A lattice study of the nucleon quark content at the physical point

Christian Torrero
for the Budapest-Marseille-Wuppertal (BMW) collaboration

Centre de Physique Théorique
Aix-Marseille Université

XXXIII International Symposium on Lattice Field Theories

Kobe - July 16, 2015

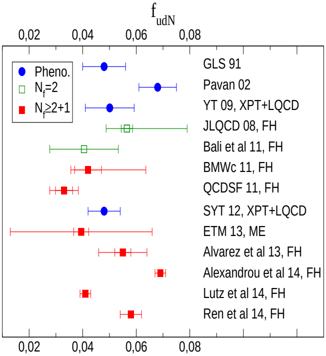
Outline

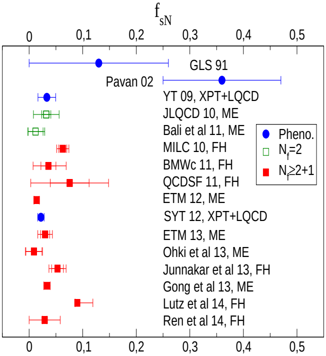
The nucleon up-down and strange quark content f_{udN} and f_{sN} are observables of great interest given their relation to

- the quark-mass ratio m_{ud}/m_s;
- $\pi - N$ and $K - N$ scattering;
- counting rates in Higgs-Boson searches;
- direct detection of dark matter (DM).

Estimates from phenomenology do not agree with each other and have large uncertainties $\longrightarrow$ **need for ab-initio computations of strong-interaction effects**.

Computations already exist (FH = Feynman-Hellmann, ME = Matrix Element):





However, most calculations employ model assumptions and/or have incomplete error analyses.

The nucleon up-down and strange quark content f_{udN} and f_{sN} are defined as

$$f_{udN} \equiv m_{ud} \frac{\langle N|\bar{u}u + \bar{d}d|N\rangle}{M_N} , \qquad f_{sN} \equiv m_s \frac{\langle N|\bar{s}s|N\rangle}{M_N} .$$

A possible strategy to compute them consists of relying on the Feynman-Hellmann theorem, i.e.,

$$f_{udN} = \frac{m_{ud}}{M_N} \frac{\partial M_N}{\partial m_{ud}}\bigg|_{\Phi} , \qquad f_{sN} = \frac{m_s}{M_N} \frac{\partial M_N}{\partial m_s}\bigg|_{\Phi} ,$$

where derivatives have to be computed at the physical point (Φ).

Main advantages of this approach:

- need for 2-point functions only;
- no disconnected contributions.

Some technical details:

- tree-level improved Symanzik gauge action (S_G) and clover-improved Wilson action (S_F);
- 2-HEX link-smearing in S_F;
- $N_f = 2 + 1$;
- 47 ensembles corresponding to about 13000 overall configurations with
  - $0.054\ fm \lesssim a \lesssim 0.116\ fm$;
  - pion mass M_π down to $\lesssim 120\ MeV$;
  - box size up to $\approx 6\ fm$;
- full non-perturbative renormalization of quark masses in the Renormalization Group Invariance (RGI) scheme (as in BMWc, JHEP 1108).

As it is well known, the mass of a given particle p can be extracted from a time correlator $C_p(t, t_S)$

$$C_p(t, t_S) = a^3 \sum_{\vec{x}} \langle O_p(x) O_p^\dagger(x_S) \rangle \ ,$$

with a being the lattice spacing, t the time component of point $x = (t, \vec{x})$ in the $4D$ discretized spacetime and where $O_p(x)$ is an interpolating operator capable of creating a hadron p out of the vacuum.

Assessing when the asymptotics sets in is a non-trivial problem, usually solved by relying on eye judgement and/or by looking for a good fit quality.

We have introduced a more systematic method which consists of carrying out the Kolmogorov-Smirnov (KS) test, quantifying the distance D between the empirical cumulative distribution function (ECDF) $F_n(x)$ and the reference cumulative distribution function $F(x)$. More precisely,

$$D = sup_x \left| F_n(x) - F(x) \right| \ .$$

In the present case, starting from the mass fits on the 47 ensembles, the KS test is carried out — for each particle — on the cumulative distribution of the fit qualities (i.e., x in the formula above corresponds to the p-value): the distance D is then used to define a significance level P that the measured distribution originates from the uniform one.

