Disconnected contributions to nucleon observables with $N_f = 2$ twisted-clover fermions at the physical light quark mass

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Introduction

- Determination of disconnected loops: Long-standing problem
  - Require all-to-all propagator, expensive
  - Flavour non-singlets receive small contributions
  - Historically neglected
- Important for
  - Flavour singlets
  - High precision determination of hadron structure constants
  - η' mass
  - Dark matter searches
  - ...

Stochastic procedures

- Exact computation of the all-to-all unfeasible nowadays
- We can use stochastic techniques
  - Invert a random set of sources $|\eta_j\rangle$ that form a basis up to stochastic errors
  - Properties $\begin{cases} \frac{1}{N}\sum_{j=1}^{N}|\eta_j\rangle = O\left(\frac{1}{\sqrt{N}}\right) \\ \frac{1}{N}\sum_{j=1}^{N}|\eta_j\rangle\langle\eta_j| = I + O\left(\frac{1}{\sqrt{N}}\right) \end{cases}$
  - In this work we use $\mathbf{Z}_2$ and $\mathbf{Z}_4$ noise sources
- So we get an unbiased estimation of the all-to-all propagator

$$M|s_j\rangle = |\eta_j\rangle \longrightarrow M_E^{-1} := \frac{1}{N}\sum_{j=1}^{N}|s_j\rangle\langle\eta_j| \approx M^{-1}$$

- Error decresases as $1/\sqrt{N}$
- Alternative: Probing

Tang, Saad 2012

Stathopoulos, Laeuchli, Orginos 2013

Strange loops: The Truncated Solver Method

- Instead of solving $M\,|s_j\rangle = |\eta_j\rangle$ exactly, we aim at a *low precision estimation*

  Bali, Collins, Schäffer 2007

  - Stop the inverter (CG) at a certain number of iterations OR at a given precision $\rho^2 \sim 10^{-4}$
- Cheap but inaccurate $\longrightarrow$ The *bias* introduced is corrected *stochastically*

$$M_E^{-1} := \frac{1}{N_{HP}} \sum_{j=1}^{N_{HP}} \left(|s_j\rangle \langle \eta_j|_{HP} - |s_j\rangle \langle \eta_j|_{LP} \right) + \frac{1}{N_{LP}} \sum_{j=N_{HP}+1}^{N_{HP}+N_{LP}} |s_j\rangle \langle \eta_j|_{LP}$$

- If the convergence in the inversions is fast, we can get away with a low N_{HP}
- Error should decrease essentially as $1/\sqrt{N_{LP}}$
- Requires loop-dependent fine-tuning

Light loops: Exact deflation

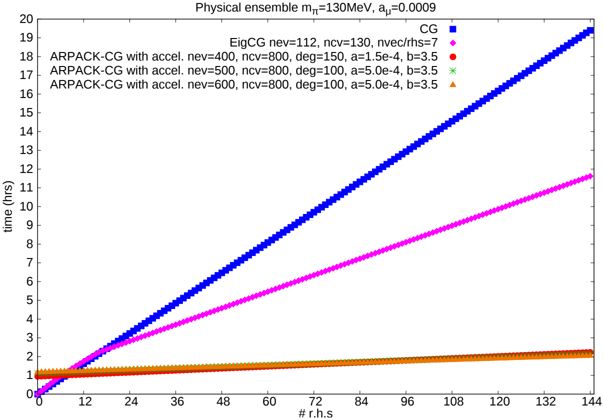
- The TSM fails for low quark masses
  - LP sources are not highly correlated with HP anymore, unless $\rho \sim 10^{-6}$
  - For $\rho \sim 10^{-6}$, the TSM performance is poor
- Deflation allows us to remove the low modes
  - Inversions become faster
  - Distinction HP-LP small in computer time $\Longrightarrow$ Drop the TSM
- Reconstruct the sources using the eigenpairs

$$|s\rangle = M^{-1} |\eta_D\rangle + \sum_{j=1}^{N} \frac{1}{\lambda_j} \langle v_j | \eta \rangle$$

$$|\eta_D\rangle = |\eta\rangle - \langle v_j | \eta \rangle |v_j\rangle$$

- Could be more optimal: exact diagonal of propagator with eigenvectors
- Problems with EO preconditioning

Light loops: Exact deflation



- Start-up time is fastly recovered
- Polynomial acceleration is fundamental for fast convergence

The one-end trick

- General trick that reduces variance, generally applied to 2pt

Foster, Michael 1998; McNeile, Michael 2006

- Propagators in tmQCD can be arranged in a way that allows the application of the one-end trick

