The electric dipole moment of the neutron from $N_f = 2 + 1 + 1$ twisted mass fermions

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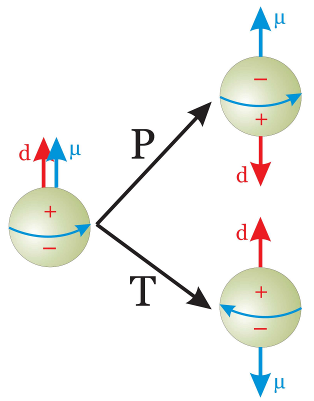
1. General Considerations

- The structure of the SM $\sim$ role of the discrete symmetries C, P, T
- Experimental observation of neutron Electric Dipole Moment (nEDM) $|\vec{d}_N|$

  $\longrightarrow$ Flags violation of P and T



  $\longrightarrow$ Due to CPT symmetry $\rightarrow$ violation of CP

- Induce CP-violation in QCD through:

$$\mathcal{L}_{\mathrm{CS}}(x) \equiv -i\theta \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[F_{\mu\nu}(x) F_{\rho\sigma}(x)\right] = -i\theta q(x) \, ,$$

- Best experimental bound: $|\vec{d}_N| < 2.9 \times 10^{-13} e \cdot \mathrm{fm}$

  [C. A. Baker *et al*, Phys. Rev. Lett. **97**, 131801 (2006) [arXiv:hep-ex/0602020]]

- Model dependend studies as well as ChPT predictions:

$$|d_N| \sim \theta \cdot \mathcal{O}\left(10^{-2} \sim 10^{-3}\right) e \cdot \mathrm{fm} \, . \quad \rightarrow \quad \theta \lesssim \mathcal{O}\left(10^{-10} \sim 10^{-11}\right)$$

2. The nEDM: Definition

- Electric Dipole Moment (EDM) can be calculated by

$$|\vec{d}_N| = \lim_{Q^2\to 0} \theta \frac{F_3(Q^2)}{2m_N} \,,$$

- Need to extract CP-odd F.F. $F_3(0)$ for $\theta \neq 0$!!!
- How: through nucleon-nucleon matrix element of the electromagnetic current $J^{\rm em}_\mu$

$\rightarrow$ CP-odd electromagnetic F.F. $F_3(Q^2)$:

$$\langle N(\vec{p}_f, s_f)|J^{\rm em}_\mu|N(\vec{p}_i, s_i)\rangle \quad = \quad \bar{u}(\vec{p}_f, s_f)\left[\cdots + \theta \frac{F_3(Q^2)}{2m_N} q_\nu \sigma_{\mu\nu}\gamma_5 + \cdots\right] u(\vec{p}_i, s_i)\,,$$

- **Assuming a theory where CP symmetry is violated:**

$$\langle \mathcal{O}(x_1, ..., x_n)\rangle_\theta = \frac{1}{Z_0}\int d[U]d[\psi_f]d[\bar{\psi}_f]\; \mathcal{O}(x_1, ..., x_n)\; e^{-S_{\rm QCD}+i\theta\int q(x)d^4x} \,.$$

- Extract $F_3(0)$ from perturbative expansion in θ:

$$\langle \mathcal{O}(x_1, ..., x_n)\rangle_\theta \quad = \quad \langle \mathcal{O}(x_1, ..., x_n)\rangle_{\theta=0} + \left\langle \mathcal{O}(x_1, ..., x_n)\left(i\theta\int d^4x q(x)\right)\right\rangle_{\theta=0} + O(\theta^2)\,,$$

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- Extract $F_3(0)$ from perturbative expansion in θ:

$$\begin{aligned}\langle \mathcal{O}(x_1, ..., x_n)\rangle_\theta \quad &= \quad \langle \mathcal{O}(x_1, ..., x_n)\rangle_{\theta=0} + \left\langle \mathcal{O}(x_1, ..., x_n)\left(i\theta\int d^4x q(x)\right)\right\rangle_{\theta=0} + O(\theta^2)\,,\\ &= \quad \langle \mathcal{O}(x_1, ..., x_n)\rangle_{\theta=0} + i\theta\,\langle \mathcal{O}(x_1, ..., x_n)\mathcal{Q}\rangle_{\theta=0} + O(\theta^2)\,,\end{aligned}$$

2. The nEDM: Extraction

- Consider the 3-pt function:

$$G_{3\text{pt}}^{\mu,(\theta)}(\vec{q}, t_f, t_i, t) \quad \equiv \quad \langle J_N^{\theta}(\vec{p}_f, t_f) J_\mu^{\text{em}}(\vec{q}, t) \overline{J}_N^{\theta}(\vec{p}_i, t_i)\rangle_\theta \ ,$$

- Had we been able to generate gauge configurations with the full action:

$$\begin{aligned} G_{3\text{pt}}^{\mu,(\theta)}(\vec{q}, t_f, t_i, t) &= |Z_N|^2 e^{-E_N^f(t_f - t)} e^{-E_N^i(t - t_i)} \frac{-i\gamma \cdot p_f + m_N(1 + 2i\alpha^1\theta\gamma_5)}{2E_N^f} \\ &\times \left[W_\mu^{\text{even}}(q) + i\theta W_\mu^{\text{odd}}(q)\right] \frac{-i\gamma \cdot p_i + m_N(1 + 2i\alpha^1\theta\gamma_5)}{2E_N^i} + O(\theta^2). \end{aligned}$$

- Treating the θ term perturbatively:

$$\begin{aligned} G_{3\text{pt}}^{\mu,(\theta)}(\vec{q}, t_f, t_i, t) &= \langle J_N^{\theta=0}(\vec{p}_f, t_f) J_\mu^{\text{em}}(\vec{q}, t) \overline{J}_N^{\theta=0}(\vec{p}_i, t_i) e^{i\theta\mathcal{Q}}\rangle_{\theta=0} \\ &= G_{3\text{pt}}^{\mu,(0)}(\vec{q}, t_f, t_i, t) + i\theta G_{3\text{pt},\mathcal{Q}}^{\mu,(0)}(\vec{q}, t_f, t_i, t) + O\left(\theta^2\right), \end{aligned}$$

with

$$\begin{aligned} G_{3\text{pt}}^{\mu,(0)}(\vec{q}, t_f, t_i, t) &= \langle J_N^{\theta=0}(\vec{p}_f, t_f) J_\mu^{\text{em}}(\vec{q}, t) \overline{J}_N^{\theta=0}(\vec{p}_i, t_i)\rangle_{\theta=0}, \\ G_{3\text{pt},\mathcal{Q}}^{\mu(0)}(\vec{q}, t_f, t_i, t) &= \langle J_N^{\theta=0}(\vec{p}_f, t_f) J_\mu^{\text{em}}(\vec{q}, t) \overline{J}_N^{\theta=0}(\vec{p}_i, t_i)\mathcal{Q}\rangle_{\theta=0}. \end{aligned}$$

2. The nEDM: Extraction

$$G_{3\text{pt}}^{\mu,(\theta)}(\vec{q},t_f,t_i,t) = |Z_N|^2 e^{-E_N^f(t_f-t)} e^{-E_N^i(t-t_i)} \frac{-i\gamma\cdot p_f + m_N(1+2i\alpha^1\theta\gamma_5)}{2E_N^f}$$
$$\times \left[W_\mu^{\text{even}}(q) + i\theta W_\mu^{\text{odd}}(q)\right] \frac{-i\gamma\cdot p_i + m_N(1+2i\alpha^1\theta\gamma_5)}{2E_N^i} + O(\theta^2).$$

$$\begin{aligned} G_{3\text{pt}}^{\mu,(\theta)}(\vec{q},t_f,t_i,t) &= \langle J_N^{\theta=0}(\vec{p}_f,t_f) J_\mu^{\text{em}}(\vec{q},t) \overline{J}_N^{\theta=0}(\vec{p}_i,t_i) e^{i\theta\mathcal{Q}}\rangle_{\theta=0} \\ &= G_{3\text{pt}}^{\mu,(0)}(\vec{q},t_f,t_i,t) + i\theta G_{3\text{pt},\mathcal{Q}}^{\mu,(0)}(\vec{q},t_f,t_i,t) + O\left(\theta^2\right), \end{aligned}$$

2. The nEDM: Extraction

- Equating the results from the two treatments:

$$
\begin{aligned}
G^{\mu,(0)}_{3\text{pt}}(\vec{q},t_f,t_i,t) &= |Z_N|^2 e^{-E^f_N(t_f-t)} e^{-E^i_N(t-t_i)} \frac{-i\gamma\cdot p_f + m_N}{2E^f_N} W^{\text{even}}_{\mu}(q) \frac{-i\gamma\cdot p_i + m_N}{2E^i_N}, \\
G^{\mu,(0)}_{3\text{pt}\mathcal{Q}}(\vec{q},t_f,t_i,t) &= |Z_N|^2 e^{-E^f_N(t_f-t)} e^{-E^i_N(t-t_i)} \Big[\frac{-i\gamma\cdot p_f + m_N}{2E^f_N} W^{\text{odd}}_{\mu}(q) \frac{-i\gamma\cdot p_i + m_N}{2E^i_N} \\
&+ \frac{\alpha^1 m_N}{E^f_N}\gamma_5 W^{\text{even}}_{\mu}(q) \frac{-i\gamma\cdot p_i + m_N}{2E^i_N} + \frac{-i\gamma\cdot p_f + m_N}{2E^f_N} W^{\text{even}}_{\mu}(q) \frac{\alpha^1 m_N}{E^i_N}\gamma_5 \Big].
\end{aligned}
$$

with

$$
W^{\text{even}}_{\mu}(q) = \gamma_\mu F_1(Q^2) - i\frac{F_2(Q^2)}{2m_N} q_\nu \sigma_{\mu\nu}, \qquad W^{\text{odd}}_{\mu}(q) = -i\frac{F_3(Q^2)}{2m_N} q_\nu \sigma_{\mu\nu}\gamma_5 + F_A(Q^2)\left(q_\mu \gamma\cdot q - \gamma_\mu Q^2\right)\gamma_5 .
$$

