Dirac spectrum representation of Polyakov loop fluctuations in lattice QCD

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in collaboration with

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Chihiro Sasaki (Wroclaw University & FIAS)

reference

"Polyakov loop fluctuations in Dirac eigenmode expansion,"
TMD, K. Redlich, C. Sasaki and H. Suganuma, arXiv: 1505.05752 [hep-lat]

Lattice 2015, July 17, 2015, Kobe

Contents

- Introduction
  - Quark confinement
  - Chiral symmetry breaking
- Previous works
  - QCD phase transition at finite temperature
  - Dirac spectrum representation of the Polyakov loop
- Our work

  Analytical part
  - Polyakov loop fluctuations
  - Dirac spectrum representation of the Polyakov loop fluctuations

  Numerical part
  - Numerical analysis for each Dirac-mode contribution to the Polyakov loop fluctuations

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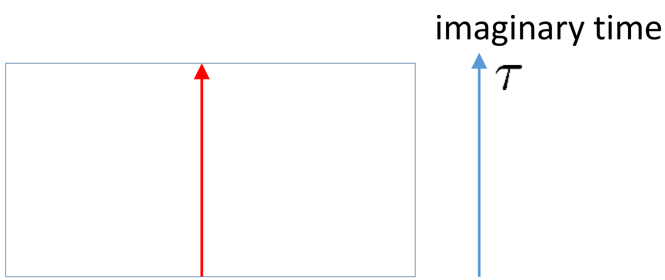
Introduction – Quark confinement

Confinement : colored state cannot be observed (quark, gluon, ･･･)
only color-singlet states can be observed (meson, baryon, ･･･)

Polyakov loop : order parameter for quark deconfinement phase transition

$$L_P(\boldsymbol{x}) = \mathrm{tr}\mathrm{T}e^{ig\int_0^\beta d\tau A_4(\boldsymbol{x},\tau)}$$ in continuum theory

$$= \mathrm{tr}\prod_{s_4=1}^{N_4} U_4(\boldsymbol{s}, s_4)$$ in lattice theory



Finite temperature :
(anti) periodic boundary condition for time direction

$$\langle L_P \rangle = \frac{1}{V}\sum_{\boldsymbol{x}} \langle L_P(\boldsymbol{x}) \rangle$$:Polyakov loop

F_q :free energy of the system with a single static quark

$$= e^{-\beta F_q} \begin{cases} = 0 & (F_q = \infty,\ \text{confinement phase}) \\ \neq 0 & (F_q : \text{finite},\ \text{deconfinement phase}) \end{cases}$$

Introduction – Chiral Symmetry Breaking

・Chiral symmetry breaking : chiral symmetry is spontaneously broken

$$\mathrm{SU(N)_L \times SU(N)_R} \underset{\text{CSB}}{\to} \mathrm{SU(N)_V}$$

for example $\mathrm{SU(2)}$

・u, d quarks get dynamical mass(constituent mass)

・Pions appear as NG bosons

・Chiral condensate : order parameter for chiral phase transition

$$\langle \bar{q}q \rangle \begin{cases} \neq 0 & \text{(chiral broken phase)} \\ = 0 & \text{(chiral restored phase)} \end{cases}$$

・Banks-Casher relation

$$\langle \bar{q}q \rangle = - \lim_{m \to 0} \lim_{V \to \infty} \pi \langle \rho(0) \rangle$$

$\hat{\not{D}}$:Dirac operator

$\hat{\not{D}}|n\rangle = i\lambda_n|n\rangle$:Dirac eigenvalue equation

$\rho(\lambda) = \frac{1}{V}\sum_n \delta(\lambda - \lambda_n)$:Dirac eigenvalue density

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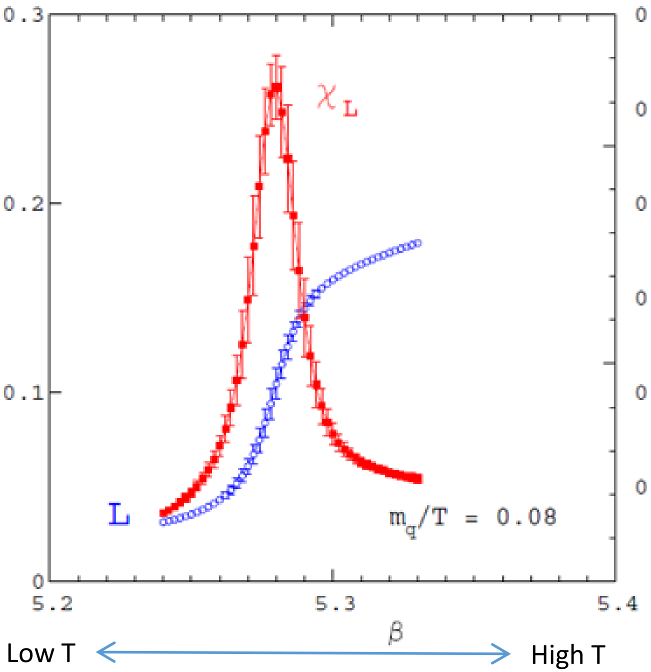
QCD phase transition at finite temperature

F. Karsch, Lect. Notes Phys. 583, 209 (2002)

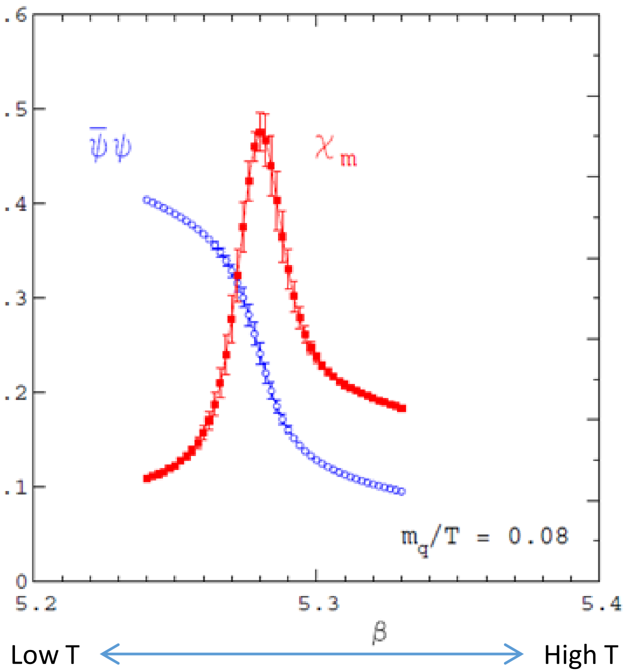
$\langle L \rangle, \chi_L$: Polyakov loop and its susceptibility