- Difference (like $\bar{\psi}\psi$)

$$\mathrm{Tr}\left[X\left(M_u^{-1} - M_d^{-1}\right)\right] =$$

$$-2i\mu \sum_r \left\langle s^\dagger X \gamma_5 s \right\rangle_r$$

- μ noise suppresion
- Volume sum enhances statistics
- Improves SNR from $\left(\frac{1}{\sqrt{V}}\right)$ to $O(1)$

- Sum (like $\bar{\psi}\gamma_5\gamma_\mu\psi$)

$$\mathrm{Tr}\left[X\left(M_u^{-1} + M_d^{-1}\right)\right] =$$

$$2 \sum_r \left\langle s^\dagger \gamma_5 X \gamma_5 D_W s \right\rangle_r$$

- No μ noise suppression
- Dirac operator introduces noise
- Similar performance to time-dilution + HPE for the strange

Implementation

- Eigenpair computation
  - PARPACK + tmLQCD on CPUs (+ QUDA...)
  - 300 Eigenpairs $u + d$ using 48 CPU-nodes during 12 hours
- Inversion + contraction
  - Optimized kernels for contractions in last QUDA release

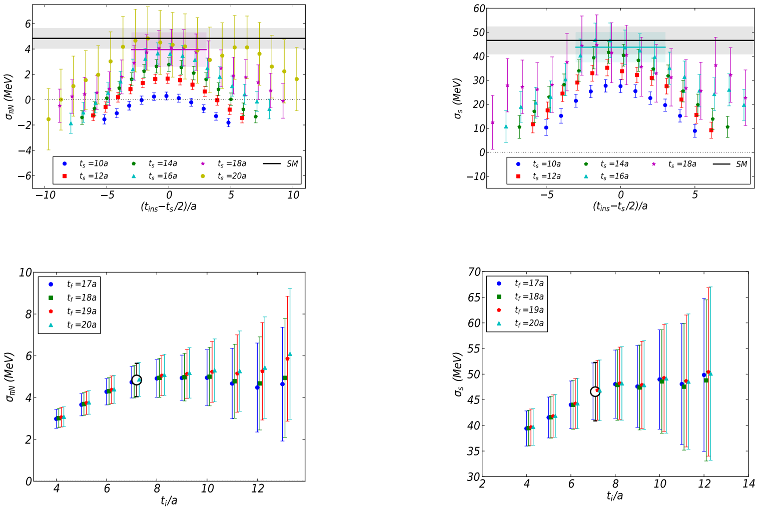
```
cudaColorSpinorField x; //Spinors to contract
cudaColorSpinorField y;
void *output;
cudaMalloc(&output, 32*Vol*sizeof(double)));
contract(x, y, output, QUDA_CONTRACT);
contract(x, y, output, QUDA_CONTRACT_TSLICE, tSlice);
```

- The output can be directly processed by cuFFT

Results: Ensemble and observables

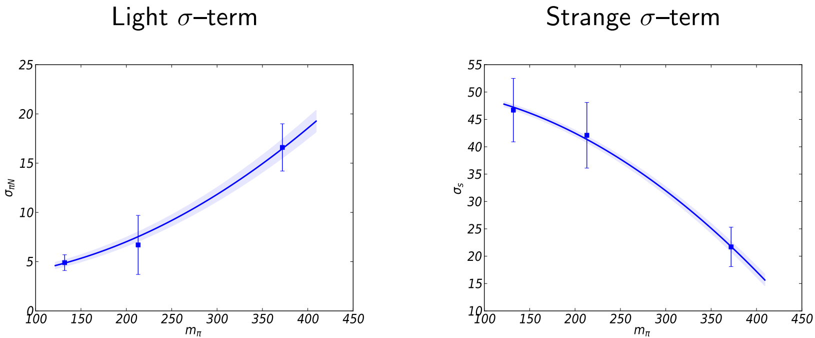
- $V = 48^3 \times 96$, $N_F = 2$ with $m_\pi \approx 130$MeV,
- Stats 1300 configurations $\times$ 100 2pt functions per configuration
- Light disconnected 2250 noise vectors, strange disconnected TSM with 63HP / 1024LP vectors
- We computed ultralocal operators (arbitrary γ insertion) for the nucleon, mainly
  - The sigma terms $\sigma_{\pi N}$ and σ_s
  - The axial charge g_A^{u+d} and g_A^s
  - The tensor charge g_T^{u+d} and g_T^s

