Non-Perturbative Gauge-Higgs Unification in Five Dimensions

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Outline

1. Gauge-Higgs Unification
2. Lattice Formulation
3. Phase Diagram
4. Dimensional Reduction
5. Spectrum

Based on `arXiv:1506.06035`

Motivation

The Standard Model Higgs

"A light Higgs has a snowballs chance in hell"

- Quadratic sensitivity to UV cut-off
- Origin of potential responsible for spontaneous symmetry breaking?

Can we solve these problems?

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An Extra-Dimensional Approach: Gauge-Higgs Unification

- Higgs field is associated with the extra-dimensional components of the gauge field
- Higgs potential is generated through quantum effects

Gauge-Higgs Unification

Perturbative Regime

- Higgs potential can break gauge symmetry via the Hosotani mechanism [Hosotani, 1983]
- Higgs mass and potential are cut-off independent at one-loop [von Gersdorff, Irges and Quiros, 2002]
- Requires the presence of fermions $\Rightarrow$ no pure gauge SSB

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Non-Perturbative Regime

- Pure gauge theory shown to exhibit SSB on a space-time lattice with an orbifold geometry [Irges, Knechtli, 2007]
- Orbifold projection: $SU(2) \to U(1)$
- SSB is due to global stick symmetry [Ishiyama *et al.*, 2010]

Lattice Formulation

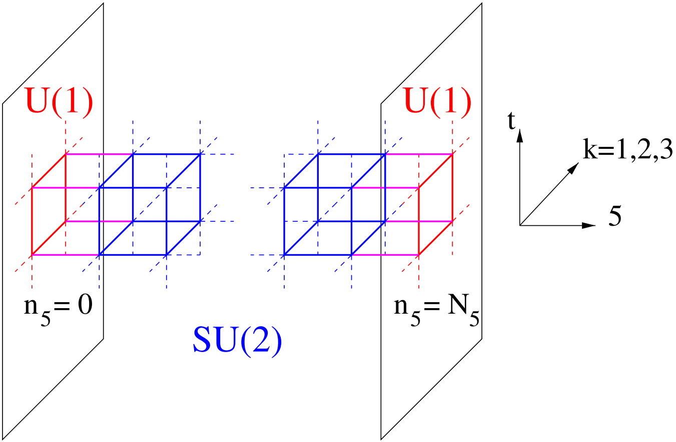
Anisotropic Wilson Action on a 5-d Euclidean orbifold

$$S_W^{orb} = \frac{\beta}{2} \sum_x \left[\frac{w}{\gamma} \sum_{\mu<\nu} \mathrm{tr}\left\{1 - U_{\mu\nu}(x)\right\} + \gamma \sum_{\mu} \mathrm{tr}\left\{1 - U_{\mu 5}(x)\right\} \right]$$

$$\mathrm{w} = \begin{cases} \frac{1}{2} & \text{plaquette on boundary} \\ 1 & \text{otherwise} \end{cases}$$

- The gauge couplings in the four and fifth dimensions are related via the anisotropy parameter $\gamma = \sqrt{\beta_5/\beta_4}$
- Lattice volume given by: $T \times L^3 \times N_5$ where N_5 is the number of links in the fifth dimension
- Three types of links: bulk, boundary and hybrid