$\langle \bar{\psi}\psi \rangle, \chi_m$: chiral condensate and its susceptibility

deconfinement transition



chiral transition



- $\mu = 0$
- two flavor QCD with light quarks

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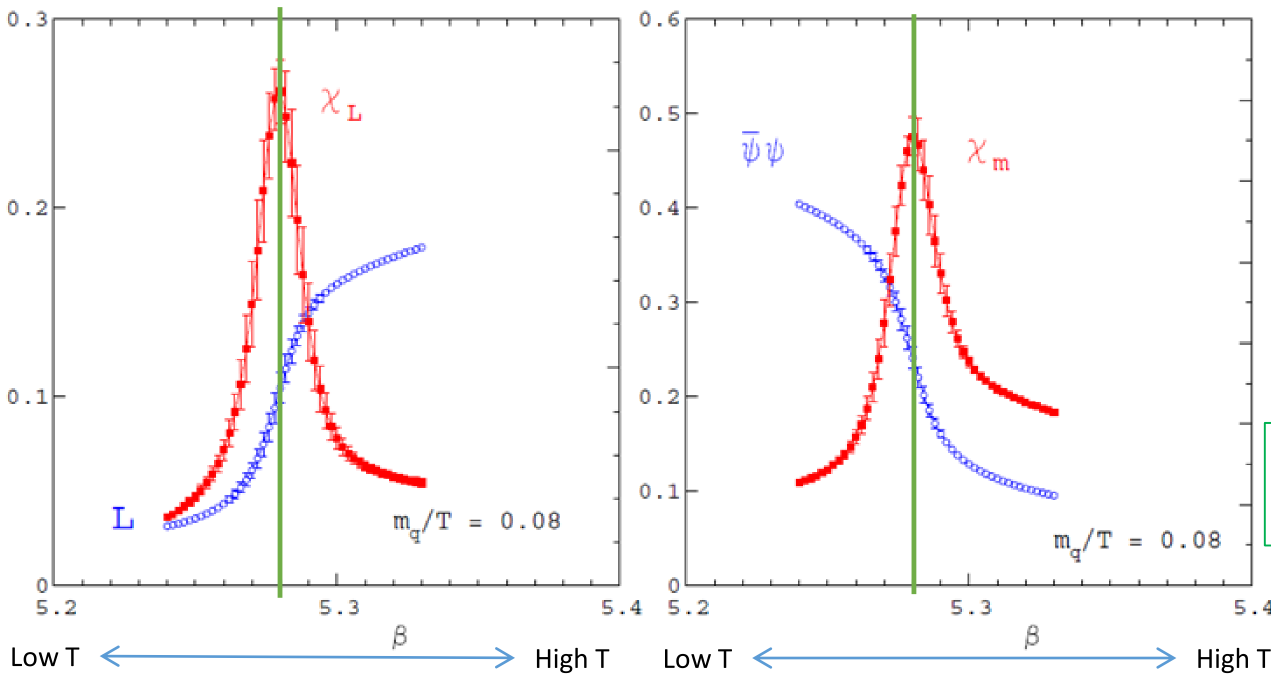

- $\mu = 0$
- two flavor QCD with light quarks

We define critical temperature as the peak of susceptibility

$\Longrightarrow$ These two phenomena are strongly correlated(?)

An analytical relation between Polyakov loop and Dirac mode on temporally odd-number lattice

TMD, H. Suganuma, T. Iritani, Phys. Rev. D 90, 094505 (2014).
H. Suganuma, TMD, T. Iritani, arXiv: 1404.6494 [hep-lat].

$$L = -\frac{(2ai)^{N_4-1}}{12V} \sum_n \lambda_n^{N_4-1} \langle n|\hat{U}_4|n\rangle$$ on temporally odd number lattice: N_4 is odd

notation:

- Polyakov loop : L
- link variable operator : $\langle s|\hat{U}_\mu|s'\rangle = U_\mu(s)\delta_{s+\hat{\mu},s'}$

  with anti p.b.c. for time direction: $\langle N_\tau, \mathbf{x}|\hat{U}_4|1,\mathbf{x}\rangle = -U_4(N_\tau,\mathbf{x})$
- Dirac eigenmode : $\hat{\not{D}}|n\rangle = i\lambda_n|n\rangle$

  Dirac operator : $\hat{\not{D}} = \frac{1}{2a}\sum_\mu \gamma_\mu(\hat{U}_\mu - \hat{U}_{-\mu})$ $\quad \sum_n |n\rangle\langle n| = 1$

properties :

- valid on only temporally odd-number lattice
- This formula is valid in full QCD and at the quenched level.

$$Z = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}U \mathrm{e}^{-S_{\mathrm{G}}[U]+\bar{q}K[U]q} = \int \mathcal{D}U \mathrm{e}^{-S_{\mathrm{G}}[U]}\mathrm{det}K[U]$$

We consider full QCD and gauge-configurations generated in MC simulation.
This formula holds for each gauge-configurations {U}
and for arbitrary fermionic kernel K[U]

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- Low-lying Dirac-modes are important for CSB (Banks-Casher relation) $(\lambda_n \sim 0)$
- Low-lying Dirac-modes have little contribution to Polyakov loop

The relation suggests **no one-to-one correspondence between Confinement and CSB in QCD.**

This conclusion agrees with the previous work by Gongyo, Iritani, Suganuma.

S. Gongyo, T. Iritani, H. Suganuma, PRD86 (2012) 034510
T. Iritani and H. Suganuma, PTEP, 2014 3, 033B03 (2014).

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The relation suggests **no one-to-one correspondence between Confinement and CSB in QCD.**

We derived the similar relation between Wilson loop and Dirac mode.
Therefore, low-lying Dirac-modes have little contribution
to the string tension σ, or the confining force.

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Polyakov loop fluctuations

P.M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki,
Phys. Rev. D88, 014506 (2013); Phys. Rev. D88, 074502 (2013)

- Polyakov loop: $L \equiv \frac{1}{N_c V} \sum_s \mathrm{tr}_c \{ \prod_{i=0}^{N_\tau - 1} U_4(s + i\hat{4}) \}$

- Z3 rotated Polyakov loop: $\tilde{L} = L e^{2\pi k i/3}$

- longitudinal Polyakov loop: $L_L \equiv \mathrm{Re}(\tilde{L})$
- Transverse Polyakov loop: $L_T \equiv \mathrm{Im}(\tilde{L})$

- Polyakov loop susceptibilities:

$$T^3 \chi_A = \frac{N_\sigma^3}{N_\tau^3} [\langle |L|^2 \rangle - \langle |L| \rangle^2],$$

$$T^3 \chi_L = \frac{N_\sigma^3}{N_\tau^3} [\langle (L_L)^2 \rangle - \langle L_L \rangle^2],$$

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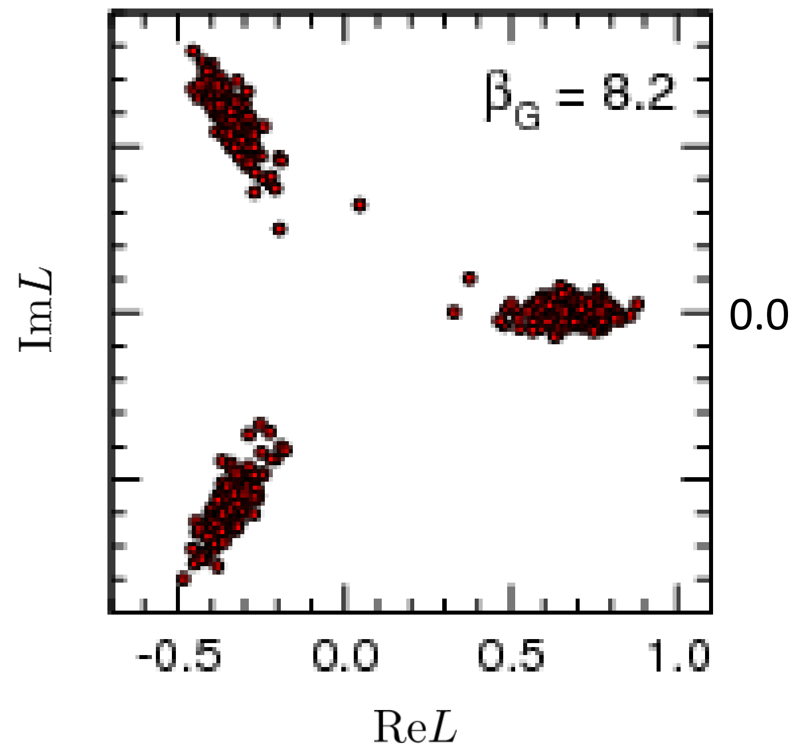
- Ratios of Polyakov loop susceptibilities:

$$R_A \equiv \frac{\chi_A}{\chi_L}, \qquad R_T \equiv \frac{\chi_T}{\chi_L}$$

An example for the Polyakov loops

C. Gattringer et al., Phys.Lett. B697 (2011) 85



T : temperature

$N_\sigma, \ N_\tau$: spatial and temporal lattice size

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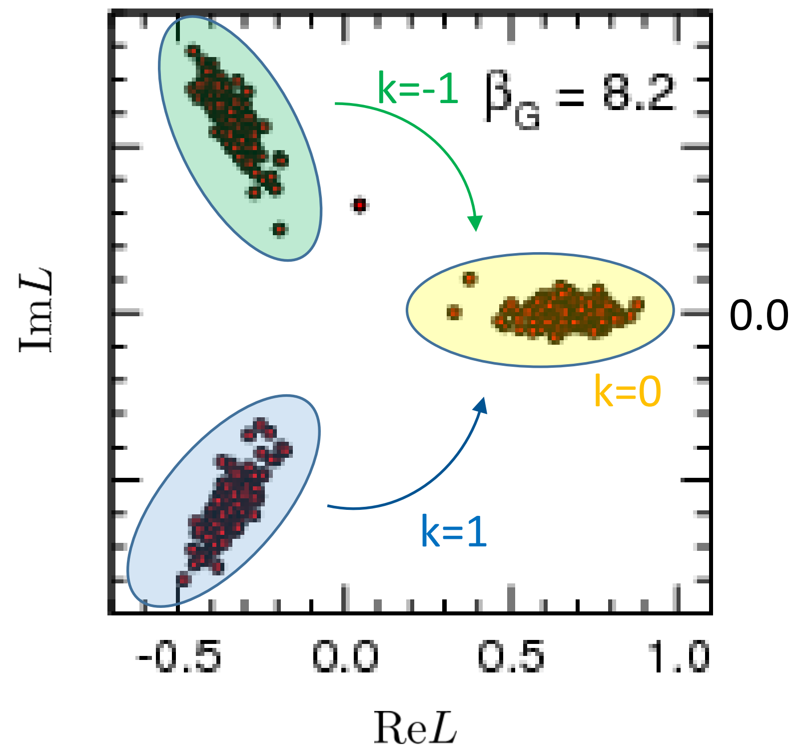

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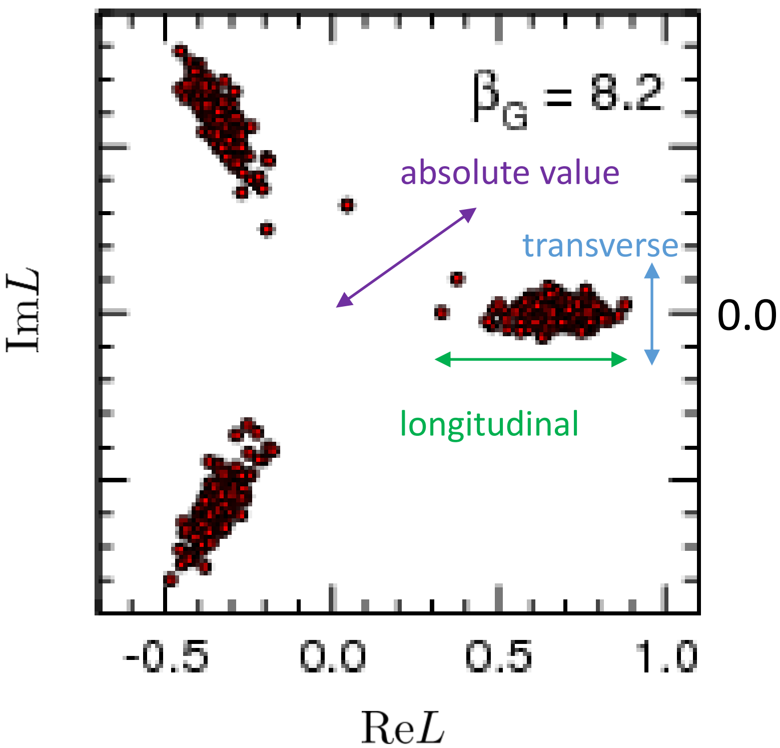

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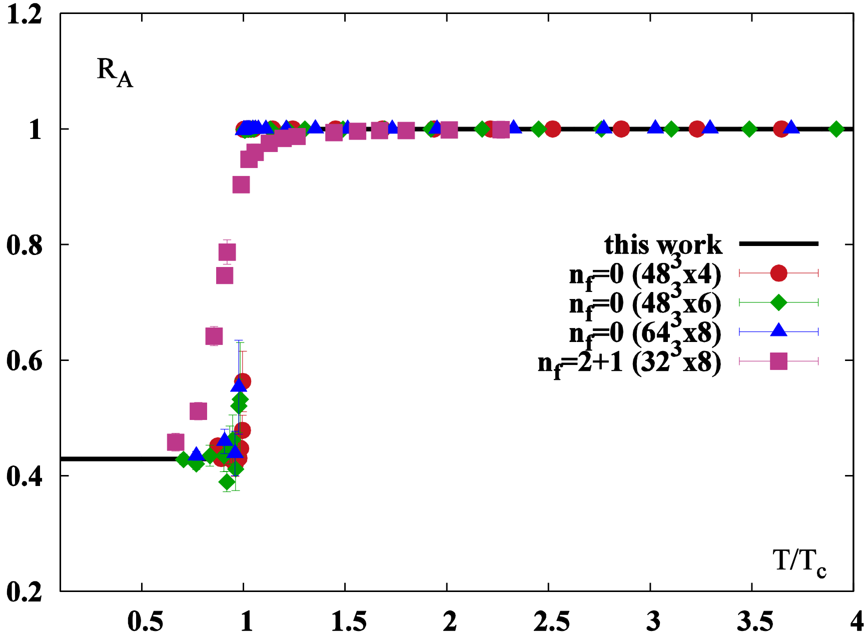
Polyakov loop fluctuations as sensitive probe

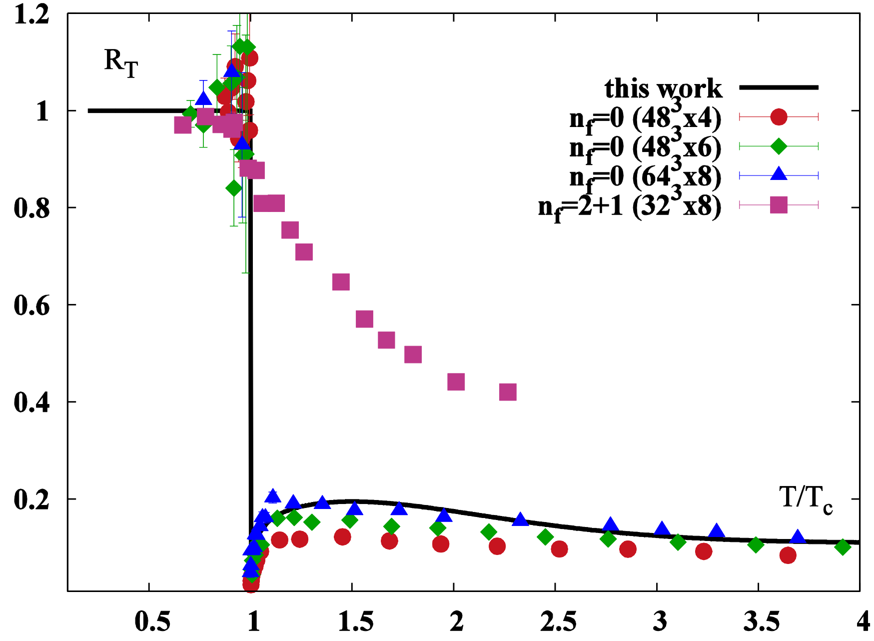
P.M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki,
Phys. Rev. D88, 014506 (2013); Phys. Rev. D88, 074502 (2013)

(R_A is) sensitive probe for deconfinement transition

$$R_A \equiv \frac{\chi_A}{\chi_L}, \qquad R_T \equiv \frac{\chi_T}{\chi_L}$$

※nf=0: quenched level
nf=2+1: (2+1)flavor full QCD(near physical point)





R_A is a good probe for deconfinement transition
even if considering dynamical quarks.