The KS test is repeated for different starting time slices t_{min} and varying plateau length L till $P > 0.3$.

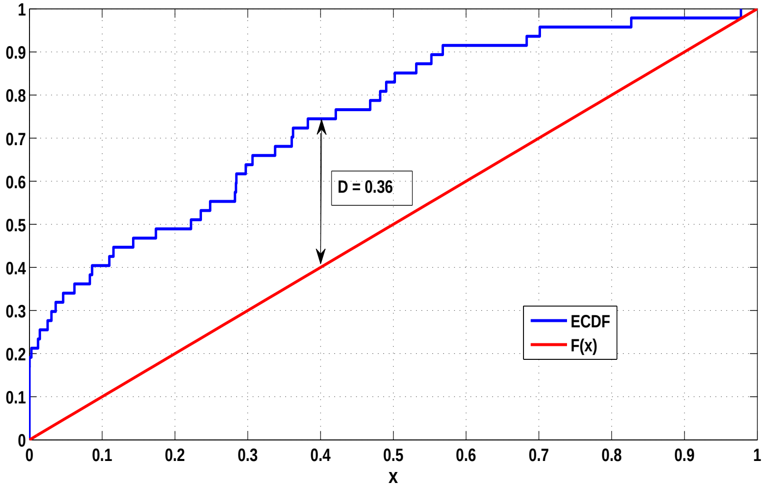


Figure 1: comparison between ECDF and $F(x)$ for the fits of M_Ω with $t_{min} = 0.8$ fm, $t_{max} \in [1.0, 1.4]$ fm, $L = 6$, $D = 0.36$ and $P = 5 \cdot 10^{-6}$.

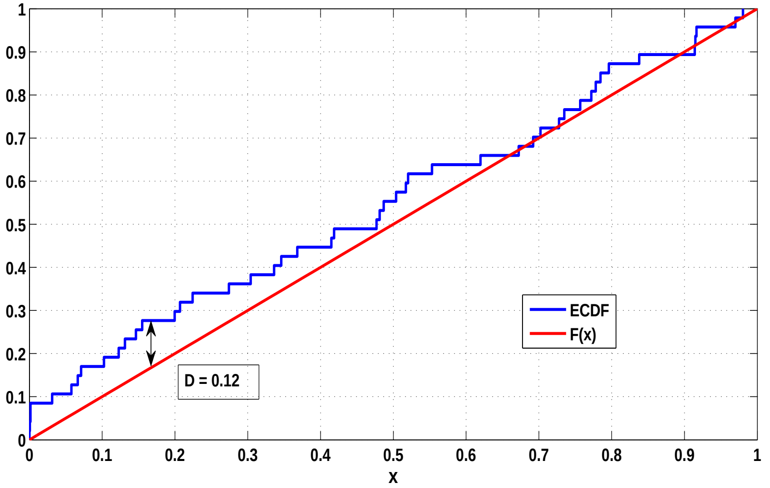


Figure 2: same as Fig. 1 but with $t_{min} = 1.0$ fm, $t_{max} \in [1.2, 1.6]$ fm, $L = 6$, $D = 0.12$, $P = 0.47$.

An example of functional form — with **experimental input** and **fit parameters** — employed in the fit reads

$$aM_X = a\left\{ M_X^{(\Phi)} + \sum_i c_{X,ud,i}\left[\frac{am_{ud}Z_s^{-1}(\beta)}{a(1+d_{ud}a^2)} - m_{ud}^{(\Phi)}\right]^i + \sum_j c_{X,s,j}\left[\frac{am_sZ_s^{-1}(\beta)}{a(1+d_sa^2)} - m_s^{(\Phi)}\right]^j \right\},$$

with $X = \Omega$ (for scale setting), N, π and K^χ with $m_{K^\chi} = m_K^2 - \frac{1}{2}m_\pi^2$. $M_N^{(\Phi)}$ was actually also considered as a fit parameter.

