$SU(4)$ symmetry of hadrons upon quasi-zero Dirac mode elimination

L. Ya. Glozman

Institut für Physik, FB Theoretische Physik, Universität Graz

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Low mode truncation

Banks-Casher:

$$< \bar{q}q >= -\pi\rho(0).$$

What we do:

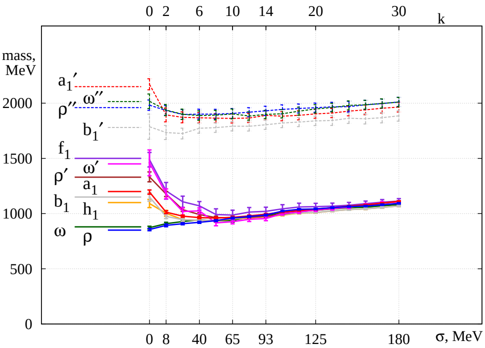
$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$

What one expects:

$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^k)\Psi$		$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^k)\Psi$	
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^0\gamma^k)\Psi$	$\overset{SU(2)_A}{\longleftrightarrow}$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^0\gamma^k)\Psi$	
$U(1)_A$ ↕				↕ $U(1)_A$
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^0\gamma^k)\Psi$	$\overset{SU(2)_A}{\longleftrightarrow}$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^0\gamma^k)\Psi$	
$(1,0)\oplus(0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^k)\Psi$	$\overset{SU(2)_A}{\longleftrightarrow}$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^k)\Psi$	

Lattice results

What one obtains given the JLQCD $N_f = 2$ gauge configurations with the overlap Dirac operator (M.Denissenya, L.Ya.G., C.B. Lang, PRD 89 (2014) 077502; 91 (2015) 034505)



We clearly see a larger degeneracy than what would be expected from the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian. What does it mean !?

Chiralspin rotations

Given the observed degeneracy pattern we reconstruct a symmetry group.
Starting point: different chiral representations contain distinct R and L combinations.
Consider rotations in an imaginary 3-dim space of doublets constructed from the Weyl spinors

$$\mathrm{U} = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \qquad \mathrm{D} = \begin{pmatrix} d_L \\ d_R \end{pmatrix}$$

$$\mathrm{U} \to \mathrm{U}' = e^{i\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}}{2}}\mathrm{U}\,, \qquad \mathrm{D} \to \mathrm{D}' = e^{i\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}}{2}}\mathrm{D}\,,$$

where $\boldsymbol{\sigma}$ are the standard Pauli matrices which obey the $\mathfrak{su}(2)$ algebra:

$$[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\,\sigma^k\,.$$

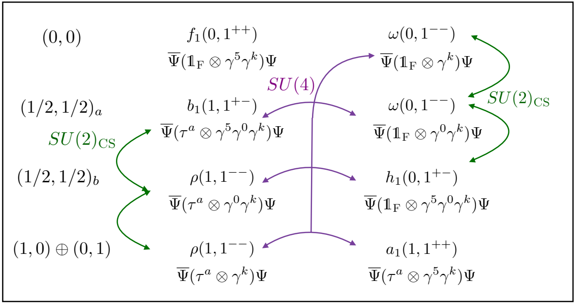
We refer to this imaginary three-dimensional space as the chiralspin space.

Instead of the Weyl spinors we can consider the left- and right-handed Dirac bispinors. Then the chiralspin rotations are generated through

$$\boldsymbol{\Sigma} = \{\gamma^0, i\gamma^5\gamma^0, -\gamma^5\}\,, \qquad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk}\,\Sigma^k\,.$$

We denote this symmetry group as $SU(2)_{\mathrm{CS}}$.

Extention to $SU(4)$



The group that contains at the same time $SU(2)_L \times SU(2)_R \times U(1)_A$ and $SU(2)_{CS}$ is $SU(4)$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\rm L} \\ u_{\rm R} \\ d_{\rm L} \\ d_{\rm R} \end{pmatrix}$$

and the following set of generators: $\{(\tau^a \otimes 1\!\!1_D), (1\!\!1_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}$

J=2 mesons and baryons

Observed degeneracy of $J = 1$ mesons implies the $SU(2)_{CS}$ and $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$ symmetries. The same symmetries are seen also in J=2 mesons and baryons (M. Denissenya, L.Ya.G., M. Pak, PRD 91 (2015) 114512; in preparation).

We conclude that these symmetries are symmetries of hadrons and QCD upon the quasi-zero-modes elimination.

We decouple by hands the valence quarks from the quark condensate of the vacuum (from the chiral symmetry breaking dynamics) and observe suddenly a new symmetry that is higher than the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian.

What does this symmetry mean and where is it hidden ?!

$SU(2)_{CS}$ and $SU(4)$ are hidden in the QCD Lagrangian

One can prove from the degeneracy pattern that there are no quark-quark interactions in the system mediated by the color-magnetic field. Only the color-electric interaction is there. Then it is natural to assume that these symmetries are symmetries of confinement in QCD and what we observe is a dynamical QCD string.

This assumption can actually be proven:

We apply a $SU(2)_{\mathrm{CS}}$ and $SU(4)$ transformation on the interaction part of the QCD Lagrangian,

$$\overline{\Psi}'\gamma^{\mu}D_{\mu}\Psi' = \overline{\Psi}\gamma^0 D_0\Psi - \overline{\Psi}\gamma^0 V^{\dagger}\gamma^0\boldsymbol{\gamma}\cdot\mathbf{D}V\Psi\ .$$

The γ^0-part is invariant under these transformations. It encodes the electric (Coulombic) charge-charge interactions in QCD.

The spatial part, $\sim \mathbf{j}\cdot\mathbf{A}$, is not invariant. It contains the magnetic interactions.

The interaction parts of the QCD Hamiltonian in Coulomb gauge

The charge-charge color-Coulombic Hamiltonian:

$$H_C = \frac{g^2}{2} \int d^3x\, d^3y\, J^{-1}\, \rho^a(\mathbf{x}) F^{ab}(\mathbf{x}, \mathbf{y})\, J\, \rho^b(\mathbf{y}) \ ,$$

is a $SU(2)_{CS}$- and $SU(4)$- singlet. It is a confining part of the QCD Hamiltonian. This Hamiltonian generates a $SU(4)$-symmetric spectrum.

The transverse part of the interaction Hamiltonian:

$$H_T = -g \int d^3x\, \Psi^\dagger(\mathbf{x}) \boldsymbol{\alpha} \cdot \mathbf{A}(\mathbf{x})\, \Psi(\mathbf{x}) \ ,$$

is not $SU(2)_{CS}$- and $SU(4)$-symmetric and therefore its expectation value vanishes in the $SU(4)$-symmetric hadron wave function.

The quasi-zero modes are due to the magnetic part of QCD. The magnetic interactions break $SU(4)$ and $SU(2)_{CS}$ symmetries of confinement explicitly and $SU(2)_L \times SU(2)_R \times U(1)_A$ - dynamically. Instanton fluctuations?

Conclusions

Observed on the lattice $SU(4)$ symmetry of hadrons upon elimination of the near-zero modes is a symmetry of confinement in QCD that is due to color-electric charge-charge interaction.

The magnetic interactions in QCD are responsible for generation of the quasi-zero modes. They break explicitly the $SU(4)$ symmetry of confinement and dynamically the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry. Instantons?

The hadron spectra observed in real world can be viewed as a result of splitting of the primary energy levels of the dynamical QCD string with the $SU(4)$ symmetry by means of dynamics associated with the quasi-zero modes.