Dirac spectrum representation of the Polyakov loop fluctuations

TMD, K. Redlich, C. Sasaki and H. Suganuma, arXiv: 1505.05752 [hep-lat]

Definition of the Polyakov loop fluctuations

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Dirac spectrum representation of the Polyakov loop

$$L = -\frac{(2ai)^{N_4 - 1}}{12V} \sum_n \lambda_n^{N_4 - 1} \langle n | \hat{U}_4 | n \rangle$$

Polyakov loop : L

Dirac eigenmode : $\hat{\not{D}} |n\rangle = i\lambda_n |n\rangle$

link variable operator : $\langle s | \hat{U}_\mu | s' \rangle = U_\mu(s) \delta_{s+\hat{\mu}, s'}$

combine

Dirac spectrum representation of the Polyakov loop fluctuations

For example,

$$L_L = -\frac{(2ai)^{N_\tau - 1}}{12V} \sum_n \lambda_n^{N_\tau - 1} \mathrm{Re} \left(e^{2\pi k i/3} \langle n | \hat{U}_4 | n \rangle \right)$$

and...

Dirac spectrum representation of the Polyakov loop fluctuations

TMD, K. Redlich, C. Sasaki and H. Suganuma, arXiv: 1505.05752 [hep-lat]

For example, the ratio R_A can be represented using Dirac modes:

$$R_A = \frac{\left\langle \left| \sum_n \lambda_n^{N_\tau - 1} \langle n|\hat{U}_4|n\rangle \right|^2 \right\rangle - \left\langle \left| \sum_n \lambda_n^{N_\tau - 1} \langle n|\hat{U}_4|n\rangle \right| \right\rangle^2}{\left\langle \left(\sum_n \lambda_n^{N_\tau - 1} \mathrm{Re}\left(\mathrm{e}^{2\pi k i/3} \langle n|\hat{U}_4|n\rangle \right) \right)^2 \right\rangle - \left\langle \sum_n \lambda_n^{N_\tau - 1} \mathrm{Re}\left(\mathrm{e}^{2\pi k i/3} \langle n|\hat{U}_4|n\rangle \right) \right\rangle^2}$$

Note 1: The ratio R_A is a good “order parameter” for deconfinement transition.

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Note 1: The ratio R_A is a good "order parameter" for deconfinement transition.

Note 2: Since the damping factor $\lambda_n^{N_\tau - 1}$ is very small with small $|\lambda_n| \simeq 0$,
low-lying Dirac modes (with small $|\lambda_n| \simeq 0$) are not important for R_A,
which are important modes for chiral symmetry breaking.

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Thus, the essential modes for chiral symmetry breaking in QCD are not important to quantify the Polyakov loop fluctuation ratios, which are sensitive observables to confinement properties in QCD.

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which are important modes for chiral symmetry breaking.

Thus, the essential modes for chiral symmetry breaking in QCD are not important to quantify the Polyakov loop fluctuation ratios, which are sensitive observables to confinement properties in QCD.

This result suggests that there is no direct, one-to-one correspondence between confinement and chiral symmetry breaking in QCD.

Introduction of the Infrared cutoff for Dirac modes

TMD, K. Redlich, C. Sasaki and H. Suganuma, arXiv: 1505.05752 [hep-lat]

Define Λ-dependent (IR-cut) susceptibilities:

$$(\chi)_\Lambda = \frac{1}{T^3}\frac{N_\sigma^3}{N_\tau^3}[\langle Y_\Lambda^2\rangle - \langle Y_\Lambda\rangle^2], \quad Y \equiv |L|,\ L_L,\ L_T$$

where, for example, $(L_L)_\Lambda = C_\tau \sum_{|\lambda_n|>\Lambda} \lambda_n^{N_\tau-1} \mathrm{Re}\left(\mathrm{e}^{2\pi k i/3}(n|\hat{U}_4|n)\right)$

Define Λ-dependent (IR-cut) ratio of susceptibilities:

$$(R_A)_\Lambda = \frac{(\chi_A)_\Lambda}{(\chi_L)_\Lambda}$$

Define Λ-dependent (IR-cut) chiral condensate:

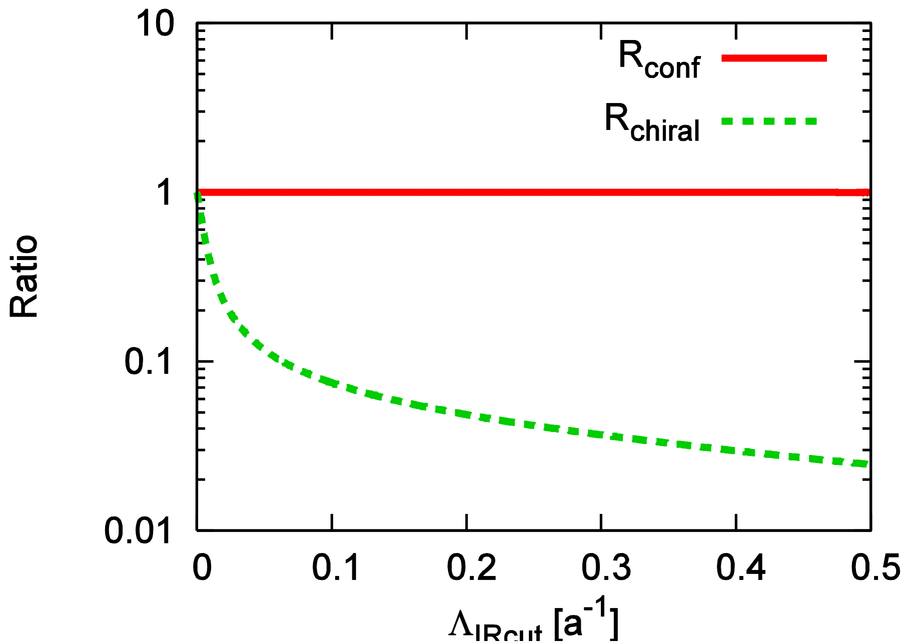
$$\langle\bar{\psi}\psi\rangle_\Lambda = -\frac{1}{V}\sum_{|\lambda_n|\geq\Lambda}\frac{2m}{\lambda_n^2+m^2}$$

Define the ratios, which indicate the influence of removing the low-lying Dirac modes:

$$R_{\mathrm{conf}} = \frac{(R_A)_\Lambda}{R_A}, \qquad R_{\mathrm{chiral}} = \frac{\langle\bar{\psi}\psi\rangle_\Lambda}{\langle\bar{\psi}\psi\rangle}$$

Numerical analysis

$$R_{\mathrm{conf}} = \frac{(R_A)_\Lambda}{R_A}, \qquad R_{\mathrm{chiral}} = \frac{\langle\bar{\psi}\psi\rangle_\Lambda}{\langle\bar{\psi}\psi\rangle}$$



lattice setup:

- quenched SU(3) lattice QCD
- standard plaquette action
- gauge coupling: $\beta = \frac{2N_{\mathrm{c}}}{g^2} = 5.6$
- lattice size: $N_{\mathrm{space}}^3 \times N_4 = 10^3 \times 5$

  ⇔ lattice spacing : $a \simeq 0.25$ fm
- periodic boundary condition for link-variables and Dirac operator

- R_{chiral} is strongly reduced by removing the low-lying Dirac modes.
- R_{conf} is almost unchanged.