- Similarly for 2-pt functions:

$$
G^{(\theta)}_{2\text{pt}}(q,t) = G^{(0)}_{2\text{pt}}(q,t) + i\theta G^{(0)}_{2\text{pt},\mathcal{Q}}(q,t) + O\left(\theta^2\right),
$$

where

$$
\begin{aligned}
G^{(0)}_{2\text{pt}}(\vec{q},t) &= \langle J^{\theta=0}_N(\vec{q},t_f)\overline{J}^{\theta=0}_N(\vec{q},t_i)\rangle_{\theta=0} = |Z_N|^2 e^{-E_N t}\frac{-i\gamma\cdot p + m_N}{2E_N}, \\
G^{(0)}_{2\text{pt},\mathcal{Q}}(\vec{q},t) &= \langle J^{\theta=0}_N(\vec{q},t_f)\overline{J}^{\theta=0}_N(\vec{q},t_i)\mathcal{Q}\rangle_{\theta=0} = |Z_N|^2 e^{-E_N t}\frac{2\alpha^1 m_N}{E_N}\gamma_5 .
\end{aligned}
$$

- **We need to measure the topological charge $\mathcal{Q}$**

2. The nEDM: Extraction

- Equating the results from the two treatments:

$$G^{\mu,(0)}_{3\text{pt}}(\vec{q},t_f,t_i,t) = |Z_N|^2 e^{-E^f_N(t_f-t)} e^{-E^i_N(t-t_i)} \frac{-i\gamma\cdot p_f + m_N}{2E^f_N} W^{\text{even}}_\mu(q) \frac{-i\gamma\cdot p_i + m_N}{2E^i_N},$$

$$G^{\mu,(0)}_{3\text{pt}\mathcal{Q}}(\vec{q},t_f,t_i,t) = |Z_N|^2 e^{-E^f_N(t_f-t)} e^{-E^i_N(t-t_i)} \Big[\frac{-i\gamma\cdot p_f + m_N}{2E^f_N} W^{\text{odd}}_\mu(q) \frac{-i\gamma\cdot p_i + m_N}{2E^i_N} + \frac{\alpha^1 m_N}{E^f_N}\gamma_5 W^{\text{even}}_\mu(q) \frac{-i\gamma\cdot p_i + m_N}{2E^i_N} + \frac{-i\gamma\cdot p_f + m_N}{2E^f_N} W^{\text{even}}_\mu(q) \frac{\alpha^1 m_N}{E^i_N}\gamma_5 \Big].$$

with

$$W^{\text{even}}_\mu(q) = \gamma_\mu F_1(Q^2) - i\frac{F_2(Q^2)}{2m_N} q_\nu \sigma_{\mu\nu}, \qquad W^{\text{odd}}_\mu(q) = -i\frac{F_3(Q^2)}{2m_N} q_\nu\sigma_{\mu\nu}\gamma_5 + F_A(Q^2)\left(q_\mu \gamma\cdot q - \gamma_\mu Q^2\right)\gamma_5 .$$

- Similarly for 2-pt functions:

$$G^{(\theta)}_{2\text{pt}}(q,t) = G^{(0)}_{2\text{pt}}(q,t) + i\theta G^{(0)}_{2\text{pt},\mathcal{Q}}(q,t) + O\left(\theta^2\right),$$

where

$$G^{(0)}_{2\text{pt}}(\vec{q},t) = \langle J^{\theta=0}_N(\vec{q},t_f)\overline{J}^{\theta=0}_N(\vec{q},t_i)\rangle_{\theta=0} = |Z_N|^2 e^{-E_N t}\frac{-i\gamma\cdot p + m_N}{2E_N},$$

$$G^{(0)}_{2\text{pt},\mathcal{Q}}(\vec{q},t) = \langle J^{\theta=0}_N(\vec{q},t_f)\overline{J}^{\theta=0}_N(\vec{q},t_i)\mathcal{Q}\rangle_{\theta=0} = |Z_N|^2 e^{-E_N t}\frac{2\alpha^1 m_N}{E_N}\gamma_5 .$$

- **We need to measure the topological charge $\mathcal{Q}$**
- **Get access to α^1 and F_3** [E. Shintani *et al*, Phys.Rev.D72:014504,2005, arXiv:hep-lat/0505022]

Pilot study:

- Test the applicability of this method.
- Use an ensemble with large number of gauge confgs.

→ B55.32, $N_f = 2+1+1$ twisted mass fermions, with **Iwasaki Gauge Action**:

- $a = 0.0823(10)$ fm
- $M_\pi = 0.372$ GeV
- $r_0/a = 5.710(41)$
- $32^3 \times 64$, $L = 2.6$ fm
- No. of confs = 4623

[JHEP 1006:111 (2010)]

If convinced that it works:

- Go to the **physical pion mass**.
- Go to the **continuum limit**.

3. Topological charge: Definition

- Calculate the topological charge:

$$\mathcal{Q} = \int d^4 x q(x) \, .$$

- Improved definition ($O(a^4)$) for $q(x)$: clover + rectangle clovers
- Smooth out ultraviolet fluctuations with **Cooling** and **Gradient-Flow**.
- Cooling exhibits equivalence with gradient flow when smoothing with Wilson Action:
  - Perturbatively $\tau \simeq n_c/3$

    [C. Bonati and M. D'Elia, Phys. Rev. D 89, 105005 (2014) [arXiv:1401.2441]]
- Generalize the equivalence for Symanzik improved actions with rectangles ($c_0 + 8c_1 = 1$):

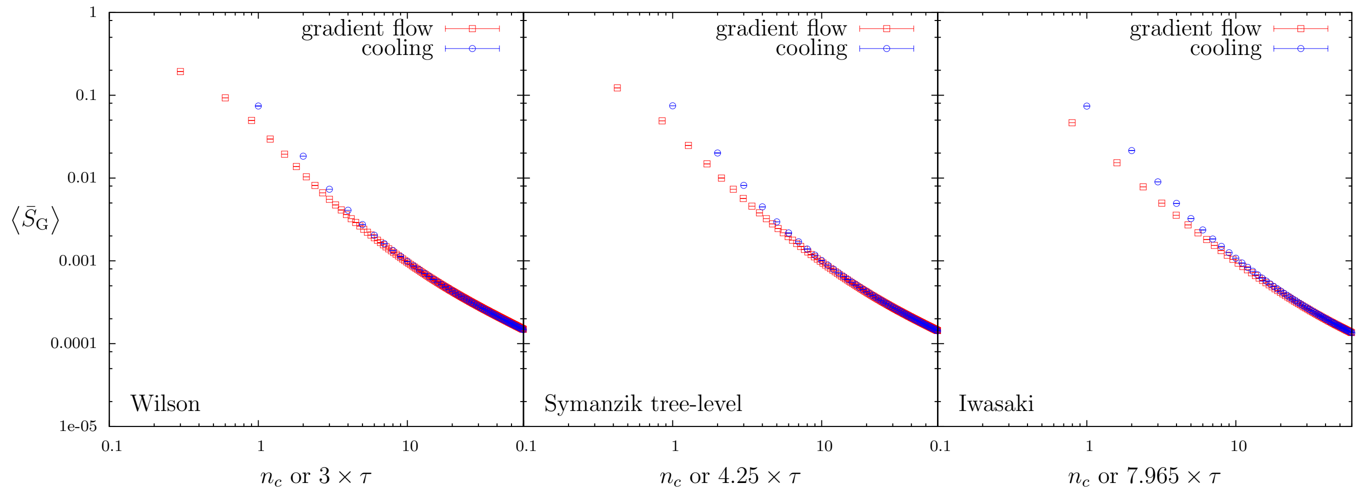
$$S_g = \frac{\beta}{N} \sum_x \left(c_0 \sum_{\substack{\mu,\nu=1 \\ 1 \leq \mu < \nu}}^{4} \left\{ 1 - \mathrm{ReTr}(U^{1\times1}_{x,\mu,\nu}) \right\} + c_1 \sum_{\substack{\mu,\nu=1 \\ \mu \neq \nu}}^{4} \left\{ 1 - \mathrm{ReTr}(U^{1\times2}_{x,\mu,\nu}) \right\} \right) ,$$

- Equivalence with $\tau \simeq n_c/(3 - 15c_1)$ [C. Alexandrou, A. A and K. Jansen, soon in arXiv]

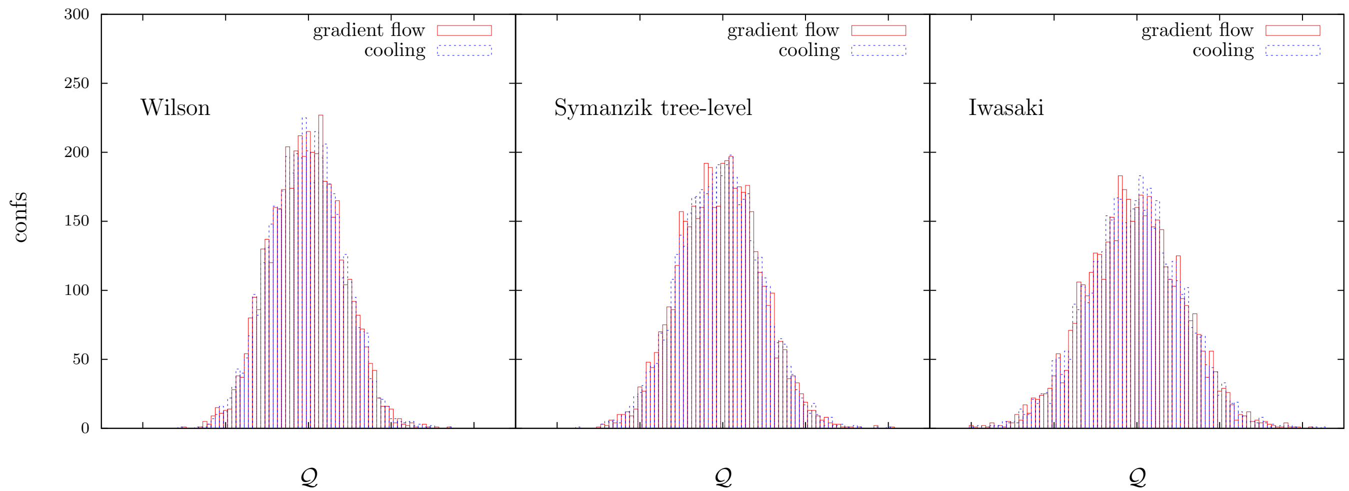
Smoothing action	c_0	c_1	n_c/τ
Wilson	1	0	$\simeq 3$
Symanzik tr.level	$\frac{5}{3}$	$-\frac{1}{12}$	$\simeq 4.25$
Iwasaki	3.648	-0.331	$\simeq 7.965$

3. Topological charge: Equivalence between smoothers

Average action density - used as a common scale between smoothers:

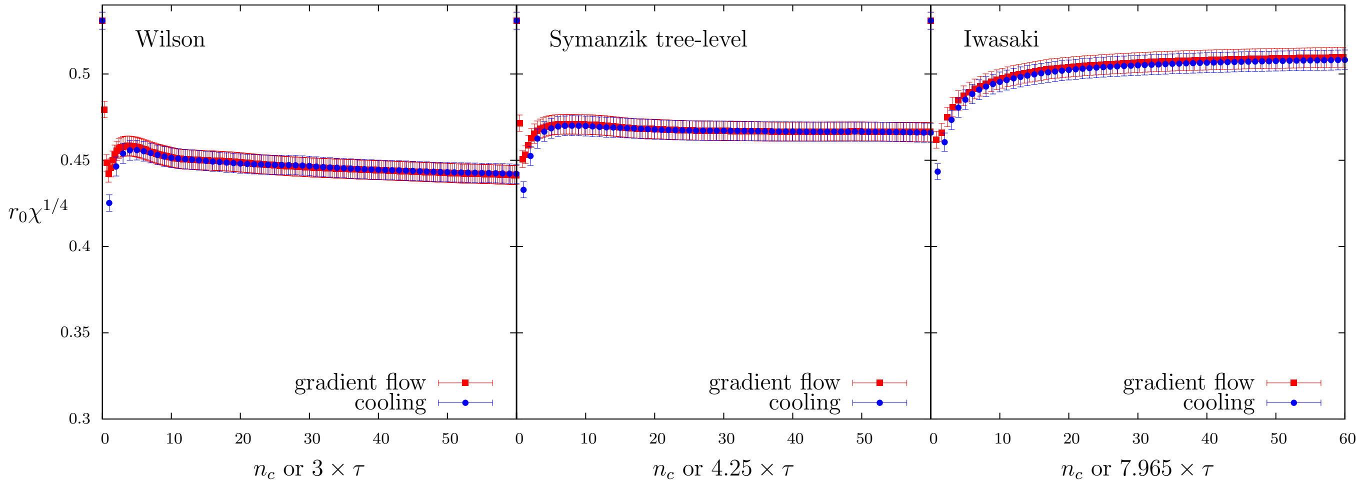


Topological Charge distribution

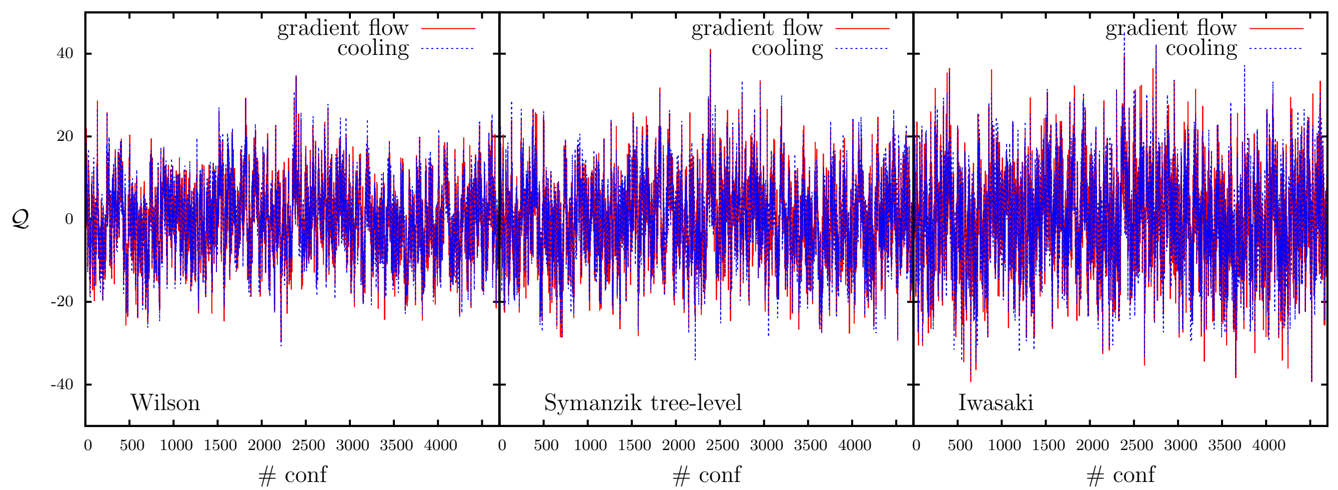


3. Topological charge: Equivalence between smoothers

Topological Susceptibility: $\chi = \langle \mathcal{Q}^2 \rangle / V$

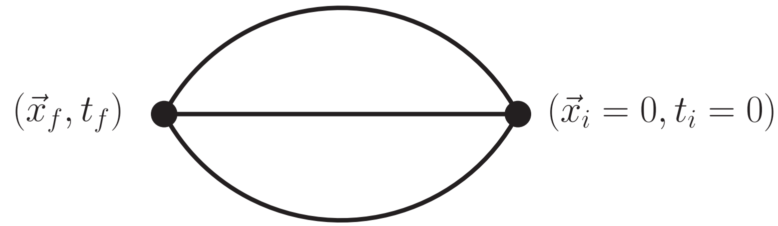


Topological Charge History



4. Results on nEDM: Correlation Functions: 2-pt functions

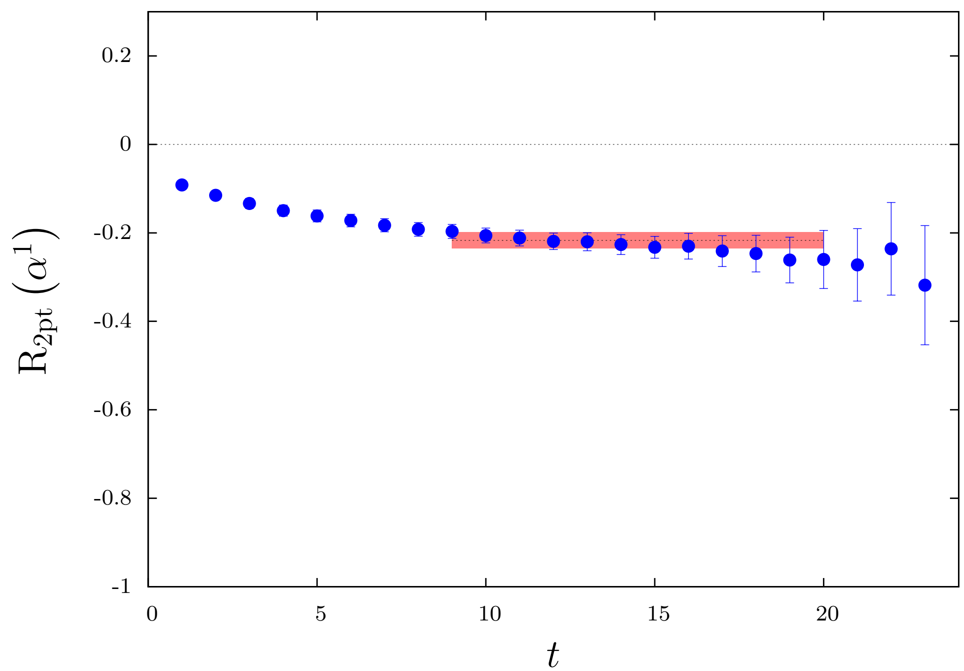
- Calculate:



- Calculate the plateau for $\mathrm{R}_{2\mathrm{pt}}(\alpha^1, t_f, t_i) = G^{(0)}_{2\mathrm{pt},\mathcal{Q}}(t_f, t_i, \gamma_5) / G^{(0)}_{2\mathrm{pt}}(t_f, t_i, (\mathbb{1} + \gamma_0)/2)$

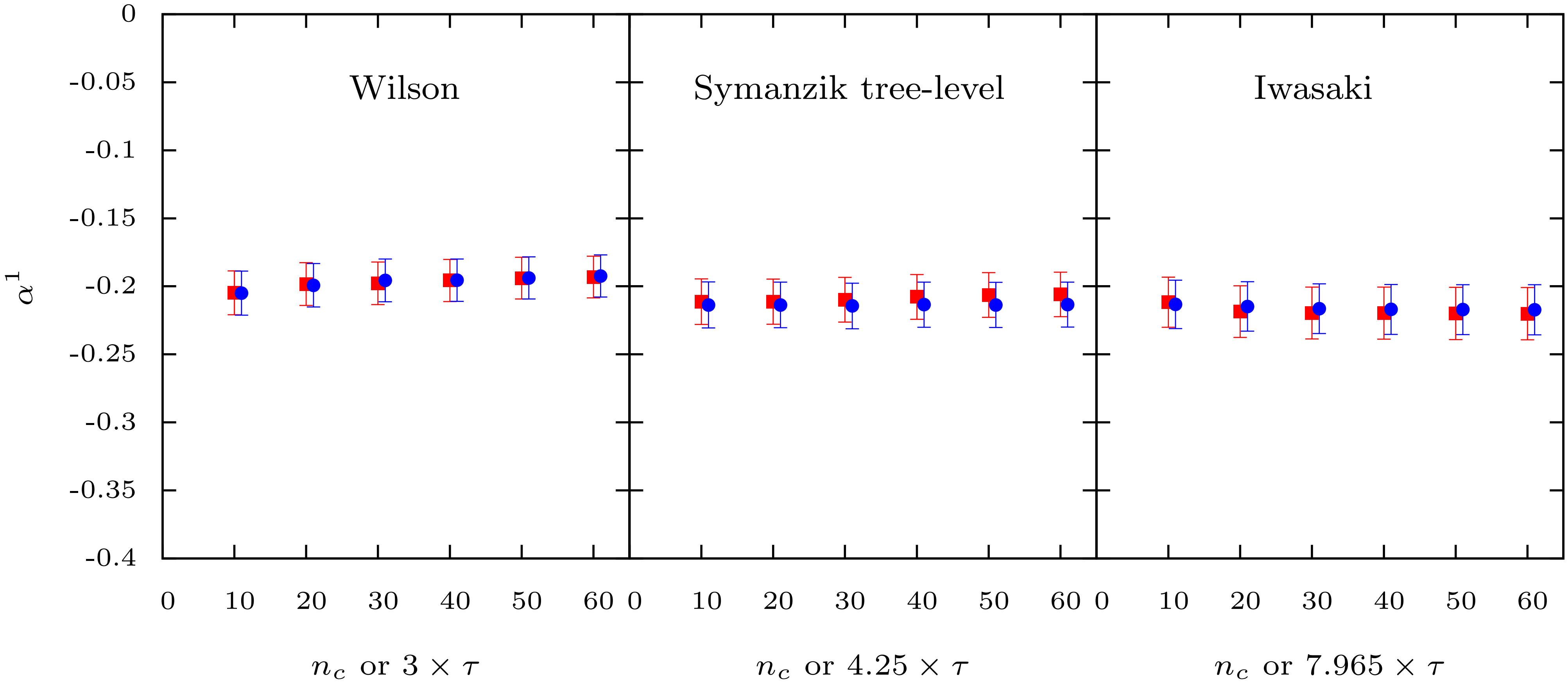
$$\Pi_{2\mathrm{pt}}(\alpha^1) = \lim_{t_f - t_i \to \infty} \mathrm{R}_{2\mathrm{pt}}(\alpha^1, t_f, t_i) = \alpha^1 ,$$

