceed via a flavour changing neutral current (FCNC). FCNCs are forbidden at tree level in the SM.
- Ideal probe for New Physics!
- In $K^+ \to \pi^+\ell^+\ell^-$ long distance effects dominant $\Rightarrow$ lattice QCD.
- Single photon exchange provides sizeable long-distance contribution.



Example long-distance contribution to $K \to \pi\ell^+\ell^-$ (QCD effects not shown).

$K \to \pi \ell^+ \ell^-$

- We compute the amplitude of $K \to \pi\gamma^*$,

$$\mathscr{A}_\mu\left(q^2\right) = \int d^4x \left\langle \pi(p) \right| T\left[J_\mu(0) H_W(x)\right] \left| K(k) \right\rangle$$

- The effective Weak Hamiltonian for a $|\Delta S| = 1$ transition is:

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left(\sum_{i=1}^{2} C_i \left(Q_i^u - Q_i^c\right) + \sum_{j=3}^{8} C_j Q_j + \mathscr{O}\left(\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right) \right).$$

- The Wilson coefficients C_1 and C_2 are substantially larger than the others, hence we require only

$$Q_1^q = \left(\bar{s}_i \gamma_\mu^L d_i\right)\left(\bar{q}_j \gamma_\mu^L q_j\right), \qquad Q_2^q = \left(\bar{s}_i \gamma_\mu^L q_i\right)\left(\bar{q}_j \gamma_\mu^L d_j\right).$$

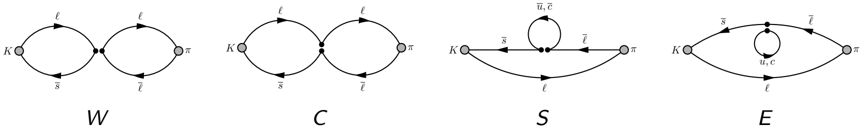
- We identify the EM current as the conserved lattice vector current,

$$J_\mu = \frac{1}{3}\left(2V_\mu^u - V_\mu^d - V_\mu^s + 2V_\mu^c\right).$$

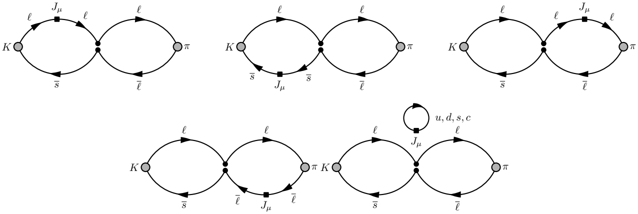
[1] G. Isidori et al., Phys. Lett. B633 (2005) 75-83. arxiv:hep-lat/0506026

Wick Contractions

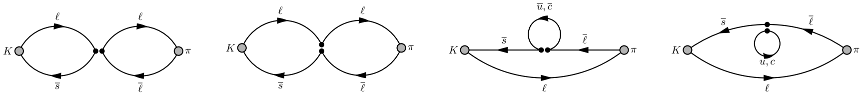
- Performing the Wick contractions for just the H_W diagrams, we obtain 4 different classes of diagrams:



- Adding in the current, we obtain 5 diagrams per H_W class:



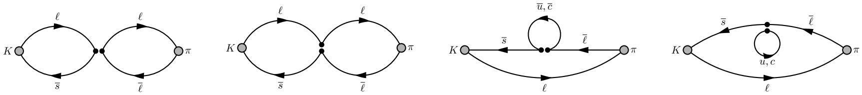
Renormalisation



- J_μ is a conserved current - no renormalisation is required.
- For H_W we renormalise the bare operators Q_1, Q_2 non-perturbatively, e.g. using RI-SMOM.
- The Wilson coefficients are known at NLO in the $\overline{\text{MS}}$ scheme; we can use continuum perturbation theory to match our renormalised operators to this scheme.

[2] RBC-UKQCD Collaboration, Phys. Rev. D88 (2013) 014508. arxiv:hep-lat/1212.5931

Renormalisation



- Divergences may arise from the contact of H_W and J_μ.
- When H_W approaches J_μ the S & E diagram classes are superficially quadratically divergent (resembles HVP).