It is also numerically confirmed that
low-lying Dirac modes are important for chiral symmetry breaking
and not important for quark confinement.

Summary

TMD, K. Redlich, C. Sasaki and H. Suganuma, arXiv: 1505.05752 [hep-lat]

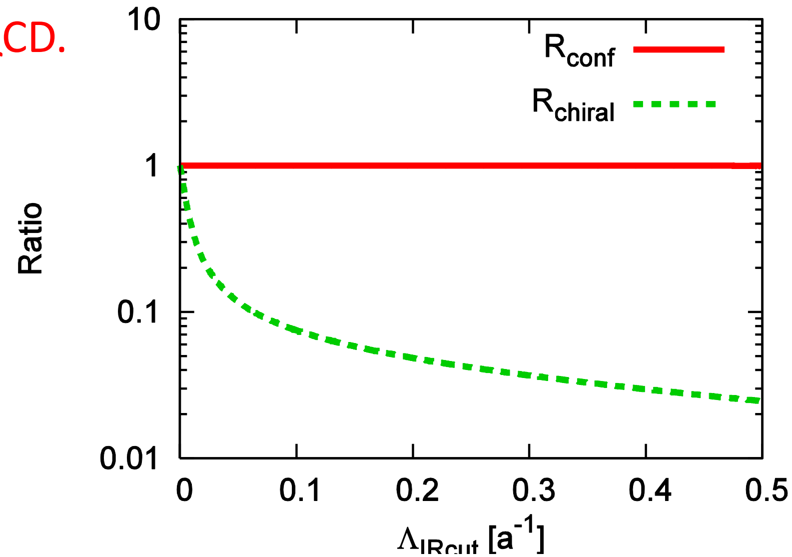
1. We have derived the analytical relation between
Polyakov loop fluctuations and Dirac eigenmodes on temporally odd-lattice lattice:

e.g.) $$R_A = \frac{\left\langle \left|\sum_n \lambda_n^{N_\tau - 1}\langle n|\hat{U}_4|n\rangle\right|^2 \right\rangle - \left\langle \left|\sum_n \lambda_n^{N_\tau - 1}\langle n|\hat{U}_4|n\rangle\right| \right\rangle^2}{\left\langle \left(\sum_n \lambda_n^{N_\tau - 1}\mathrm{Re}\left(\mathrm{e}^{2\pi k i/3}\langle n|\hat{U}_4|n\rangle\right)\right)^2 \right\rangle - \left\langle \sum_n \lambda_n^{N_\tau - 1}\mathrm{Re}\left(\mathrm{e}^{2\pi k i/3}\langle n|\hat{U}_4|n\rangle\right) \right\rangle^2}$$

N_τ : odd
Dirac eigenmode : $\hat{\not{D}}|n\rangle = i\lambda_n|n\rangle$
Link variable operator : $\langle s|\hat{U}_\mu|s'\rangle = U_\mu(s)\delta_{s+\hat{\mu},s'}$

2. We have semi-analytically and numerically confirmed that low-lying Dirac modes are not important to quantify the Polyakov loop fluctuation ratios, which are sensitive observables to confinement properties in QCD.

3. Our results suggest that there is
no direct one-to-one correspondence between confinement and chiral symmetry breaking in QCD.



Appendix

Why Polyakov loop fluctuations?

P.M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki,
Phys. Rev. D88, 014506 (2013); Phys. Rev. D88, 074502 (2013)

Ans. 1: Avoiding ambiguities of the Polyakov loop renormalization

$$L^{\rm ren} = Z(g^2) L^{\rm bare}, \qquad L^{\rm bare} \equiv \frac{1}{N_c V} \sum_s {\rm tr}_c \{ \prod_{i=0}^{N_\tau - 1} U_4(s + i\hat{4}) \}$$

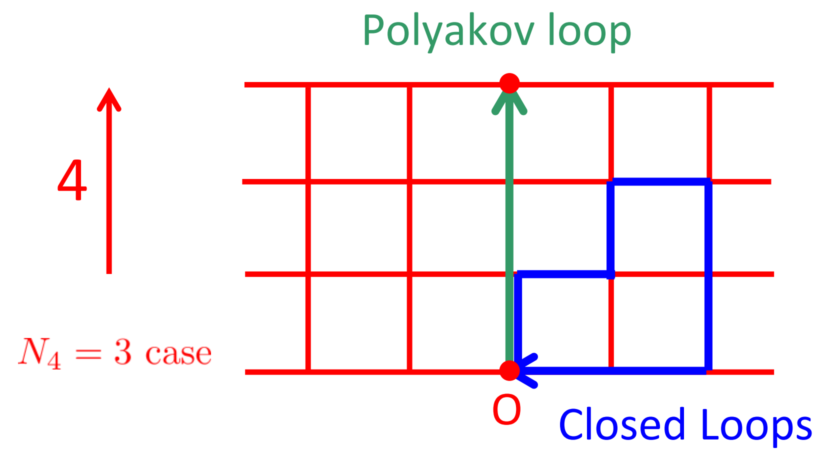
$Z(g^2)$: renormalization function for the Polyakov loop, which is still unknown

Avoid the ambiguity of renormalization function
by considering the ratios of Polyakov loop susceptibilities:

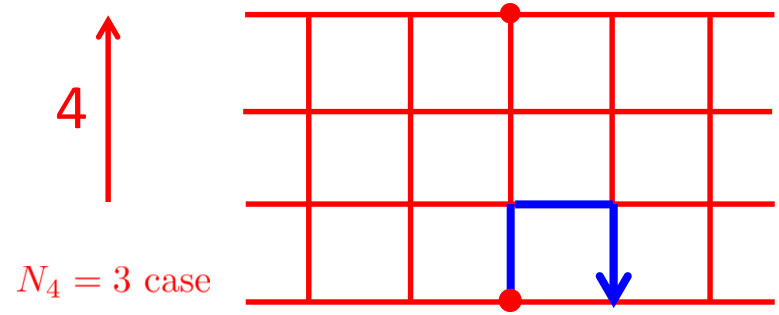
$$R_A \equiv \frac{\chi_A}{\chi_L}, \qquad R_T \equiv \frac{\chi_T}{\chi_L}$$

An analytical relation between Polyakov loop and Dirac mode on temporally odd-number lattice

In general, only gauge-invariant quantities such as Closed Loops and the Polyakov loop survive in QCD. (Elitzur's Theorem)



All the non-closed lines are gauge-variant and their expectation values are zero.



e.g.

$$\mathrm{Tr}\hat{U}_4\hat{U}_1\hat{U}_{-4} = \sum_s \mathrm{tr}\{U_4(s)U_1(s+\hat{4})U_4^\dagger(s+\hat{1})\}\underline{\delta_{s,s+\hat{1}}} = 0$$

gauge-variant

(Tr  =0)

Key point

Note: any closed loop needs even-number link-variables on the square lattice.