- Example of a plateau for Iwasaki action, gradient flow at $\tau = 6.3$



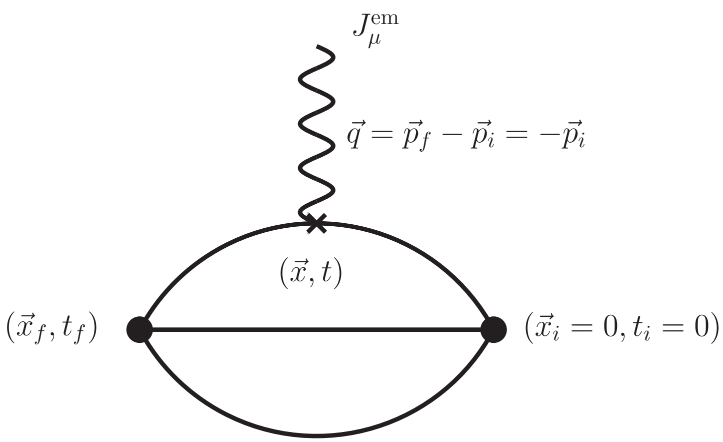
4. Results on nEDM: Calculation of α^1

α^1 via cooling (n_c) and gradient flow (τ) for Wilson, Symanzik tree-level and Iwasaki action.



4. Results on nEDM: Correlation Functions: 3-pt functions

- Calculate:



- Build ratio $\mathrm{R}^{\mu}_{3\mathrm{pt}}\left(\vec{q}, t_f, t_i, t, \Gamma_k\right)$ with plateau:

$$\begin{aligned}\Pi^{0}_{3\mathrm{pt}}(\Gamma_k) &= \lim_{t_f - t \to \infty} \lim_{t - t_i \to \infty} \mathrm{R}^{0}_{3\mathrm{pt}}\left(\vec{q}, t_f, t_i, t\right) . \\ &= \sqrt{\frac{2m_N^2}{E_N\left(E_N + m_N\right)}}\, q_k \left[\frac{\alpha^1 F_1\left(Q^2\right)}{2m_N} + \frac{(E_N + 3m_N)\alpha^1 F_2\left(Q^2\right)}{4m_N^2} + \frac{(E_N + m_N) F_3\left(Q^2\right)}{4m_N^2}\right] ,\end{aligned}$$

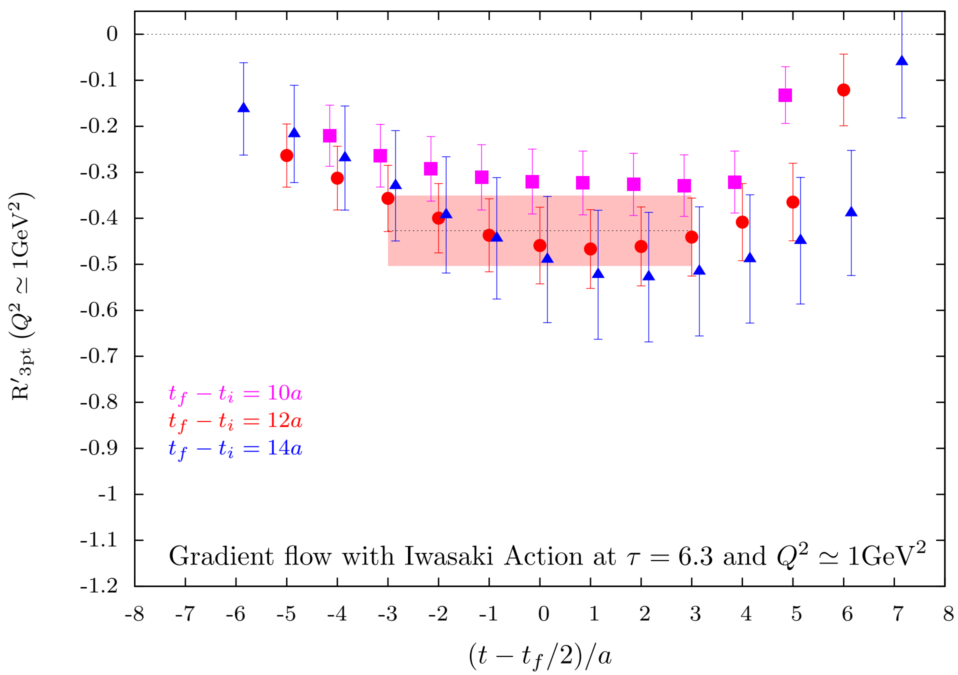
- Momentum q_k hinders a direct evaluation of $F_3(Q^2 = 0)$
  - Usual parametrization in Q^2 and then fit with Dipole ansatz
  - Continuum Derivative [D. Guadagnoli *et al*, JHEP 0304 (2003) 019, [arXiv:hep-lat/0210044]]
  - Momentum Elimination [C. Alexandrou *et al*, PoS LATTICE2014 (2014) 075, [arXiv:1410.8818]]

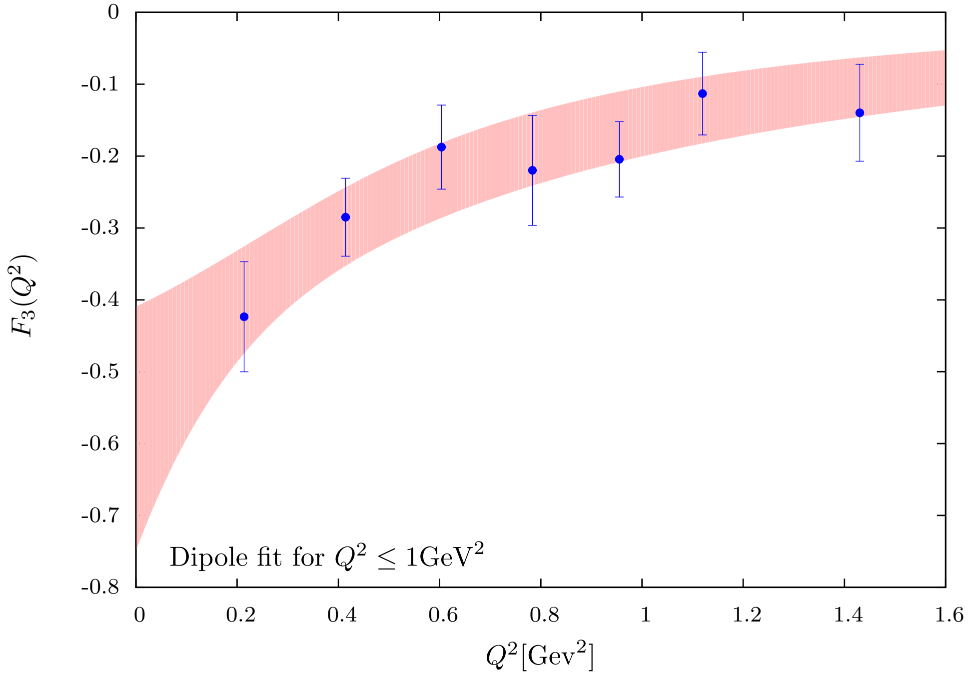
4. Results on nEDM: Extraction of $F_3(0)$ via Dipole Fit

- Extract the plateau $\Pi^0_{3\mathrm{pt}}\left(Q^2, \Gamma_k\right)$ for momentum transfer Q^2
- Use $F_1(Q^2)$, $F_2(Q^2)$ and α^1 to extract $F_3(Q^2)$
- Parametrizing F_3 in momentum transfer Q^2 and perform a dipole fit:

$$F_3\left(Q^2\right) = \frac{F_3\left(0\right)}{\left(1 + \frac{Q^2}{m^2_{F_3}}\right)^2},$$

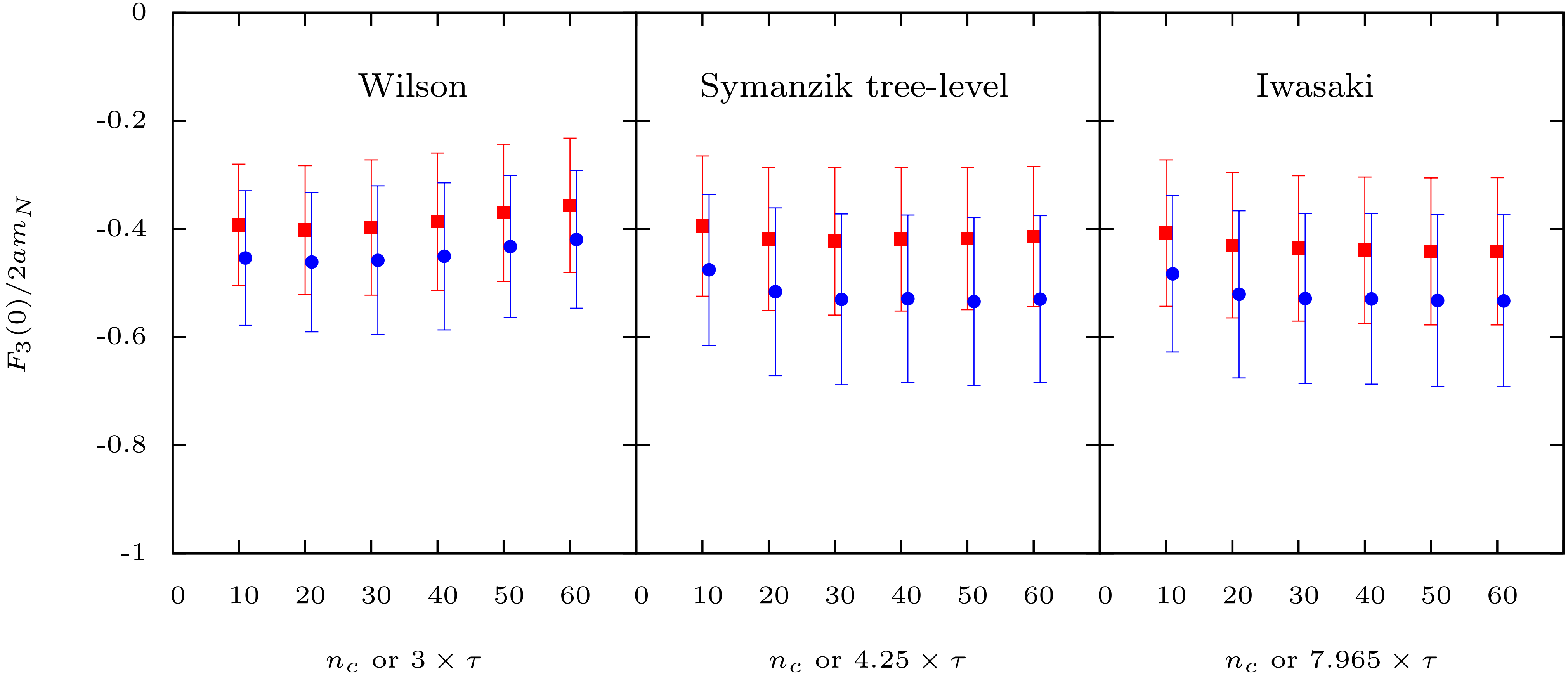
- Examples:





The nEDM: Extraction of $F_3(0)$ via Dipole fit

$F_3(0)$ via cooling (n_c) and gradient flow (τ) for Wilson, Symanzik tree-level and Iwasaki smoothing actions:



4. Results on nEDM: Extraction of $F_3(0)$ via Continuum Derivative

- Using the method of continuum derivative:

$$\lim_{q^2\to 0} \frac{\partial}{\partial q_j} \Pi^0_{3\text{pt}}\left(\vec{q}, \Gamma_k\right) = \sqrt{\frac{2m_N^2}{E_N\left(E_N + m_N\right)}}\delta_{kj}\left[\frac{\alpha^1 F_1(0)}{2m_N} + \frac{\alpha^1 F_2(0)}{m_N} + \frac{F_3(0)}{2m_N}\right].$$

- Build ratios:

$$\lim_{q^2\to 0} \frac{\partial}{\partial q_j} \mathrm{R}^{\mu}_{3\text{pt}}\left(\vec{q}, t_f, t_i, t, \Gamma_k\right) = \lim_{L\to\infty} \frac{1}{G^{(0)}_{2\text{pt}}(\vec{q}, t_f, t_i, \Gamma_5)} \cdot \sum_{x=-L/2+a}^{L/2-a} i x_j G^{\mu,(0)}_{3\text{pt},\mathcal{Q}}(\vec{x}, t_f, t_i, t, \Gamma_k),$$

- Requires position space correlators $G^{(0)}_{\mu J_N J^{\text{em}}_{\mu} \mathcal{Q} J_N}\left(\vec{x}, t_f, t_i, t, \Gamma_k\right)$

- In finite volume this approximates the derivative of δ-function:

$$a^3 \sum_{\vec{x}} i x_j G^{\mu,(0)}_{3\text{pt},\mathcal{Q}}(\vec{x}, t_f, t_i, t, \Gamma_k) = \frac{1}{V} \sum_{\vec{k}} \left(a^3 \sum_{\vec{x}} i x_j \exp\left(i\vec{k}\vec{x}\right)\right) G^{\mu,(0)}_{3\text{pt},\mathcal{Q}}(\vec{q}, t_f, t_i, t, \Gamma_k),$$

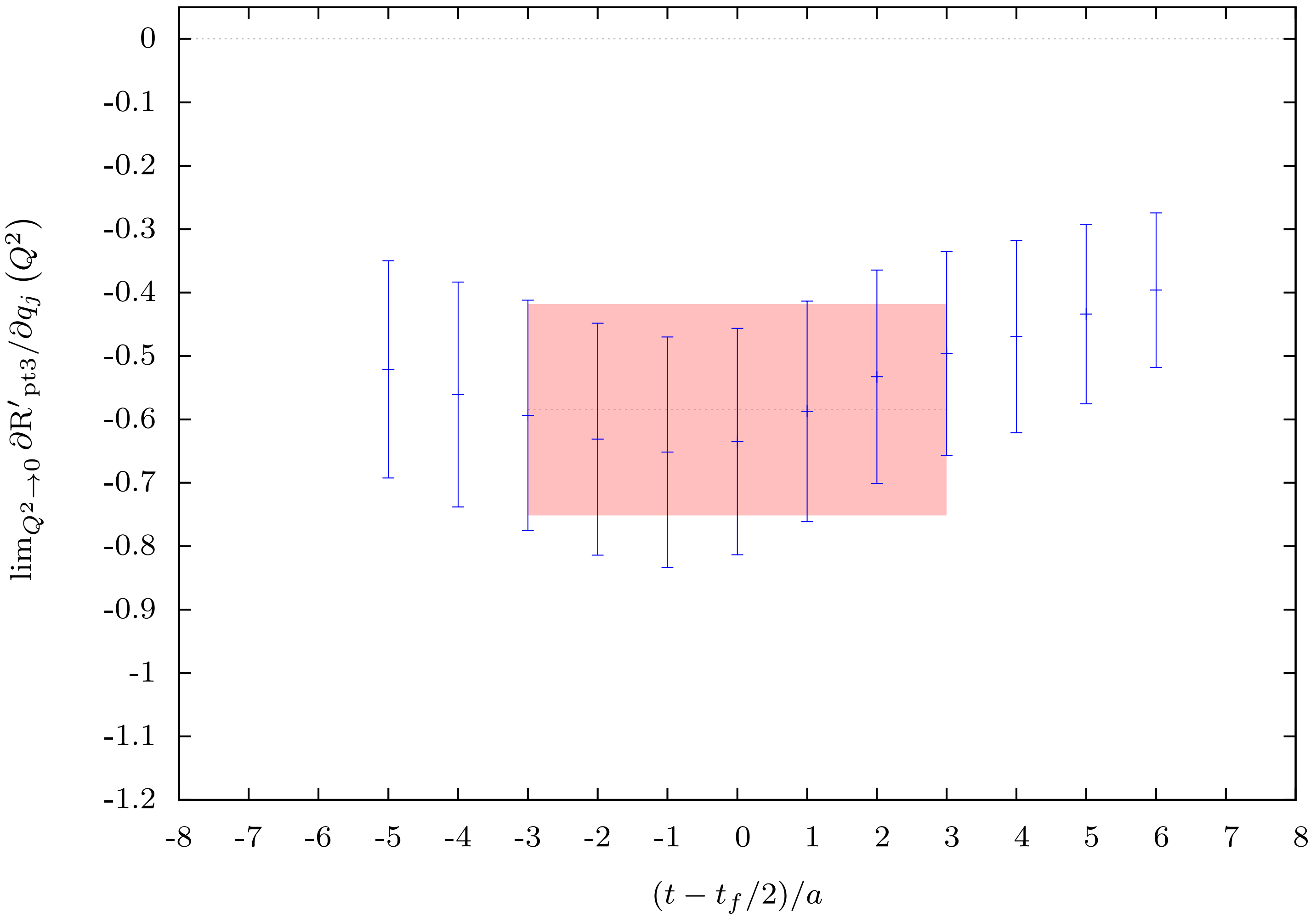
$$= \frac{1}{(2\pi)^3} \int d^3\vec{k} \frac{\partial}{\partial k_j} \delta^{(3)}\left(\vec{k}\right) G^{\mu,(0)}_{3\text{pt},\mathcal{Q}}(\vec{k}, t_f, t_i, t, \Gamma_k).$$

- Residual t-dependence $G^{\mu,(0)}_{3\text{pt},\mathcal{Q}}(\vec{q}, t_f, t_i, t, \Gamma_k) \sim \exp\left(-\Delta E_N t\right)$, with $\Delta E_N \to 0$ for $L \to \infty$

  [W. Wilcox, Phys.Rev. D66 (2002) 017502, [arXiv:hep-lat/0204024]]

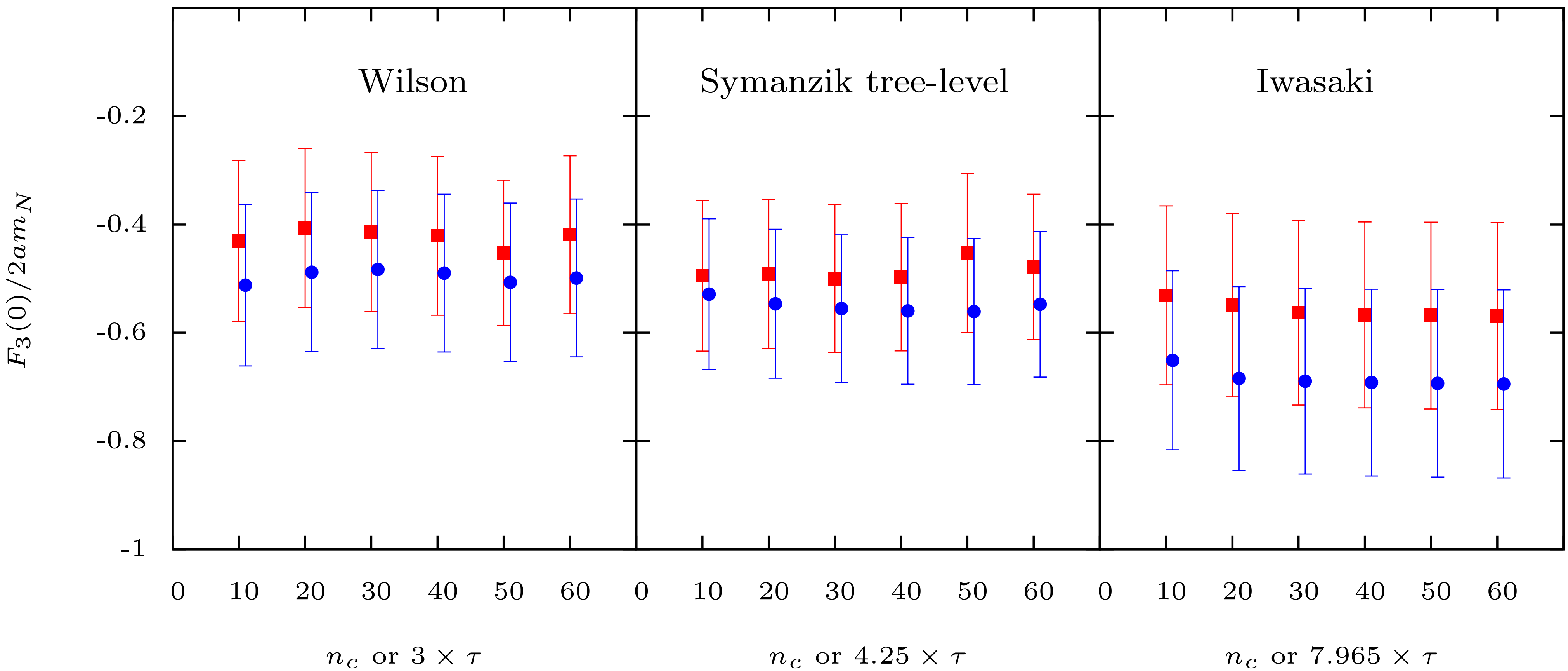
4. Results on nEDM: Extraction of $F_3(0)$ via Continuum Derivative

Example of a plateau for Iwasaki action, gradient flow at $\tau = 6.3$.



