Pseudoscalar Meson Spectrum with Staggered Wilson Fermions

33rd International Symposium on Lattice Field Theory
Kobe International Conference Center, Kobe, Japan

Christian Zielinski
Joint work with David H. Adams

Division of Mathematical Sciences
Nanyang Technological University
Singapore

This talk

- This talk is about the computation of the **pseudoscalar meson spectrum** with two-flavor **staggered Wilson fermions**
- We discuss how the pseudoscalar meson spectrum for usual staggered fermions changes after the introduction of the staggered Wilson term
- We present preliminary numerical results for the resulting spectrum
- The presented spectrum calculations are of exploratory nature

Outline

Outline

What are staggered Wilson fermions?

- Staggered Wilson fermions are a novel fermion formulation [Adams '10, '11]
    - Staggered version of Wilson fermions
    - Constructed by adding a "Wilson term" to staggered fermions
    - Number of flavors reduced from four to two
- One-flavor version later proposed by C. Hoelbling [Hoelbling, '11)]
    - However, fine-tuning required to cancel new counterterms
- Here we focus on the original two-flavor version

Fermion action

- The staggered Wilson action is of the form

$$S_{\rm sw} = \bar{\chi}\,(D_{\rm st} + m_q + W_{\rm st})\,\chi$$

  with staggered Dirac operator $D_{\rm st} = \eta_\mu \nabla_\mu$ and bare mass m_q
- In the two-flavor case the staggered Wilson term reads

$$W_{\rm st} = \frac{r}{a}\,(\mathbb{1} - \Gamma_{55}\Gamma_5)\,, \quad r > 0,$$

  where $\Gamma_{55} \cong \gamma_5 \otimes \xi_5$, $\Gamma_5 \cong \gamma_5 \otimes \mathbb{1} + \mathcal{O}\left(a^2\right)$ in spin $\otimes$ flavor interpretation
- The new term is of the form $\mathbb{1} - \Gamma_{55}\Gamma_5 \cong \mathbb{1} \otimes \mathbb{1} - \mathbb{1} \otimes \xi_5 + \mathcal{O}\left(a^2\right)$
  - Gives mass $\propto \frac{1}{a}$ to the two negative flavor-chirality species
  - Two positive flavor-chirality species remain massless

Properties

- Advantages
  - Reduced number of flavors compared to usual staggered fermions
  - Computationally more efficient than Wilson fermions [LAT13, LAT14]
  - Can be used as a kernel to construct staggered versions of domain wall fermions and overlap fermions
- Lattice artifacts break SU(4) flavor symmetry of usual stag. fermions
  - Similarly here SU(2) symmetry of the two physical flavors broken

Outline

Hadron spectroscopy

- We are interested in hadron spectroscopy using stag. Wilson fermions
  - For usual staggered fermions meson and baryon rest-frame operators are known [Golterman & Smit, Sharpe, ...]
  - Can adapt these operators to the two-flavor case
  - However, physical interpretation changes!
- We illustrate spectrum calculations for the case of pseudoscalar mesons
- The case of usual staggered fermions
  - There are 16 pseudoscalar mesons
  - They have spin $\otimes$ flavor structure $\gamma_5 \otimes \xi_F$ with $\xi_F \in \{\mathbb{1}, \xi_5, \xi_\mu, \xi_\mu \xi_5, \xi_\mu \xi_\nu\}$

Pseudoscalar mesons

- States fall into eight irreducible representations (irreps.) of the lattice timeslice group with flavors [Goltermann '86]:
  - $\mathbb{1}$, ξ_5, ξ_i, ξ_4, $\xi_i\xi_5$, $\xi_4\xi_5$, $\xi_i\xi_j$, $\xi_i\xi_4$
- There are two types of states in these irreps. [Bae *et al.* '08]
  - Some states propagate with a factor of $(-1)^t$ and some do not
  - In general the timeslice operators excite two states
  - If the operator couples to $\gamma_S \otimes \xi_F$, then also to the "time-parity partners" $\gamma_4\gamma_5\gamma_S \otimes \xi_4\xi_5\xi_F$ (here $\gamma_S = \gamma_5$ or $\gamma_S = \gamma_4\gamma_5$)
- The time-time correlation function can be parametrized by

$$R_+ \cosh\left[m_+ \left(N_t/2 - t\right)\right] + (-1)^t R_- \cosh\left[m_- \left(N_t/2 - t\right)\right]$$

if one-particle states dominate [Goltermann '86]

Staggered Wilson Dirac operator

- As the staggered Wilson term makes two flavors heavy, ξ_F is of the following structure

$$\xi_F \cong \begin{bmatrix} \text{light+light} & \text{light+heavy} \\ \text{heavy+light} & \text{heavy+heavy} \end{bmatrix}$$