An analytical relation between Polyakov loop and Dirac mode on temporally odd-number lattice

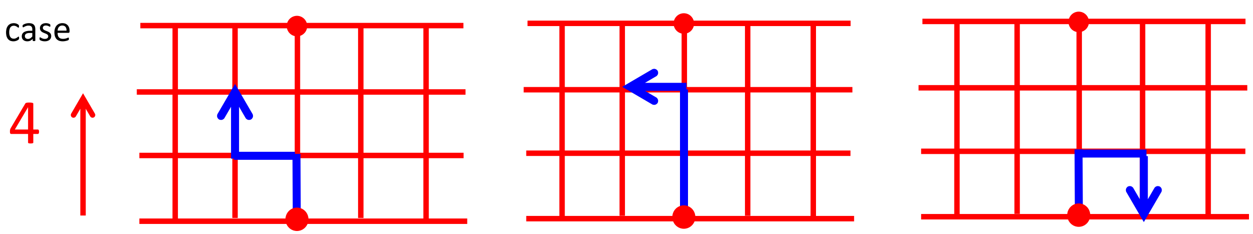
$$I \equiv \mathrm{Tr}_{c,\gamma}(\hat{U}_4 \, \hat{\not{D}}^{N_4-1}) \qquad (N_4 : \mathrm{odd})$$

Dirac operator : $\hat{\not{D}} = \frac{1}{2}\sum_{\mu} \gamma_\mu (\hat{U}_\mu - \hat{U}_{-\mu})$

In this functional trace $I \equiv \mathrm{Tr}_{c,\gamma}(\hat{U}_4 \, \hat{\not{D}}^{N_4-1})$,
it is impossible to form a closed loop on the square lattice,
because the length of the trajectories, N_4, is odd.

Almost all trajectories are gauge-variant & give no contribution.

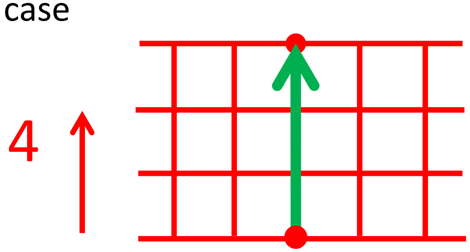
$N_4 = 3$ case



gauge variant
(no contribution)

Only the exception is the Polyakov loop.

$N_4 = 3$ case



gauge invariant !!

I is proportional to the Polyakov loop.

$$I \propto L$$

L : Polyakov loop

Analytical relation between Polyakov loop and Dirac modes with twisted boundary condition

C. Gattringer, Phys. Rev. Lett. 97 (2006) 032003.

$$L = \frac{1}{8V}\left(2\sum_{\lambda}\lambda^{N_4} - (1+i)\sum_{\lambda_+}\lambda_+^{N_4} - (1-i)\sum_{\lambda_-}\lambda_-^{N_4}\right)$$

twisted boundary condition:

$$U_4(\mathbf{x}, N_4) \to \pm i U_4(\mathbf{x}, N_4), \qquad \forall \mathbf{x}$$

$$D(x, y) \to D_\pm(x, y)$$

λ : Eigenvalue of $D(x|y)$

$\lambda_\pm$: Eigenvalue of $D_\pm(x|y)$

$$D(x|y) = (4+m)\delta_{x,y} - \frac{1}{2}\sum_{\mu=\pm 1}^{\pm 4}[1 \mp \gamma_\mu]U_\mu(x)\delta_{x+\mu,y}$$: Wilson Dirac operator

The twisted boundary condition is not the periodic boundary condition.

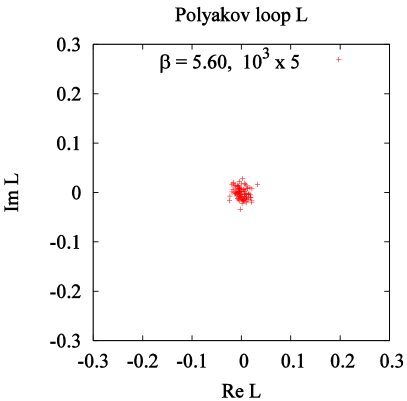
However,
the temporal periodic boundary condition is physically important
for the imaginary-time formalism at finite temperature.

(The b.c. for link-variables is p.b.c., but the b.c. for Dirac operator is twisted b.c.)

λ_n V.S. $(n|\hat{U}_4|n)$, $\lambda_n^{N_4-1}(n|\hat{U}_4|n)$

$$L_P = \frac{(2i)^{N_4-1}}{3V}\sum_n \lambda_n^{N_4-1}(n|\hat{U}_4|n)$$

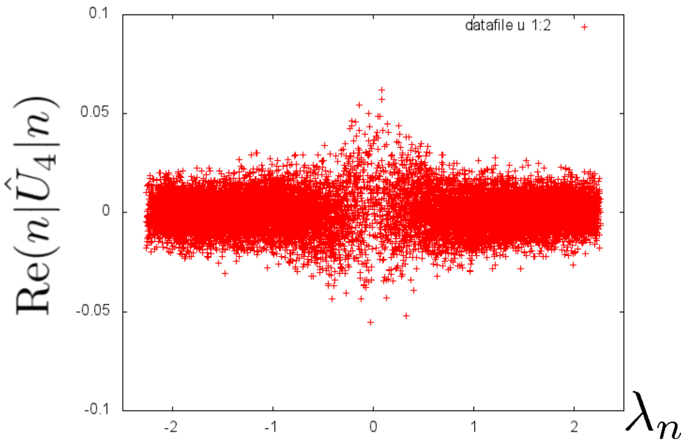
$\beta = 5.6$
lattice size : $10^3 \times 5$

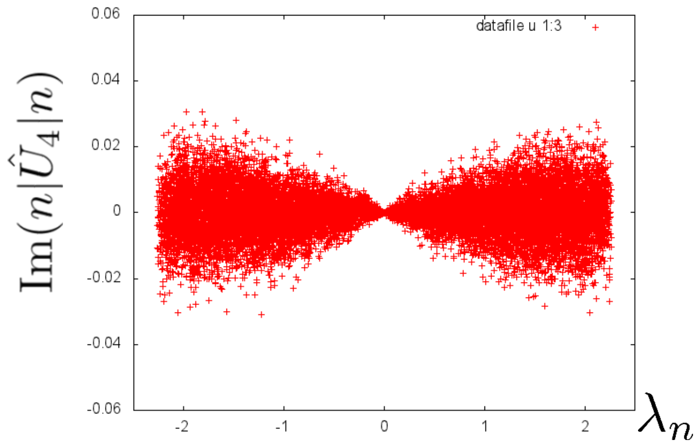


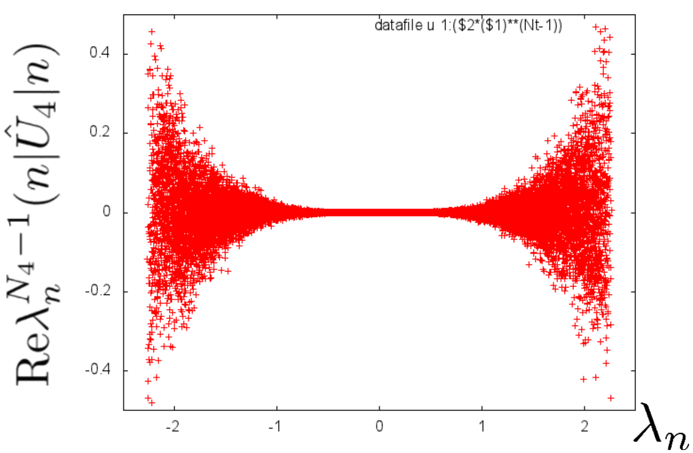
$\langle L_P \rangle = 0$
(confined phase)

