Beating the sign problem in finite density lattice QCD

Zn Collaboration

Y. Taniguchi, Tukuba
Thu. 8:30-8:50

S.Oka, Rikkyo
Thu. 8:50-9:10

R.Fukuda, Tokyo/ETH
Thu. 09:10 - 09:30

A.Nakamura
RCNP, Osaka
Riken, FEFU

S.Sakai, Kyoto

A.Suzuki, Tsukuba
Thu. 9:30-9:50

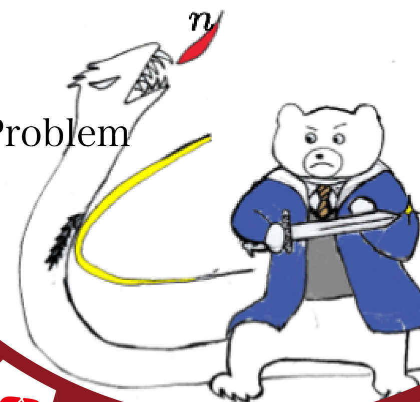
K.Nagata, KEK

Sign Problem

$$Z=\int \mathcal{D}U \prod_f \det\Delta(m_f,\mu_f)\, e^{-\beta S_G}$$

Complex

$$Z(\mu,T)=\sum_n Z_n(T)\left(e^{\mu/T}\right)^n$$



Grand-Canonical Partition Function

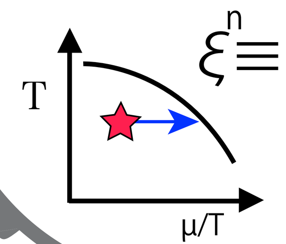
⟺ Canonical Partition Function

$$Z(\mu,T)=\mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$
$$=\sum_n \langle n|e^{-(H-\mu\hat{N})/T}|n\rangle$$
$$=\sum_n \langle n|e^{-H/T}|n\rangle e^{\mu n/T}$$
$$=\sum_n \langle n|e^{-H/T}|n\rangle e^{\mu n/T}$$
$$=\sum_n Z_n(T)\xi^n,\quad (\xi=e^{\mu/T})$$

We assume $[H,\hat{N}]=0$

Key-Relation

$$Z(\mu,T)=\sum_n Z_n\xi^n$$
$$\xi\equiv e^{\mu/T}$$



$$\det\Delta(\mu)$$
$$=\xi^{-N_r/2}C_0\det(\xi+Q)$$

Diagonalize Q

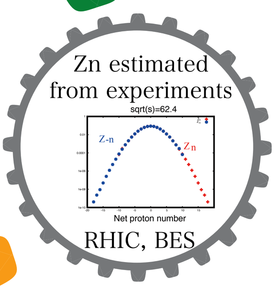
$$\det\Delta(\mu)=C_0\sum c_n\xi^n$$

$$Z(\mu,T)=\int\mathcal{D}U\left(\sum C_n\xi^n\right)^{N_f}e^{-\beta S_G}$$
$$=\sum Z_N\xi^n$$

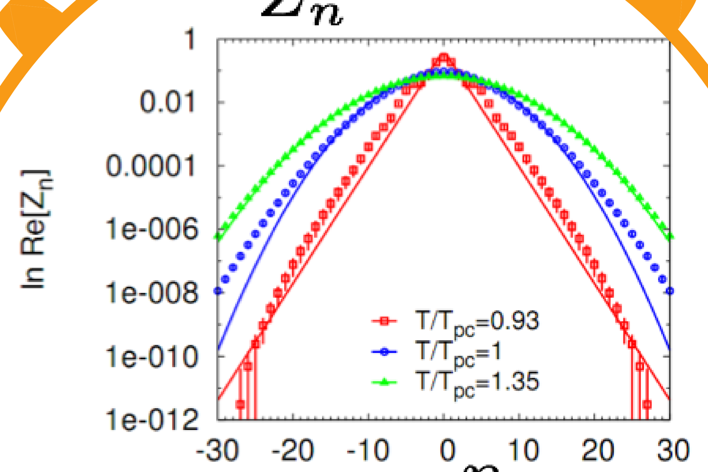
Nagata-AN, 2012

Or $$Z_n=\int\frac{d\theta}{2\pi}e^{i\theta n}\times Z\left(\theta\equiv\frac{\mu_I}{T}\right)$$

A.Hasenfrartz-Toussant, 1993



Z_n



We can construct at real μ

$\log Z$ (Pressure) $\quad T\frac{\partial}{\partial\mu}\log Z$ (Number density)

$\left(T\frac{\partial}{\partial\mu}\right)^2\log Z$ (Susceptibility)