4. Results on nEDM: Extraction of $F_3(0)$ via Continuum Derivative

$F_3(0)$ via cooling (n_c) and gradient flow (τ) for Wilson, Symanzik tree-level and Iwasaki smoothing actions:



4. Results on nEDM: Extraction of $F_3(0)$ via Momentum Elimination

- Define a suitable ratio such that:

$$\Pi(q) \equiv \Pi^{0}_{3\mathrm{pt}}(\Gamma_k) + \frac{i}{2}\alpha_1 \Pi^{i,\theta=0}_{3\mathrm{pt}}(\Gamma_5) + 2\alpha_1 \Pi^{i,\theta=0}_{3\mathrm{pt}}(\Gamma_k) = \frac{q_k}{4}\sqrt{\frac{(E_N+m_N)}{2E_N m_N^2}}F_3(Q^2)\,,$$

- On-axis momenta e.g. $\vec{q} = (\pm q, 0, 0)^T$
- Fourier transform on $\Pi(q) \to \Pi(y)$ in position space; with $(n = y/a)$

$$\Pi(y) = \begin{cases} +\Pi(n), & n = 0, \ldots, N/2 \\ -\Pi(N-n), & n = N/2+1, \ldots, N-1, \;\; N = L/a \end{cases},$$

- Average over pos. and neg. y we get $\to \overline{\Pi}(n)$
- Finally transform back and introduce continuous momenta k:

$$\Pi(k) = \Big[\exp(ikn)\overline{\Pi}(n)\Big]_{n=0,\,N/2} + 2i\sum_{n=1}^{N/2-1}\overline{\Pi}(n)\sin\left(\frac{k}{2}\cdot(2n)\right)$$

- We define $\hat{k} \equiv 2\sin(\frac{k}{2})$ and $P_n(\hat{k}^2) = P_n((2\sin(\frac{k}{2}))^2) = \sin(nk)/\sin(\frac{k}{2})$ and obtain:

$$\Pi(\hat{k}) - \Pi(0) = i\sum_{n=1}^{N/2-1}\hat{k}\,P_n(\hat{k}^2)\overline{\Pi}(n)\,.$$

- $P_n(\hat{k}^2)$ is related to Chebyshev polynomials of the 2nd kind

4. Results on nEDM: Extraction of $F_3(0)$ via Momentum Elimination

- By applying the derivative in respect to $\hat{k}$:

$$\frac{F_3(\hat{k}^2)}{2m_N} = i \sum_{n=1}^{N/2-1} P_n(\hat{k}^2)\,\overline{\Pi}(n)\,,$$

- Generalize to off-axis momentum classes $M(q, q_{\rm off}^2) = \{\vec{q} \mid \vec{q} = \{\pm q, q_1, q_2\}\,,\; q_1^2 + q_2^2 = q_{\rm off}^2\}$ where $\{\pm q, q_1, q_2\}$ denotes all possible permutations of $\pm q$, q_1 and q_2
- To combine results for $F_3(Q^2)/(2m_N)$ for different momentum classes $q_{\rm off}$ and arrive at (Euclidean) $Q^2(\hat{k}, q_{\rm off}^2) = 0$: Analytic Continuation:

$$\begin{aligned} k &\to i\kappa \\ \hat{k} &\to i\hat{\kappa} = -2\sinh\left(\frac{\kappa}{2}\right) \\ P_n(\hat{k}^2) \to P_n\left(\hat{\kappa}^2\right) &= \sinh(n\kappa)/\sinh\left(\frac{\kappa}{2}\right) \end{aligned}$$

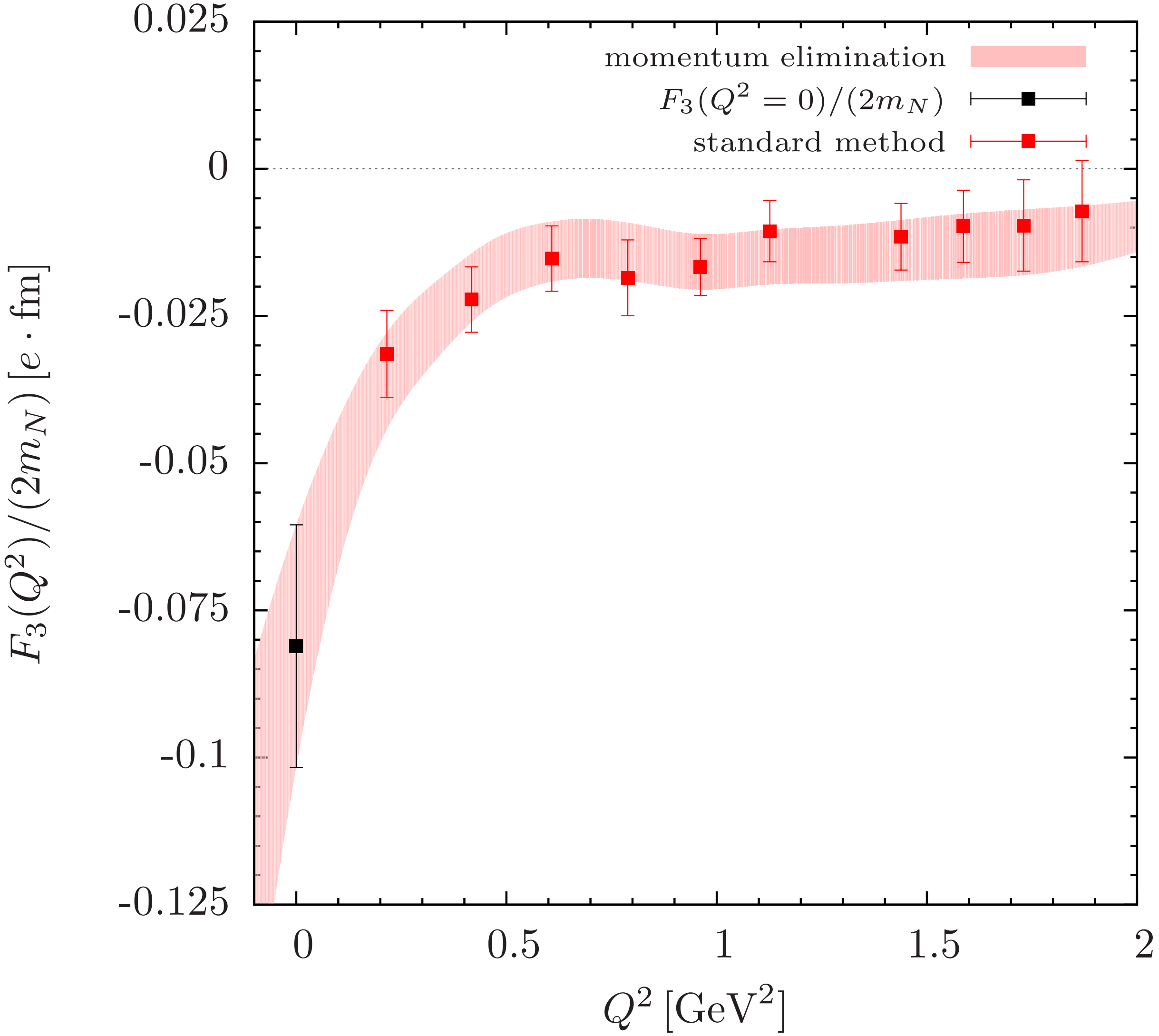
  Final formular takes similar form:

$$\frac{F_3(\hat{\kappa}^2)}{2m_N} = i \sum_{n=1}^{N/2-1} P_n(\hat{\kappa}^2)\,\overline{\Pi}(n)\,,$$

  Combine results from different sets of $M(q, q_{\rm off}^2)$ by taking the error weighted average.

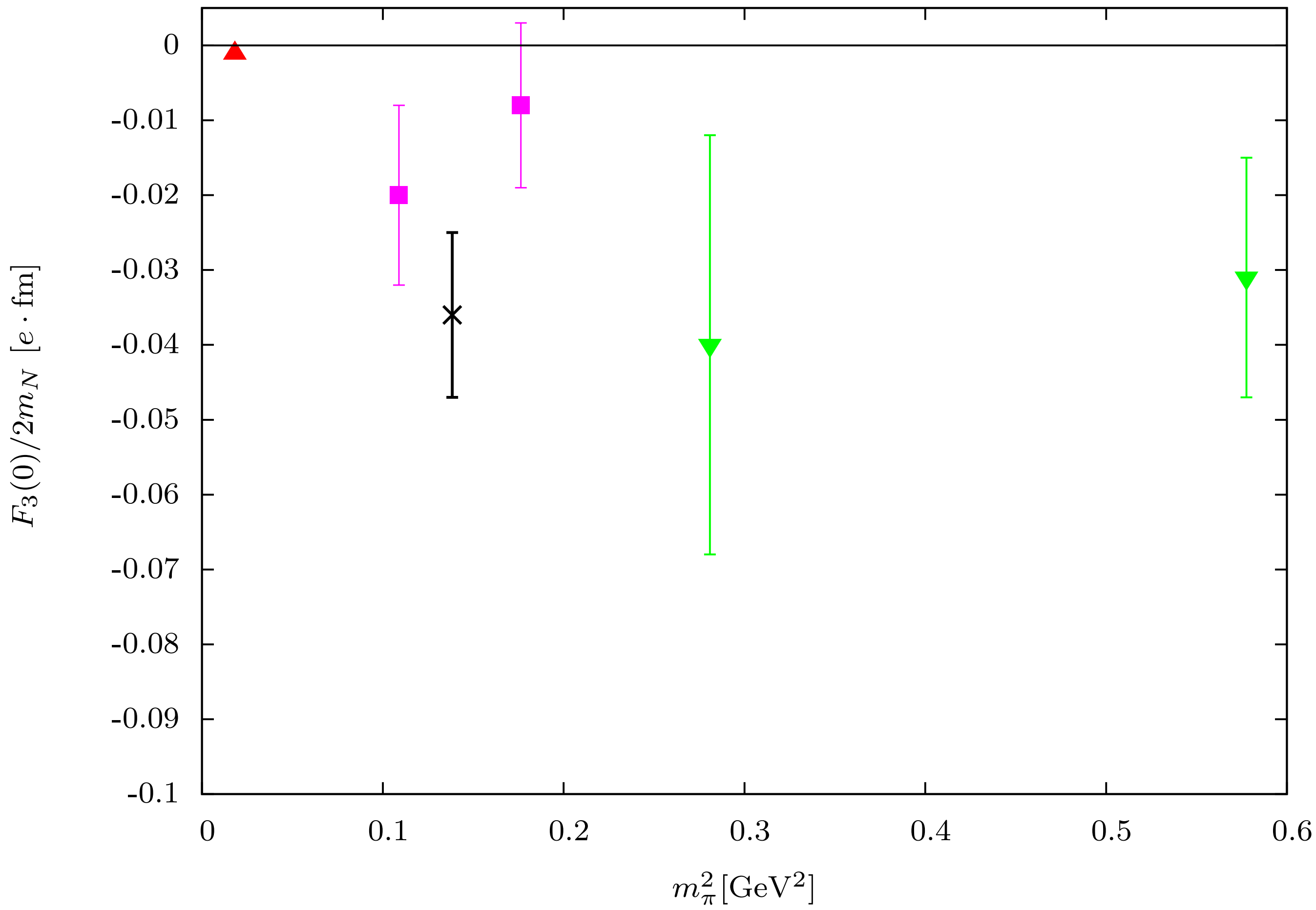
4. Results on nEDM: Extraction of $F_3(0)$ via Momentum Elimination