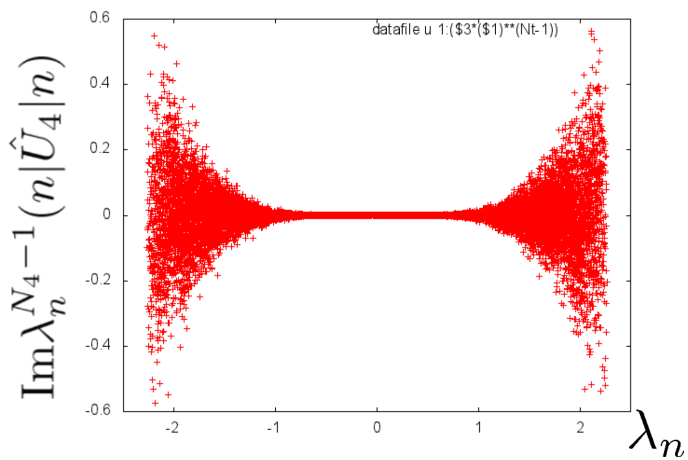
$$\not{D}|n\rangle = i\lambda_n|n\rangle$$

Dirac eigenvalue: $i\lambda_n$





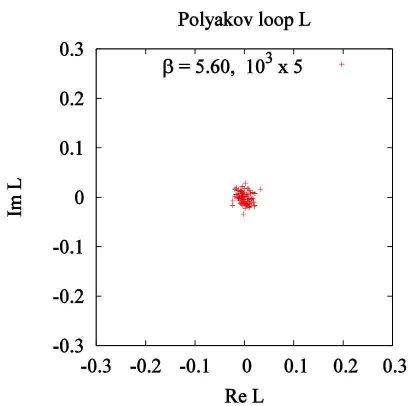




λ_n V.S. $(n|\hat{U}_4|n)$, $\lambda_n^{N_4-1}(n|\hat{U}_4|n)$

$$L_P = \frac{(2i)^{N_4-1}}{3V} \sum_n \lambda_n^{N_4-1}(n|\hat{U}_4|n)$$

$\beta = 5.6$
lattice size : $10^3 \times 5$

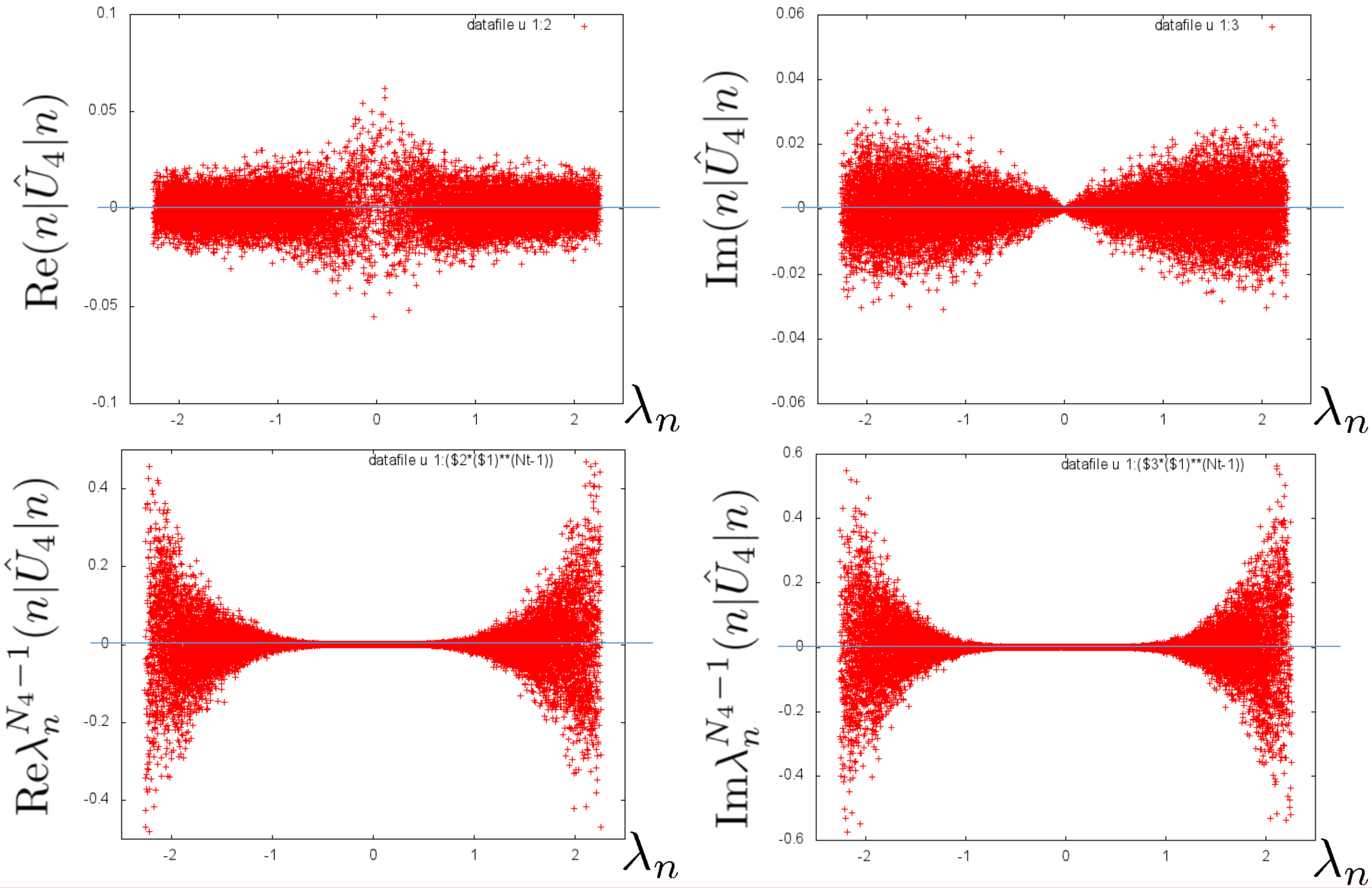


$\langle L_P \rangle = 0$
(confined phase)

$$\not{D}|n\rangle = i\lambda_n|n\rangle$$

Dirac eigenvalue: $i\lambda_n$

confined phase



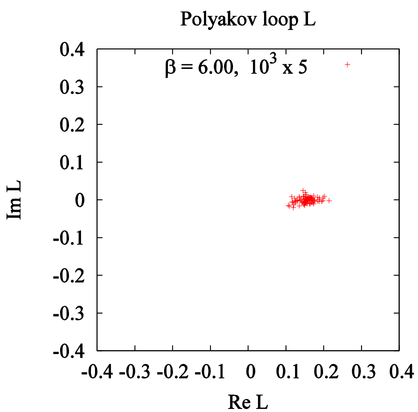
$\langle L \rangle = 0$ is due to the symmetric distribution of positive/negative value of $(n|\hat{U}_4|n)$, $\lambda_n^{N_4-1}(n|\hat{U}_4|n)$

Low-lying Dirac modes have little contribution to Polyakov loop.