Lattice Formulation

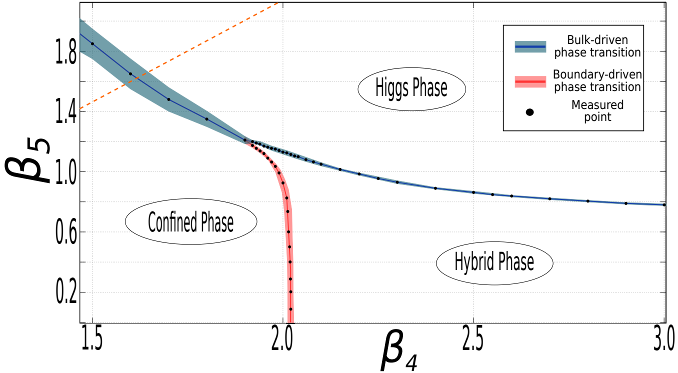


Boundary: $\Omega^{U(1)} U \Omega^{\dagger U(1)}$

Bulk: $\Omega^{SU(2)} U \Omega^{\dagger SU(2)}$

Hybrid: $\Omega^{U(1)} U \Omega^{\dagger SU(2)}$

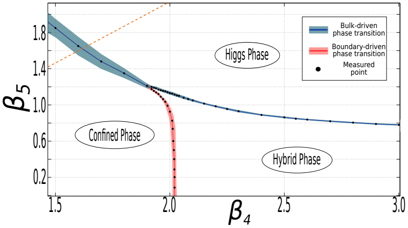
Phase Diagram



- Lattice volume: $32^4 \times 4$
- Volume independent once $L \geq 24$ and $N_5 \gtrsim 4$

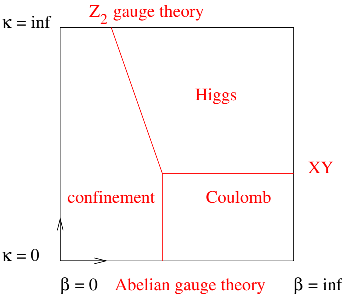
Phase Diagram

5-d GHU on orbifold



4-d Abelian Higgs model

[Fradkin and Shenker, 1979]



- Expected reduction to 4-d Abelian Higgs model

Dimensional Reduction

Use Wilson loops to extract the potential V(r) between a pair of static charges along layers orthogonal to the extra dimension

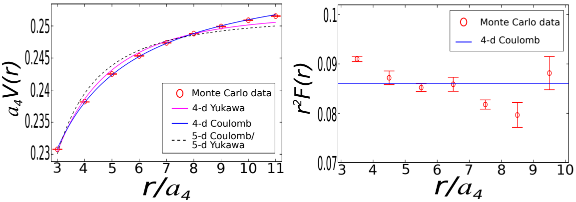
Global Fits to Potential

Dimension	Yukawa	Coulomb	Confining
4	$c_0 - c_1 \frac{e^{-m_Z r}}{r}$	$c_0 - \frac{c_1}{r}$	$c_0 + \sigma r - \frac{c_1}{r}$
5	$c_0 - c_1 \frac{K_1(m_Z r)}{r}$	$c_0 - \frac{c_1}{r^2}$	$c_0 + \sigma r - \frac{c_1}{r}$

- Use the static force $F(r) = V'(r)$ as a shape parameter to determine any small deviations
- $r^2 F(r)$ is renormalized and dimensionless in 4-d

Dimensional Reduction - Hybrid Phase

Boundary



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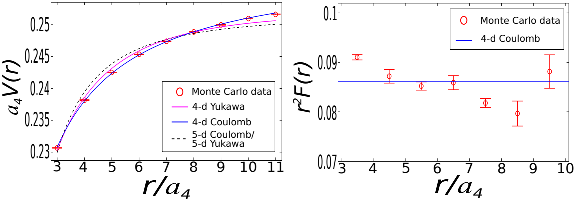
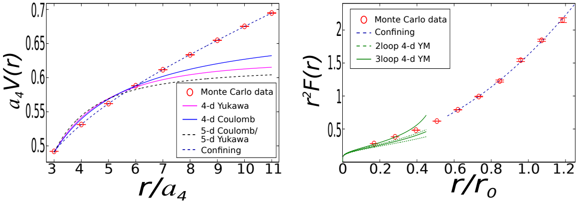


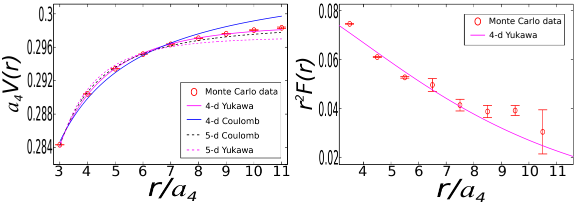
Bulk



- Dimensional reduction into 4-d layers ([Fu and Nielson, 1984])

Dimensional Reduction - Higgs Phase

Boundary



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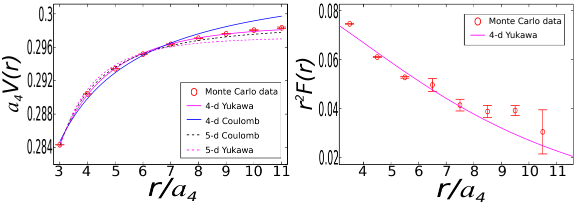