The masses of these four particles are fitted at the same time, i.e. the corresponding functionals share the same fit parameters - with the exception of the $c_{X,ud,i}$'s and $c_{X,s,i}$'s.

Quark masses in the functional above are obtained through the ratio-difference method (BMWc, JHEP 1108).

Fit parameters $c = \{a, m_{ud}^{(\phi)}, m_s^{(\phi)}, \dots\}$ of functions $f^{(i)}(c,x)$ — with $i = 1,2,3,4$ and $x = \{am_{ud}, am_s\}$ — are determined by minimizing a χ^2 function defined as

$$\chi^2 = V^T C^{-1} V \ ,$$

where C is the covariance matrix associated to the entries of the column vector V whose structure reads

$$V = (y_1^{(1)} - \ f^{(1)}(c,x_1)\ , \ \dots \ , \ y_n^{(4)} - \ f^{(4)}(c,x_n)\ , \ x_1 - q_1\ , \ x_2 - q_2\ , \ \dots\ , \ x_n - q_n)\ ,$$

where q_i is the value of variable x_i obtained in simulation i.

Entries of matrix C are obtained via a bootstrap procedure with $n_{boot} = 2000$.

All fits are correlated.

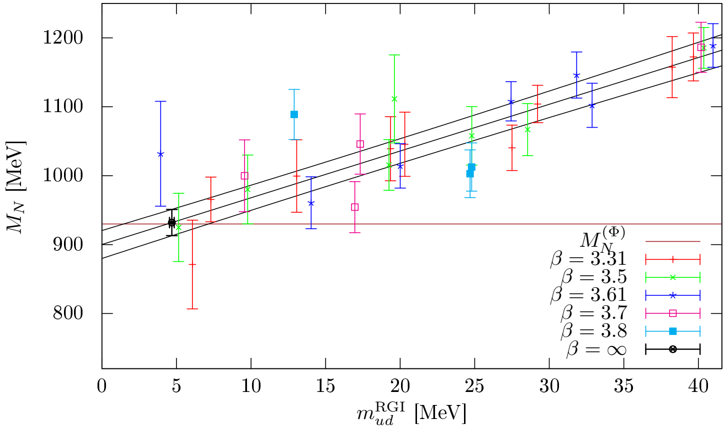


Figure 3: typical dependence of M_N vs. m_{ud}^{RGI}. The black point corresponds to the physical point while the horizontal line in brown to $M_N^{(\Phi)}$.

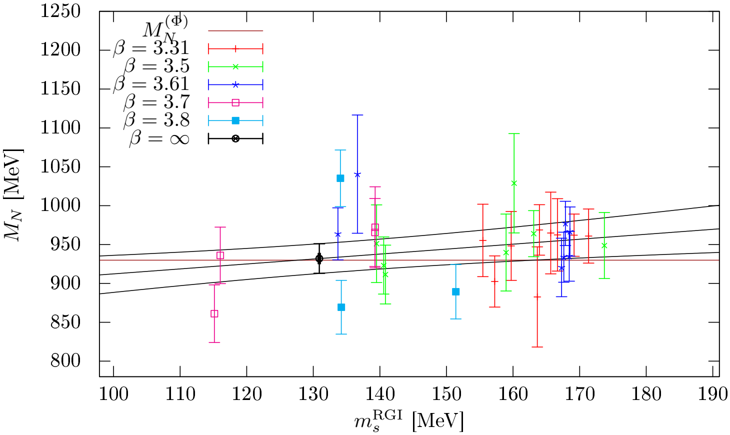


Figure 4: same as Fig. 3 but vs. m_s^{RGI}.

To estimate the systematic uncertainties on results, different strategies have been considered in the fitting procedure:

- choosing two different time intervals for the asymptotic behaviour of $C_p(t, 0)$;
- pruning the data with two cuts in the pion mass (at 380 MeV and 480 MeV);
- taking into account six different procedures in computing Z_S (as in BMWc, JHEP 1108);
- considering different parametrizations for m_{ud} and m_s dependence of baryons (polynomials vs. Padé);
- allowing for different cutoff effects, i.e.

$$\frac{am_{ud}Z_s^{-1}(\beta)}{a(1+d_{ud}a^2)} \longrightarrow \frac{am_{ud}Z_s^{-1}(\beta)}{a(1+d_{ud}\alpha_s a)} .$$

This results in $2 \cdot 2 \cdot 6 \cdot 2 \cdot 2 = \mathbf{96}$ fitting strategies altogether.