Results: Sigma terms



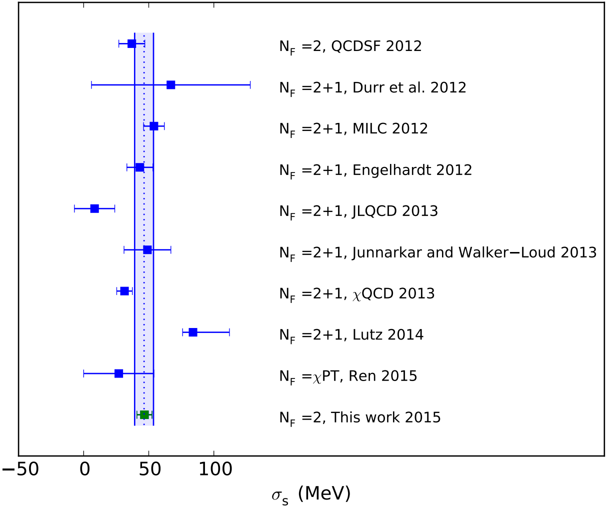
- Excited states contamination until large source-sink separations ($t_s \approx 1.8$ fm)

Results: Compare with simple chiral extrapolation



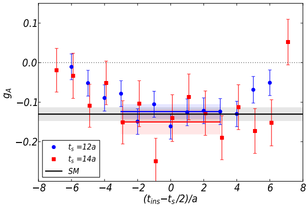
- In good agreement with a simple chiral extrapolation
- $\sigma(m_\pi) = \sigma_\infty + K m_\pi^2$

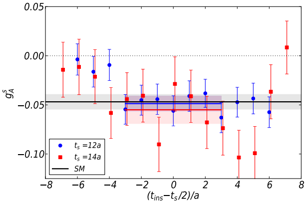
Results: Compare with previous results σ_s

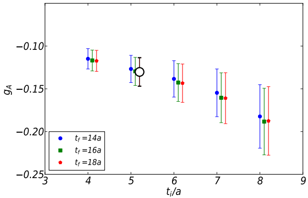


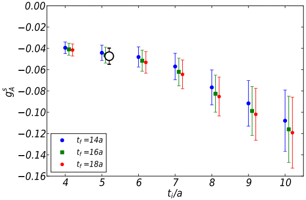
- Excellent agreement with previous results

Results: g_A



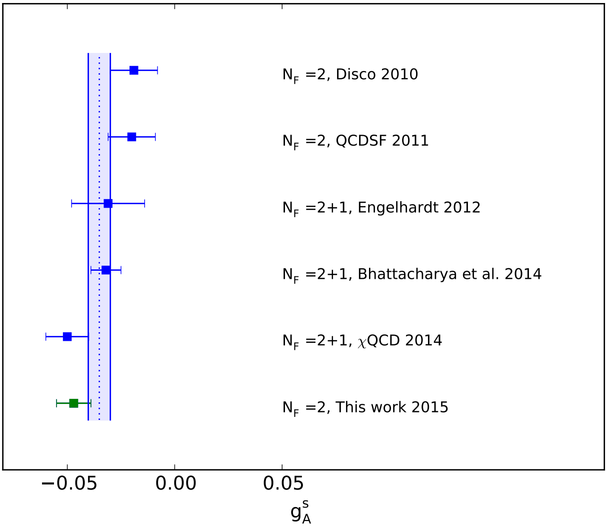






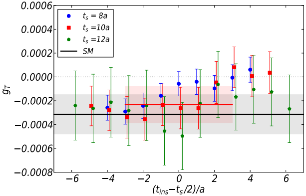
- No strong indication of excited states at the source-sink separations tested

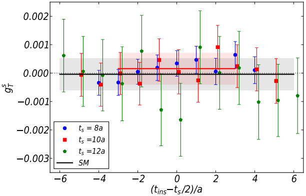
Results: Compare with previous results g_A^s

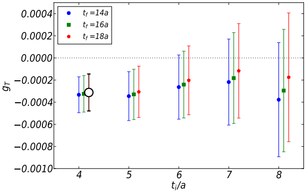


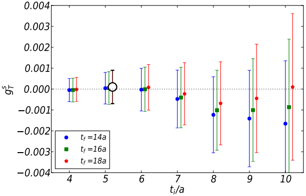
- Good agreement with previous results

Results: g_T









- No strong indication of excited states at the source-sink separations tested

Summary and Conclusions

- High precision computation of disconnected loops at the physical point
- Deflation becomes a must to tackle the light mass
- TSM performance degrades for very light masses: loss of correlations HP-LP
- Plans to improve the results and optimize the procedure
  - Could AMA and LMA further reduce the variance?
  - Systematic effects (Volume, lattice spacing)
  - Careful renormalization
  - Low-mode exact reconstruction of the inverse
  - Move eigensolvers to GPU (DONE!)