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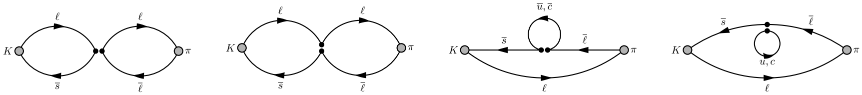

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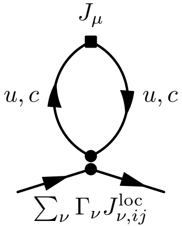

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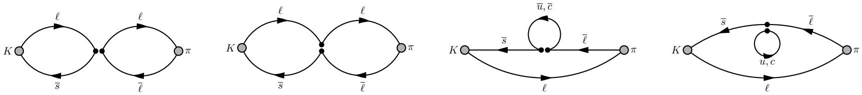

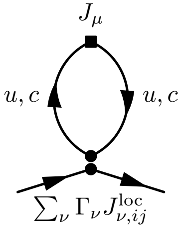

- Divergences may arise from the contact of H_W and J_μ.
- When H_W approaches J_μ the S & E diagram classes are superficially quadratically divergent (resembles HVP).
- Electromagnetic gauge invariance implies that this divergence can at most be logarithmic.
- The logarithmic divergence is cancelled by the GIM mechanism.

Lattice Correlators

- We measure the 4pt correlator:

$$\Gamma_\mu^{(4)}(t_H, t_J, \mathbf{k}, \mathbf{p}) = \sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{q}\cdot\mathbf{x}} \left\langle \mathscr{O}(t_\pi, \mathbf{p}) \right| T\left[J_\mu(t_J, \mathbf{x}) H_W(t_H, \mathbf{y}) \right] \left| \mathscr{O}_K(0, \mathbf{k}) \right\rangle .$$

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- We extract the amplitude from the integrated correlator in the limit $T_A, T_B \to \infty$:

$$I_\mu(T_a, T_b, \mathbf{k}, \mathbf{p}) = e^{-(E_\pi(\mathbf{p}) - E_K(\mathbf{k}))t_J} \int_{t_J - T_a}^{t_J + T_b} dt_H \tilde{\Gamma}_\mu^{(4)}(t_H, t_J, \mathbf{k}, \mathbf{p}),$$

where $$\tilde{\Gamma}_\mu^{(4)} = \frac{\Gamma_\mu^{(4)}}{Z_{\pi K}}, Z_{\pi K} = \frac{Z_\pi Z_K^\dagger}{4E_\pi(\mathbf{p}) E_K(\mathbf{k})} e^{-t_\pi E_\pi(\mathbf{p})}.$$

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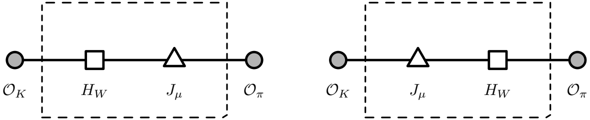

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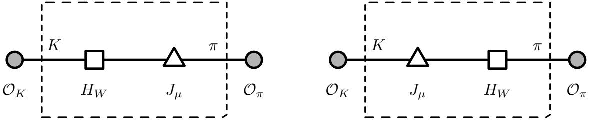

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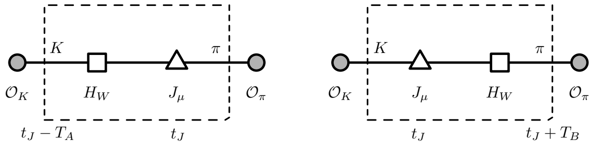

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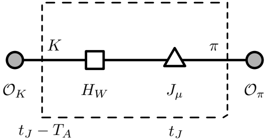

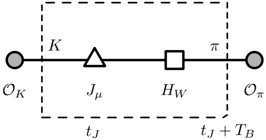

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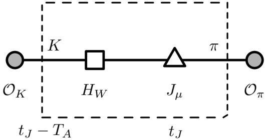

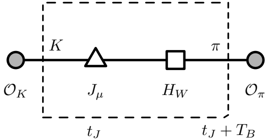

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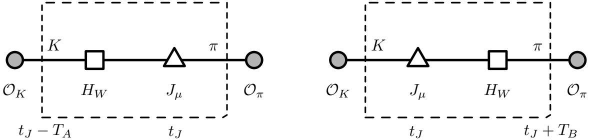
Integrated Correlator



- The spectral representation of the integrated correlator is given by:

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\begin{aligned}
I_\mu\left(T_a, T_b, \mathbf{k}, \mathbf{p}\right) \quad = \quad & -\sum_n \frac{1}{2E_n} \frac{\left\langle \pi\left(\mathbf{p}\right)\middle| J_\mu \middle| n^{s=\mathbf{0}}, \mathbf{k}\right\rangle \left\langle n^{s=\mathbf{0}}, \mathbf{k}\middle| H_W \middle| K\left(\mathbf{k}\right)\right\rangle}{E_K\left(\mathbf{k}\right) - E_n} \left(1 - e^{\left(E_K(\mathbf{k}) - E_n\right) T_a}\right) \\
& + \sum_m \frac{1}{2E_m} \frac{\left\langle \pi\left(\mathbf{p}\right)\middle| H_W \middle| m^{s=\mathbf{1}}, \mathbf{p}\right\rangle \left\langle m^{s=\mathbf{1}}, \mathbf{p}\middle| J_\mu \middle| K\left(\mathbf{k}\right)\right\rangle}{E_m - E_\pi\left(\mathbf{p}\right)} \left(1 - e^{-\left(E_m - E_\pi(\mathbf{p})\right) T_b}\right)
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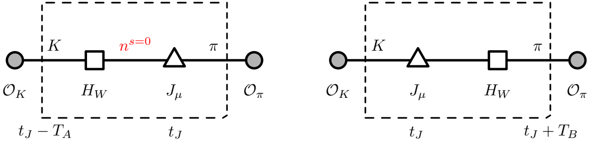