$F_3(Q^2)/2m_N$ via cooling ($n_c = 50$) for Iwasaki smoothing action extracted for $q^2_{\text{offmax}} = 5(2\pi/L)^2$



5. Conclusions

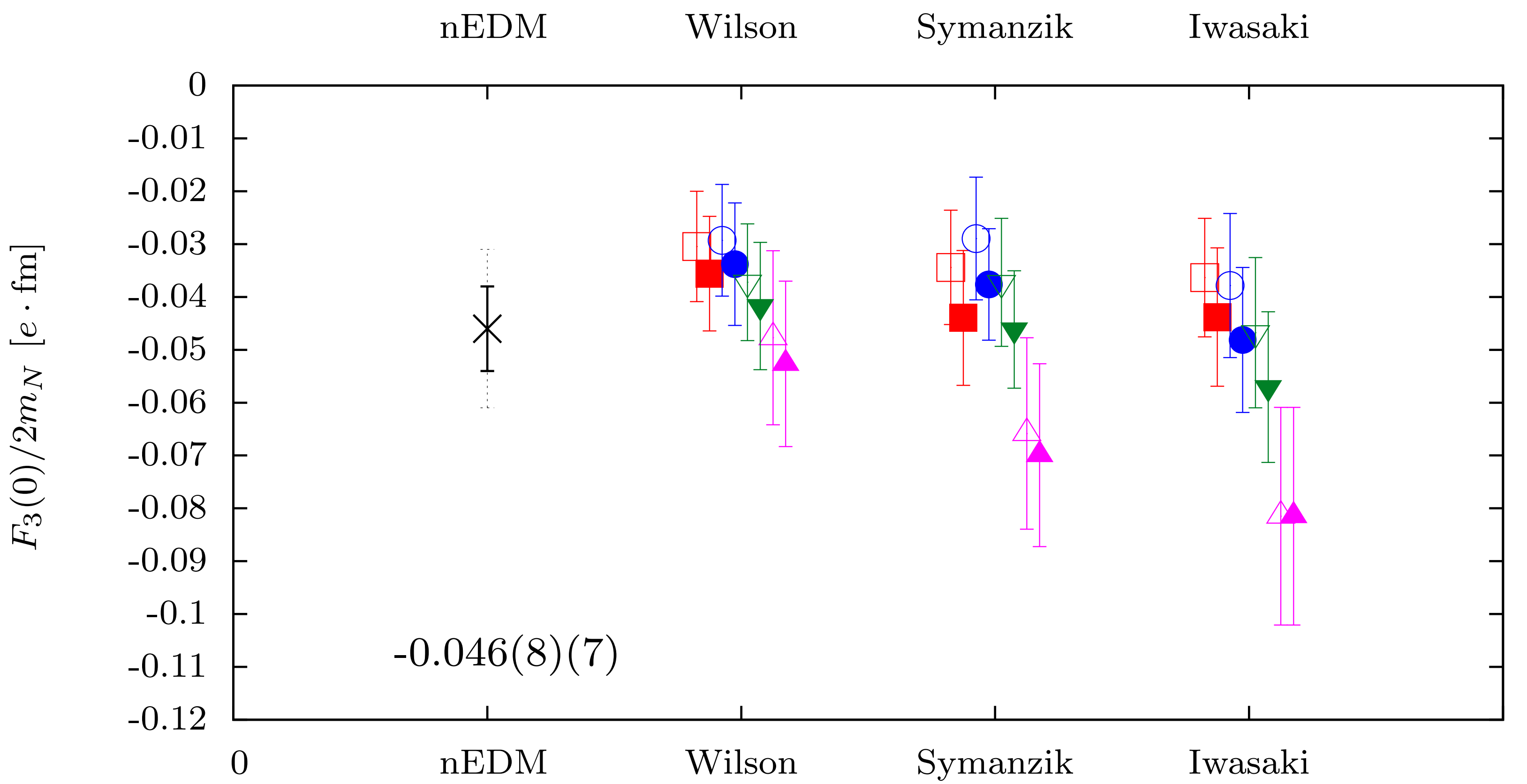
Our result (×) compare to other works:



(▲) Ottnad *et. al.* ChPT, [arXiv:0911.3981 [hep-ph]], (■) Shintani *et al*, PoS(LATTICE 2013)298, (▼) Shintani *et al* [arXiv:0803.0797].

5. Conclusions

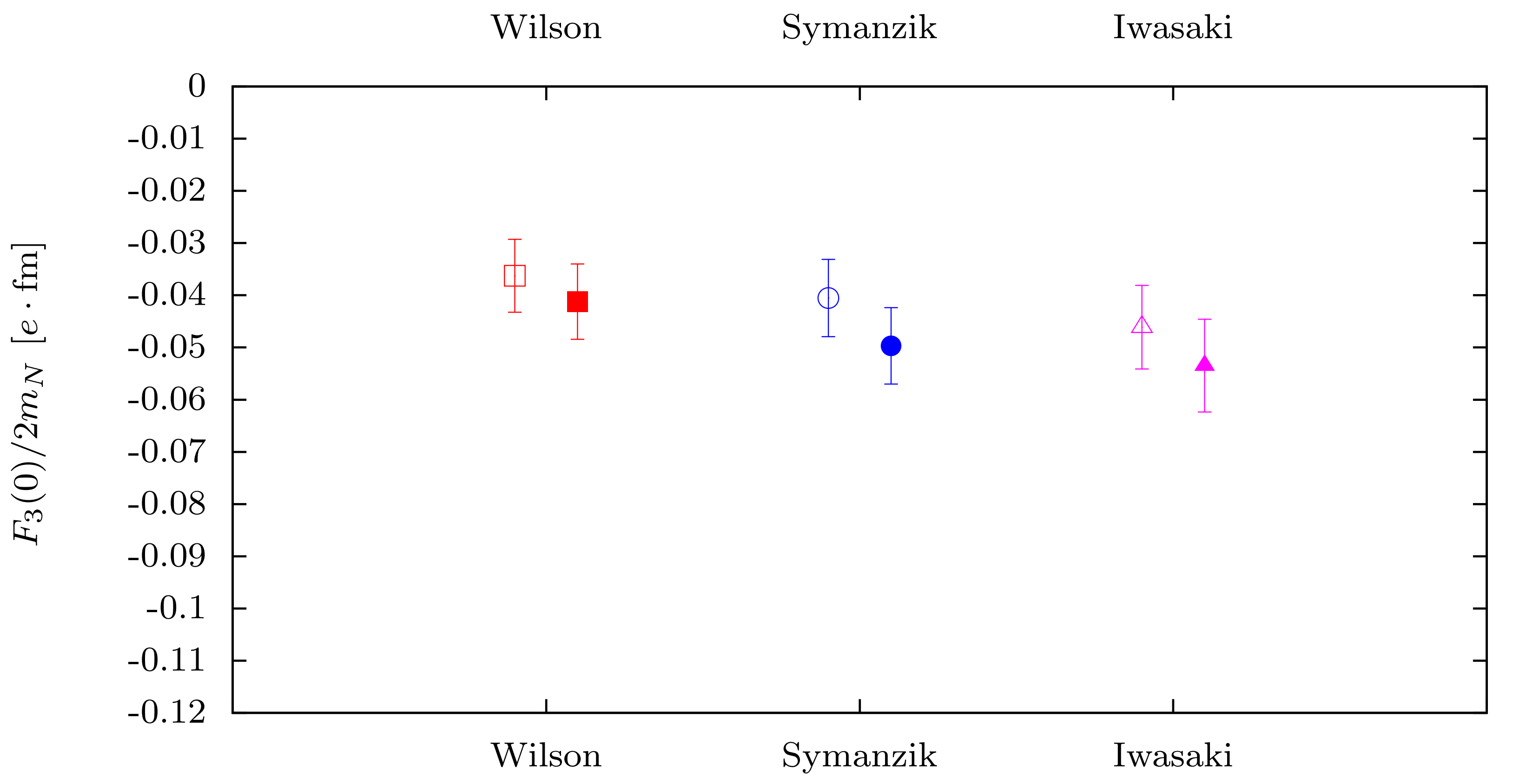
Different configurations of **momentum treatments**, **smoothers**, **smoothing actions**



- Open symbols: cooling, Filled symbols: gradient flow
- Dipole Fit, Continuum Derivative, $F_2(0)$ via dipole fit, Continuum Derivative, $F_2(0)$ via momentum elimination, Momentum Elimination