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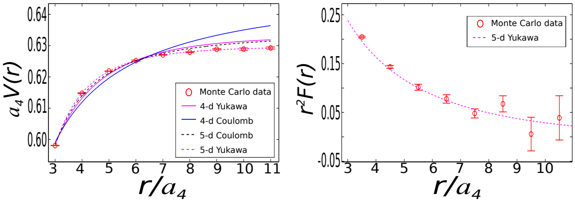

- Dimensional reduction via localisation (near the P.T.)

Spectrum

Spectroscopic Extraction

Solve a GEVP for a matrix of 2-pt correlation functions

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

- Eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$
- Eigenvectors: $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$
- Construct a set of scalar and vector operators
- Enlarge basis via use of smearing

A Standard Model-Like Spectrum

- Standard Model: $\rho = M_H / M_Z \approx 1.38$
- Always find $M_Z > M_H$; $\rho \approx 0.65$ when $\beta_4 = \beta_5$

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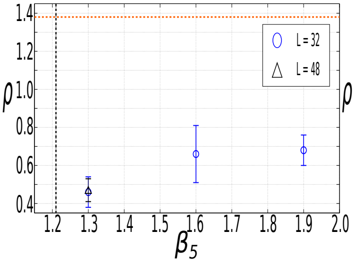
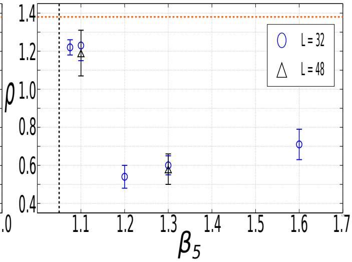
- Always find $M_Z > M_H$; $\rho \approx 0.65$ when $\beta_4 = \beta_5$
- Move towards the region where we find dimensional reduction: $\beta_4 > \beta_5$ close to the phase transition

$\beta_4 = 1.9$ (Higgs-Confined)

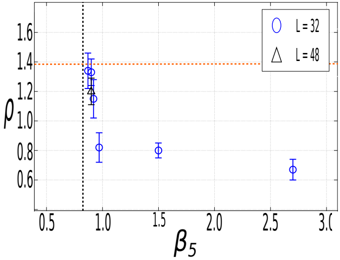


$\beta_4 = 2.1$ (Higgs-Hybrid)

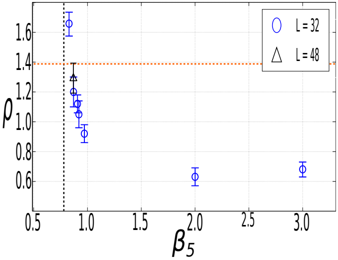


A Standard Model-Like Spectrum

$\beta_4 = 2.7$ (Higgs-Hybrid)



$\beta_4 = 3.0$ (Higgs-Hybrid)



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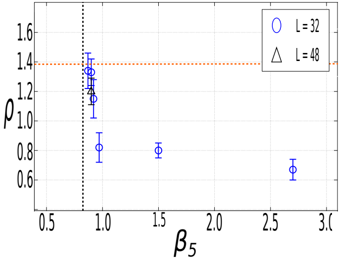

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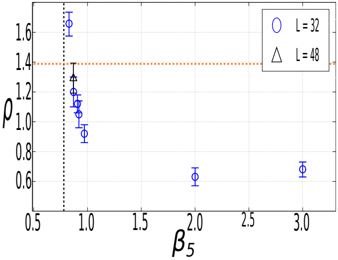
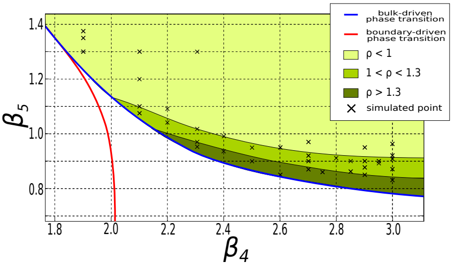
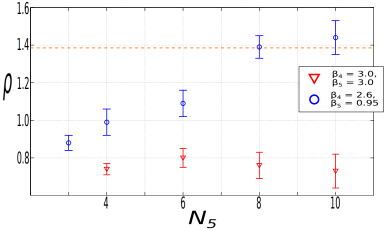