The systematics is subsequently evaluated by means of the **Akaike Information Criterion**, i.e. for the i^{th} fitting procedure the AIC value AIC_i

$$AIC_i = 2k_i - 2ln(L_i) = 2k_i + \chi^2 \ ,$$

is computed, being k_i the number of fit parameters and L_i the value of the likelihood function at its minimum.

The mean value and systematic error of a generic fit parameter c_i are obtained by computing, respectively, the weighted mean standard deviation of the values of c_i resulting from the different fitting procedures with the weight ω_i given by

$$\omega_i = exp[(AIC_{min} - AIC_i)/2] \ ,$$

where AIC_{min} is the lowest of the AIC_i's.

The bootstrap error on the mean provides the statistical error.

Latest results read

$$f_{udN} = 0.0386(19)(7) \; , \qquad f_{sN} = 0.103(44)(2) \; ,$$

while the estimate obtained for M_N is given by

$$M_N = 942(13)(2) \text{ MeV} \; ,$$

in agreement with the experimental value $M_N = 938.9 \text{ MeV}$.

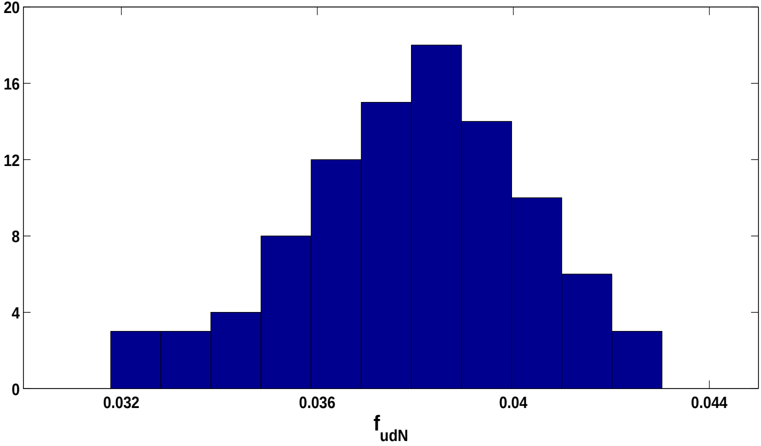


Figure 5: AIC-weighted distribution of the values of f_{udN} combining statistical and systematic error.

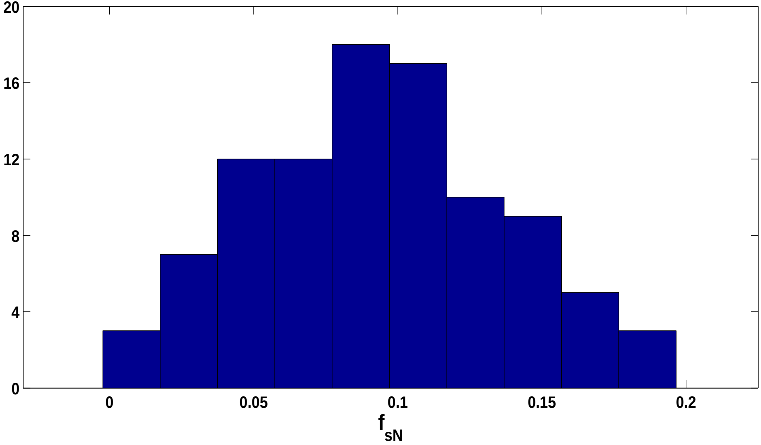


Figure 6: same as Fig. 5 but for f_{sN}.

Conclusions

- a complete investigation of different sources of systematic uncertainties has been carried out;
- errorbars are still a bit large, at least on f_{sN}.

Outlook

- more lever arm on m_s and smaller statisticl errors are needed to improve the precision on f_{sN};
- work on spin-dependent coupling has begun.