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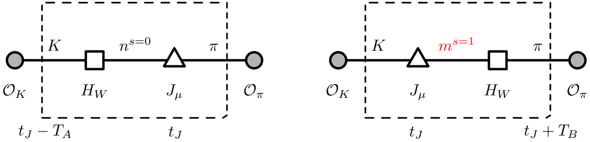

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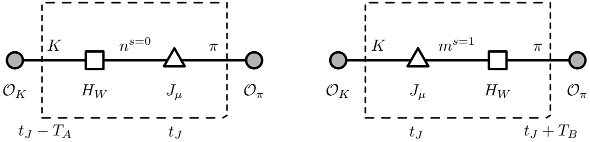

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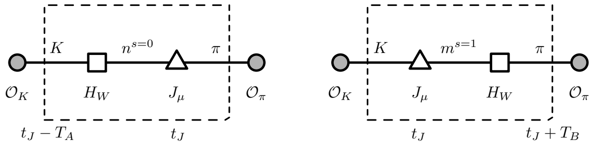

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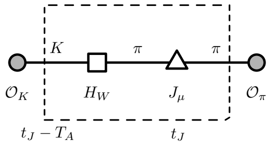
- When $E_n < E_K\left(\mathbf{k}\right)$, the exponential in T_a diverges. For physical masses this is true for $n = \pi$, $n = \pi\pi$ or $n = \pi\pi\pi$.

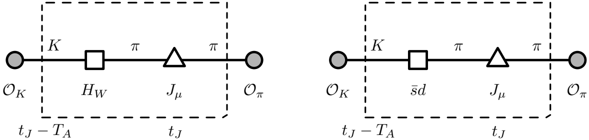
Removal of Single Pion Divergence



- For our exploratory studies with $m_\pi = 421$ MeV only the the π state is divergent:

$$D_\mu\left(T_a, \mathbf{k}, \mathbf{p}\right) = -\frac{\langle \pi\left(\mathbf{p}\right)|J_\mu|\pi\left(\mathbf{k}\right)\rangle \langle \pi\left(\mathbf{k}\right)|H_W|K\left(\mathbf{k}\right)\rangle}{2E_\pi\left(\mathbf{k}\right)\left(E_K\left(\mathbf{k}\right) - E_\pi\left(\mathbf{k}\right)\right)} e^{\left(E_K(\mathbf{k}) - E_\pi(\mathbf{k})\right)T_a}$$

Removal of Single Pion Divergence



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- We remove it by performing the scalar shift

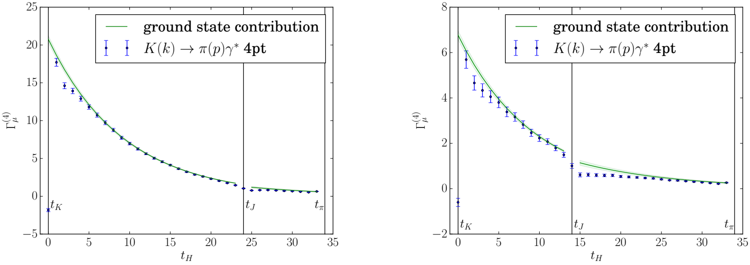
$$H_W \to H'_W = H_W + c_s \bar{s}d,$$

  This shift is unphysical owing to the chiral Ward Identity $i(m_s - m_d)\bar{s}d = \partial_\mu V^\mu_{\bar{s}d}$.
- We tune the parameter c_s to achieve $\langle \pi(\mathbf{k}) | H'_W | K(\mathbf{k}) \rangle = 0$.

[3] RBC-UKQCD Collaboration, Phys. Rev. Lett. 133 (2014) 112003. arxiv:hep-lat/1406.0916

4pt Correlator

- We simulated the decay $K(k) \to \pi(p) + \gamma^*(q)$ on a $24^3 \times 64$ lattice using Domain Wall Fermions with Iwasaki gauge action, and an inverse lattice spacing of $a^{-1} \simeq 1.73$ GeV, $m_\pi \simeq 421$ MeV, $m_K \simeq 600$ MeV, using 256 configurations.
- $t_K = 0$, $t_\pi = 34$, two separate current insertions at $t_J = 24$ and $t_J = 14$, $\mathbf{k} = (0,0,0)$, $\mathbf{p} = (1,0,0)$.