$N_5 = 4$



Increasing the Size of the Extra Dimension



- ρ remains constant when $\beta_4 = \beta_5$
- M_Z falls faster than M_H in Standard Model-like region

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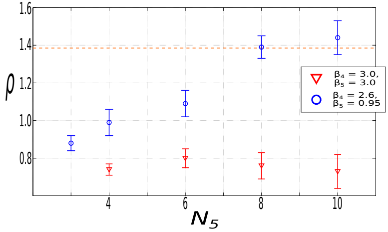
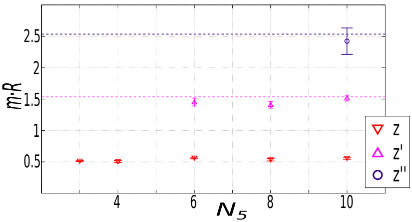


$\beta_4 = 2.6\ ,\ \beta_5 = 0.95$



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K-K behaviour in Z channel:

- $M_i \sim 1/R$
- $M_i^{KK} = (i + \text{const})/R$

Hint for a $H' \sim 1/R$ above ground state

Conclusions and Outlook

Summary

Concentrated on the region where $\gamma \leq 1$ and find:

- Confined, Higgs and Hybrid phases
- SSB in Higgs phase due to breaking of global symmetry
- Close to the Higgs/Hybrid phase transition:
  - A Standard Model-like hierarchy of Higgs and Z masses
  - Dimensional reduction via localisation on the boundaries

Outlook

- Construct lines of constant physics (effective theory)
- Study relationship with 4-d Abelian Higgs model
- Extend study to where we expect compactification ($\gamma > 1$)

What Causes Spontaneous Symmetry Breaking?

The 'Stick Symmetry' [Ishiyama *et al.*, 2010]

Global ($\mathbb{Z}_4$) symmetry $\mathcal{S}_L$

$$U(n_5 = 0, 5) \rightarrow g_s^{-1}\, U(n_5 = 0, 5)$$
$$U(n_5 = 0, \mu) \rightarrow g_s^{-1}\, U(n_5 = 0, \mu)\, g_s$$

where $g_s \in W_{SU(2)}(U(1))$ and $\{g_{orb}, g_s\} = 0$

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where $g_s \in W_{SU(2)}(U(1))$ and $\{g_{orb}, g_s\} = 0$

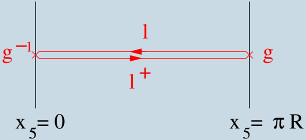
- $\mathcal{S}_L^2$ is a centre transf. ($g_s^2 = -I$); operators along fifth dimension are invariant $\Rightarrow$ not finite T (Debye) masses
- $\mathbb{Z}_4/\mathbb{Z}_2^c \simeq \mathbb{Z}_2$ is non-trivial $\Rightarrow$ responsible for SSB
- Appears to be no continuum stick symmetry $\Rightarrow$ SSB is a property of an effective theory at finite cut-off

Operators

Higgs Operators

Higgs d.o.f come from **Polyakov loops** in extra dimension

- $p = l\ g\ l^\dagger\ g^\dagger$
- $h = [p - p^\dagger, g]/(4N_5)$



Basis: $\mathcal{H} = \mathrm{tr}\,[hh^\dagger]$; $\mathcal{P} = \mathrm{tr}\,[p]$ (increase via smearing)

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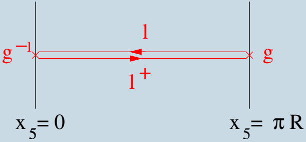

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Z Boson Operators

Z boson d.o.f come from **vector** Polyakov loops

- $\mathcal{Z} = \mathrm{tr}\,[g \; \alpha \; U^\dagger \; \alpha]$
- $\mathcal{Z}' = \mathrm{tr}\,[g \; U \; l \; U^\dagger \; l^\dagger]$
- Order parameters for SSB