- In the continuum limit heavy contributions decouple
  - The "light+light" part of ξ_F then determines the physical interpretation

Example

$$\xi_F = \xi_3 \xi_4 = \begin{bmatrix} -\mathrm{i}\sigma_3 & 0 \\ 0 & \mathrm{i}\sigma_3 \end{bmatrix}$$

"Light+light" part is $-\mathrm{i}\sigma_3$, hence $-\mathrm{i}\bar{\chi}(\gamma_5 \otimes \xi_3 \xi_4)\chi$ corresponds to a π^0 operator

Continuum pseudoscalar mesons

- Out of the 16 pseudoscalar mesons eight become heavy
  - In the continuum limit we are left with eight pseudoscalar mesons
  - We find four physical particles with two-fold degeneracy
- The two physical flavors $q \equiv (q_1, q_2)^\intercal$ forms the following mesons:

ξ_F	Composition	Particle
$\xi_k\xi_4$	$\overline{q}\sigma_k q$	$\pi^\pm$, π^0
$\xi_i\xi_j$		
ξ_5	$\overline{q}q$	η
$\mathbb{1}$		

Numerical tests

- We implemented staggered Wilson fermions in the Chroma/QDP software package [Edwards et al. '05]
  - Spectrum calculations are carried out with program 'spectrum_s'
  - Pseudoscalar meson operators identical to the ones used in Bae *et al.* '08
- Quenched study on a $16^3 \times 64$ lattice at $\beta = 6$
  - 200 configurations in Coulomb gauge
  - Evaluate eight staggered wall sources per configuration
  - Combine them appropriately to project onto irreps. of the lattice timeslice group
- Cross-checked our implementation for usual staggered fermions $(r = 0)$
  - Match the masses reported in Bae *et al.* '08
- Numerical results are **preliminary** for $r = 1$
  - We find $m^2 \propto m_q$ in accordance with chiral perturbation theory
  - All degeneracies as expected

Usual staggered fermions: Pseudoscalar meson spectrum

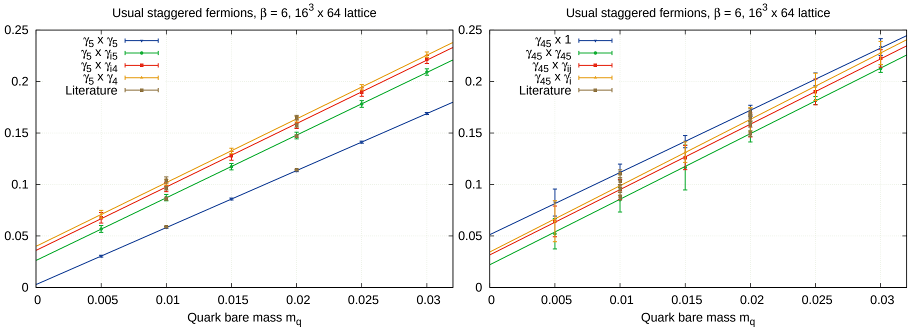

Figure: Pseudoscalar meson mass $m^2(m_q)$

Staggered Wilson fermions: Effective masses

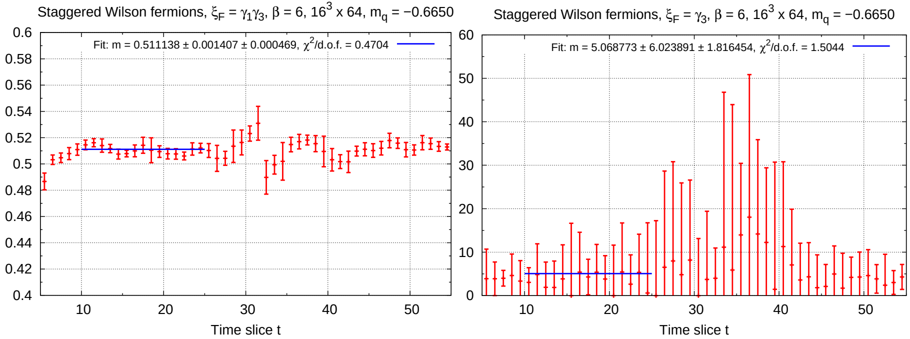

Figure: Effective mass $m_{\rm eff}(t)$ for a light and a heavy pseudoscalar meson

Staggered Wilson fermions: Light spectrum

Outline

Summary

- We showed how the pseudoscalar meson spectrum changes after the introduction of the staggered Wilson term
- We determined the light part of the spectrum and identified the corresponding particles
- We demonstrated the feasibility of numerical spectrum calculations with staggered Wilson fermions
- We hope that this proof-of-concept study paves the way to more realistic applications of staggered Wilson fermions