- For this calculation we consider only the C and W classes of diagrams (i.e. no loops).

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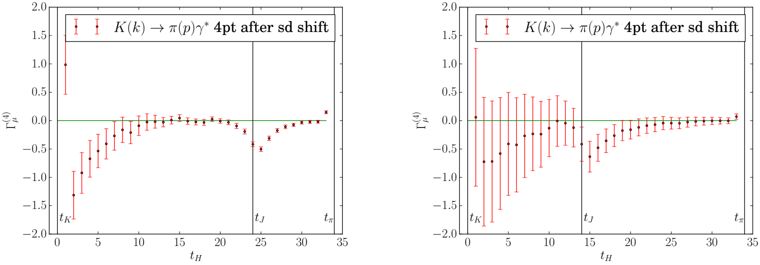

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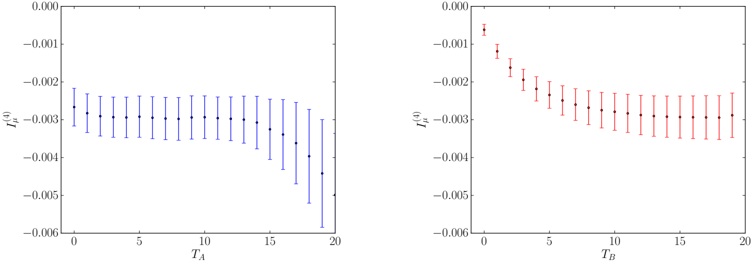
Integrated Correlator

- We integrate both sides of the integral independently and combine the two to obtain the matrix element.



$I_\mu = \int_{t_J - T_A}^{t_J+14} dt_H \left(\tilde{\Gamma}_\mu^{H_W} - c_s \tilde{\Gamma}_\mu^{\bar{s}d} \right)$ $\qquad$ $I_\mu = \int_{t_J - 7}^{t_J+T_B} dt_H \left(\tilde{\Gamma}_\mu^{H_W} - c_s \tilde{\Gamma}_\mu^{\bar{s}d} \right)$

- In lattice units we obtain $\mathscr{M}_\mu = -0.0030(6)$.

Summary

- We have developed theoretical techniques for calculating the long distance contribution for $K^+ \to \pi^+ \ell^+ \ell^-$.
- Our exploratory calculations show that we can successfully extract the matrix element in practice.

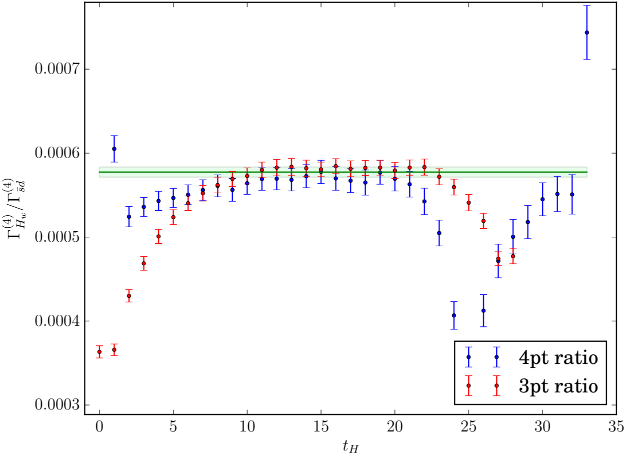
- Outlook
  - Short term:
    - Include S & E classes of diagrams.
    - More kinematics.
  - Long term:
    - Lighter quark masses.
    - Infinite volume & continuum limits.
  - Longer term:
    - Disconnected diagrams.

Thank you!

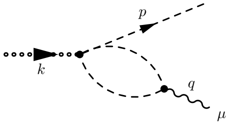
Backup Slides

Backup Slides

c_s determination



Chiral Perturbation Theory



- Form factor for the decay:

$$\mathscr{A}_\mu\left(q^2\right) = \frac{V(z)}{(4\pi^2)}\left[q^2\left(k+p\right)_\mu - \left(m_K^2 - m_\pi^2\right)q_\mu\right],\ z = \frac{q^2}{m_K^2}.$$

- Form factor has been parametrised in chiral perturbation theory (ChPT):

$$V(z) = a + bz + V_{\pi\pi}(z).$$

- Opportunity for lattice QCD to check ChPT result from first principles.

[4] C. Dib et al., Phys. Rev. D39 (1989) 2639.
[5] G. D'Ambrosio et al., JHEP 9808 (1998) 004. arxiv:hep-ph/9808289