$\langle\bar{\psi}\psi\rangle$ Hadron propagators

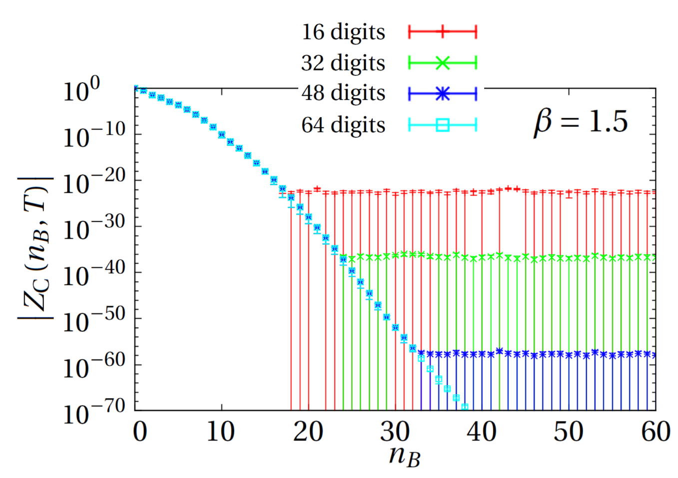
Hasenfrartz-Toussant method

Large n

➡ Rapid Oscillation in Fourier trans.

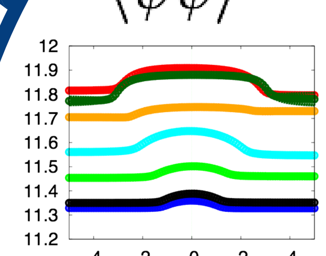
➡ 'Sign Problem'

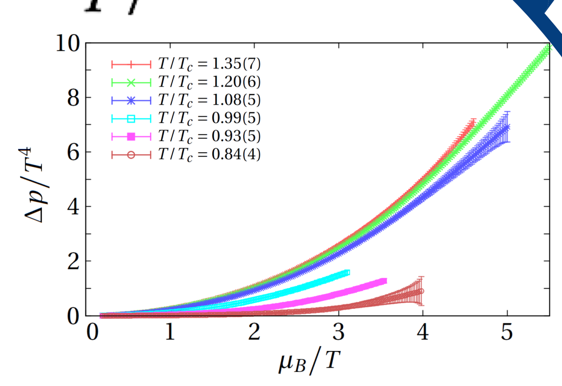
Multi-Precision Arithmetic ➡

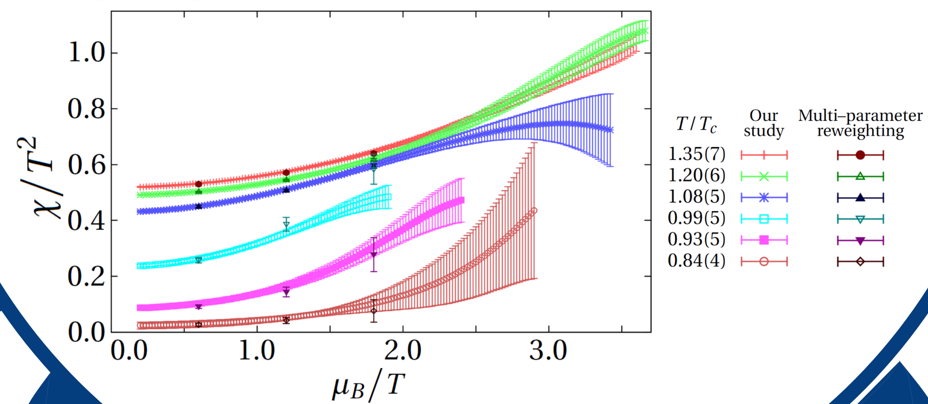


Canonical Partition Functions

$\Delta p/T^4$





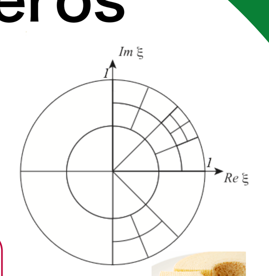


Lee-Yang Zeros

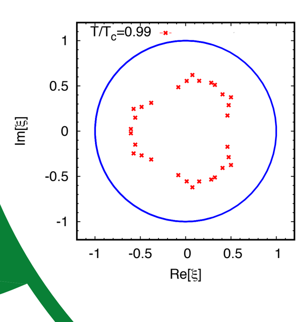
$$Z(\alpha_k)=0$$

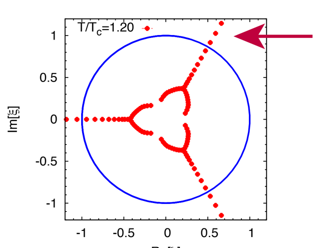
$$Z(\xi,T)\propto\prod_k(\xi-\alpha_k)\equiv f(\xi)$$

$$\frac{f'}{f}=\sum_k\frac{1}{\xi-\alpha_k}\qquad \frac{1}{2\pi}\oint_C\frac{f'(\xi)}{f(\xi)}d\xi=\text{No. of Zeros in C}$$



cut Baum-Kuchen





Roberge-Weise Phase Transition !