5. Conclusions

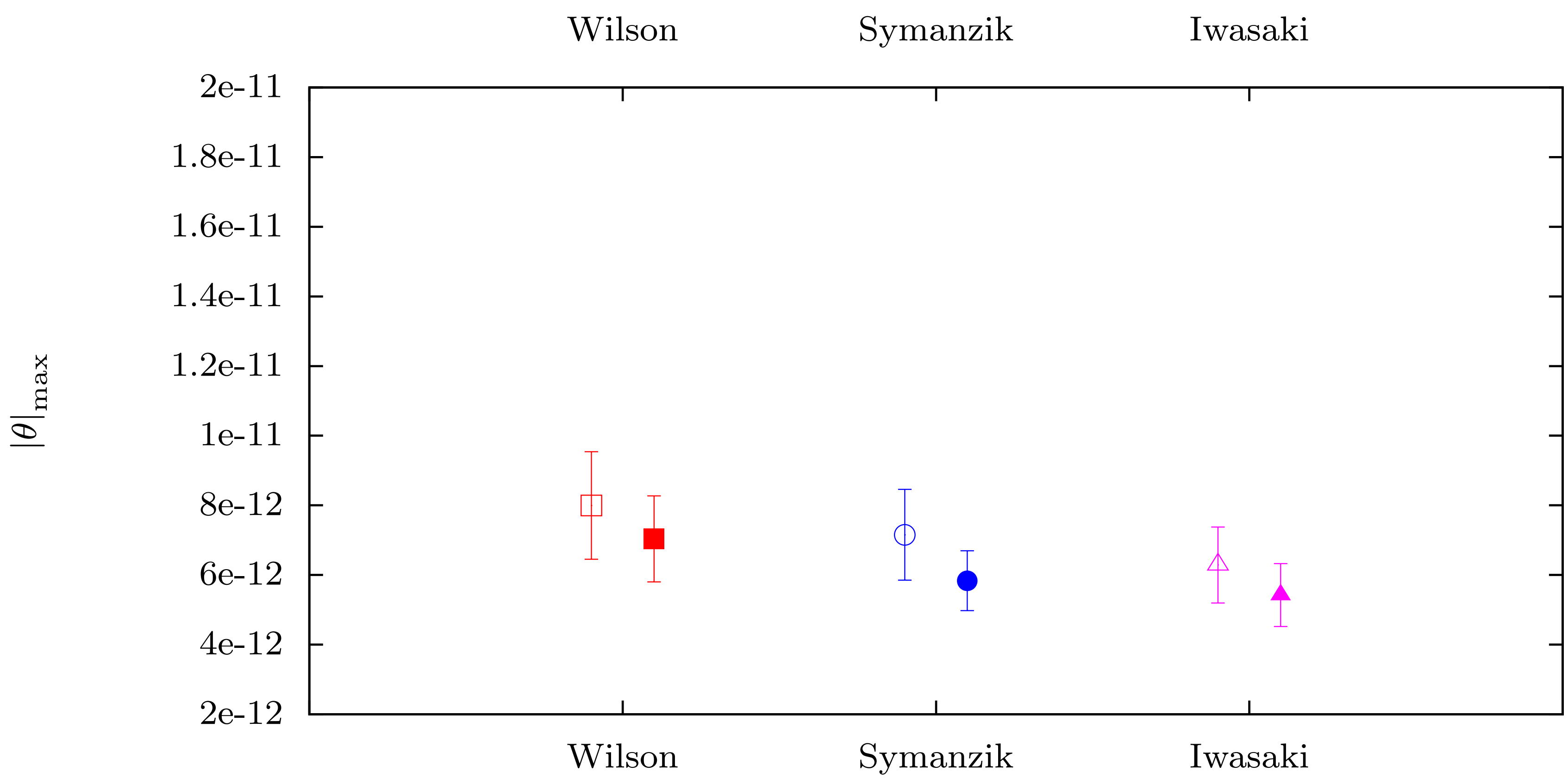
Different configurations of **smoothers**, **smoothing actions**



- Open symbols: cooling, Filled symbols: gradient flow

5. Conclusions

Now using $|\vec{d}_N| < 2.9 \times 10^{-13} e \cdot \text{fm}$ ...



- Open symbols: cooling, Filled symbols: gradient flow

5. Conclusions

- Calculate the nEDM for B55.32 at $M_\pi \simeq 370$ MeV.
- Implemented three momentum dependence treating techniques:
  - Dipole fit
  - Continuum Derivative
  - Momentum Elimination
- Implemented two smoothers and demonstrated equivalence
  - Cooling
  - Gradient Flow
- Used three smoothing actions:
  - Wilson
  - Symanzik tree-level improved
  - Iwasaki
- All combinations give similar results!
- Our result agrees with older estimations

THANK YOU!

Appendix 1: Topological charge

- Calculate the topological charge:

$$\mathcal{Q} = \int d^4x q(x)\,.$$

- On the lattice:

$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left\{F_{\mu\nu}F_{\rho\sigma}\right\}\,.$$

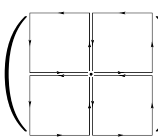
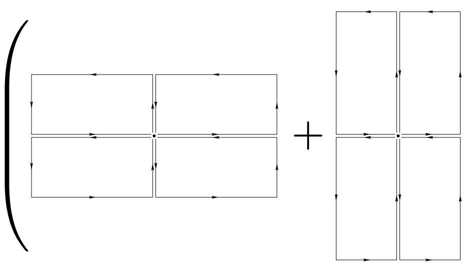
- Improved definition $O(a^4)$:

$$q(x) = c_0 q_L^{\mathrm{clov}}(x) + c_1 q_L^{\mathrm{rect}}(x)\,,$$

with $c_0 = 5/3$ and $c_1 = -1/12$ as well as

$$q_L^{\mathrm{clov}}(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(C_{\mu\nu}^{\mathrm{clov}}C_{\rho\sigma}^{\mathrm{clov}}\right) \quad \text{and} \quad q_L^{\mathrm{rect}}(x) = \frac{2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(C_{\mu\nu}^{\mathrm{rect}}C_{\rho\sigma}^{\mathrm{rect}}\right)\,,$$

where

$C_{\mu\nu}^{\mathrm{clov}}(x) = \frac{1}{4}\mathrm{Im}\Bigg($  $\Bigg)$ and $C_{\mu\nu}^{\mathrm{rect}}(x) = \frac{1}{8}\mathrm{Im}\Bigg($  $\Bigg)$.

- Smooth out ultraviolet fluctuations...

Appendix 2: Extraction of $F_3(0)$ via Momentum Elimination

- First use:

$$\Pi^{i,\theta=0}_{3\mathrm{pt}}\left(\Gamma_5\right) = -\frac{i}{2m_N}\sqrt{\frac{2m_N^2}{E_N(E_N+m_N)}}q_i\left(F_1\left(Q^2\right)+\frac{q^2}{4m_N^2}F_2\left(Q^2\right)\right),$$

$$\Pi^{i,\theta=0}_{3\mathrm{pt}}\left(\Gamma_k\right) = -\frac{1}{4m_N}\sqrt{\frac{2m_N^2}{E_N(E_N+m_N)}}\epsilon_{ijk}q_j\left(F_1\left(Q^2\right)+F_2\left(Q^2\right)\right),$$

- In order to define:

$$\Pi(q) \equiv \Pi^0_{3\mathrm{pt}}(\Gamma_k) + \frac{i}{2}\alpha_1\Pi^{i,\theta=0}_{3\mathrm{pt}}\left(\Gamma_5\right) + 2\alpha_1\Pi^{i,\theta=0}_{3\mathrm{pt}}\left(\Gamma_k\right) = \frac{q_k}{4}\sqrt{\frac{(E_N+m_N)}{2E_N m_N^2}}F_3(Q^2),$$

- On-axis momenta e.g. $\vec{q} = (\pm q, 0, 0)^T$
- Fourier transform on $\Pi(q) \rightarrow \Pi(y)$ in position space; with $(n = y/a)$

$$\Pi(y) = \begin{cases} +\Pi(n), & n = 0,\ldots,N/2 \\ -\Pi(N-n), & n = N/2+1,\ldots,N-1,\; N = L/a \end{cases},$$

- Average over pos. and neg. y we get $\rightarrow \overline{\Pi}(n)$

Appendix 2: Extraction of $F_3(0)$ via Momentum Elimination

- Finally transform back and introduce continuous momenta k:

$$\Pi(k) = \left[\exp(ikn)\overline{\Pi}(n)\right]_{n=0,\,N/2} + \sum_{n=1}^{N/2-1} \exp(ikn)\overline{\Pi}(n) + \sum_{n=N-1}^{N/2+1} \exp(ik(N-n))\overline{\Pi}(n)$$

$$= \left[\exp(ikn)\overline{\Pi}(n)\right]_{n=0,\,N/2} + 2i \sum_{n=1}^{N/2-1} \overline{\Pi}(n) \sin\left(\frac{k}{2}\cdot(2n)\right)$$

- We define $\hat{k} \equiv 2\sin\left(\frac{k}{2}\right)$ and $P_n\left(\hat{k}^2\right) = P_n\left(\left(2\sin\left(\frac{k}{2}\right)\right)^2\right) = \sin(nk)/\sin\left(\frac{k}{2}\right)$ and obtain:

$$\Pi(\hat{k}) - \Pi(0) = i \sum_{n=1}^{N/2-1} \hat{k}\, P_n\left(\hat{k}^2\right)\overline{\Pi}(n)\,.$$

- $P_n\left(\hat{k}^2\right)$ is related to Chebyshev polynomials of the 2nd kind

- By applying the derivative in respect to $\hat{k}$:

$$\frac{F_3(\hat{k}^2)}{2m_N} = i \sum_{n=1}^{N/2-1} P_n(\hat{k}^2)\,\overline{\Pi}(n)\,,$$

Appendix 2: Extraction of $F_3(0)$ via Momentum Elimination

- Can generalize to off-axis momentum classes

$$M(q, q_{\text{off}}^2) = \{\vec{q} \mid \vec{q} = \{\pm q, q_1, q_2\}\,,\ q_1^2 + q_2^2 = q_{\text{off}}^2\}$$

where $\{\pm q, q_1, q_2\}$ denotes all possible permutations of $\pm q$, q_1 and q_2

- To combine results for $F_3(Q^2)/(2m_N)$ for different momentum classes q_{off} and arrive at (Euclidean) $Q^2(\hat{k}, q_{\text{off}}^2) = 0$: Analytic Continuation:

$$\begin{aligned} k &\to i\kappa \\ \hat{k} &\to i\hat{\kappa} = -2\sinh\left(\frac{\kappa}{2}\right) \\ P_n(\hat{k}^2) \to P_n\left(\hat{\kappa}^2\right) &= \sinh(n\kappa)/\sinh\left(\frac{\kappa}{2}\right) \end{aligned}$$

Final formular takes similar form:

$$\frac{F_3(\hat{\kappa}^2)}{2m_N} = i \sum_{n=1}^{N/2-1} P_n(\hat{\kappa}^2)\, \overline{\Pi}(n)\,,$$

Combine results from different sets of $M(q, q_{\text{off}}^2)$ by taking the error weighted average.