λ_n V.S. $(n|\hat{U}_4|n)$, $\lambda_n^{N_4-1}(n|\hat{U}_4|n)$

$$L_P = \frac{(2i)^{N_4-1}}{3V}\sum_n \lambda_n^{N_4-1}(n|\hat{U}_4|n)$$

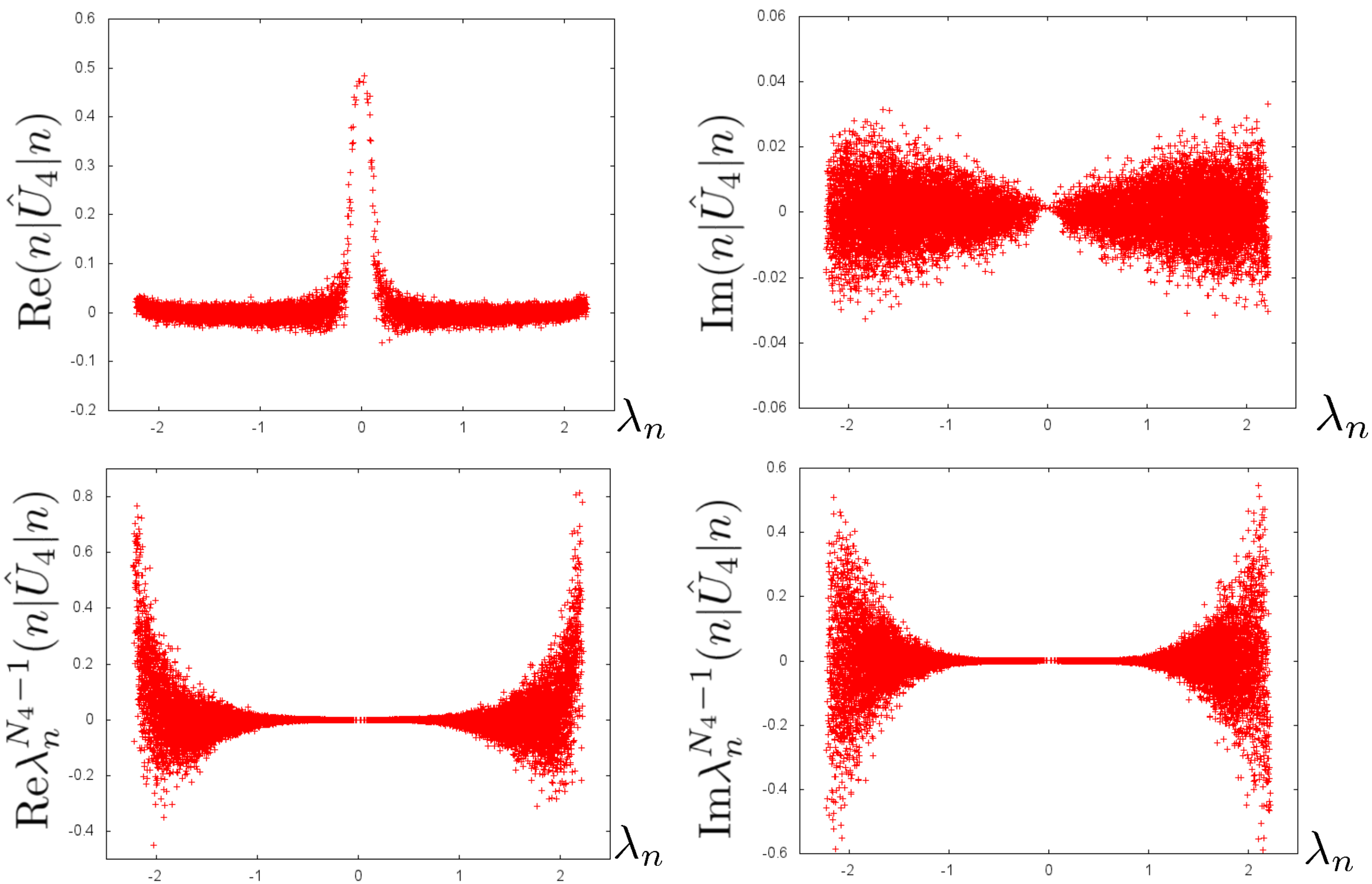
$\beta = 6.0$
lattice size : $10^3 \times 5$



$\langle L_P \rangle \neq 0$
(deconfined phase)

$$\not{D}|n\rangle = i\lambda_n|n\rangle$$

Dirac eigenvalue: $i\lambda_n$



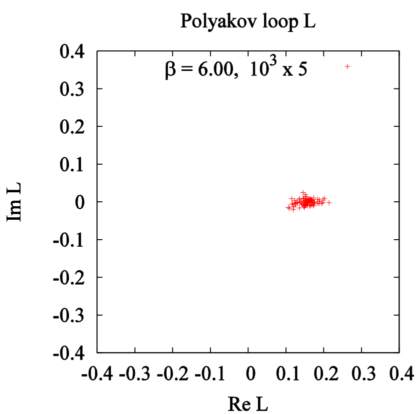
We mainly investigate the real Polyakov-loop vacuum, where the Polyakov loop is real, so only real part is different from it in confined phase.

λ_n V.S. $(n|\hat{U}_4|n)$, $\lambda_n^{N_4-1}(n|\hat{U}_4|n)$

$$L_P = \frac{(2i)^{N_4-1}}{3V}\sum_n \lambda_n^{N_4-1}(n|\hat{U}_4|n)$$

$\beta = 6.0$

lattice size : $10^3 \times 5$

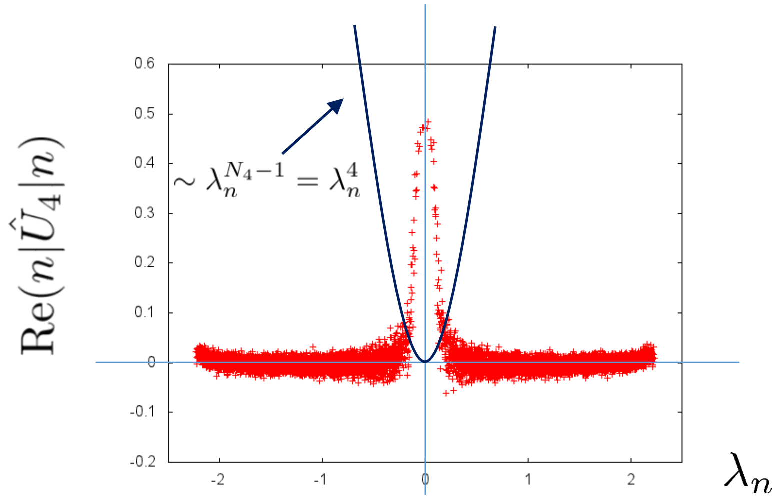


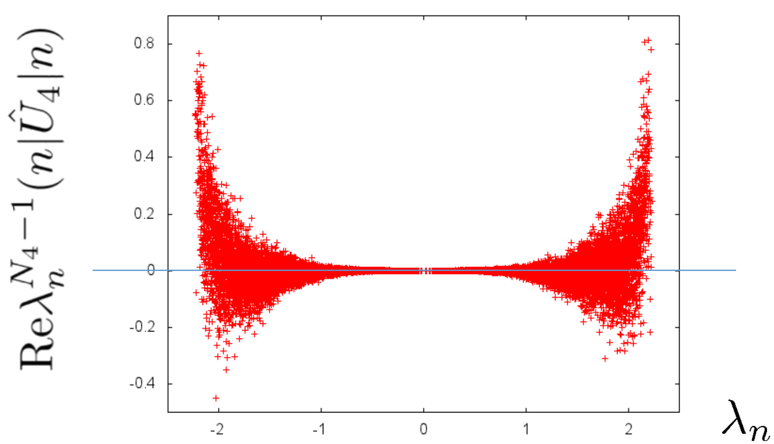
$\langle L_P \rangle \neq 0$

(deconfined phase)

$$\not{D}|n\rangle = i\lambda_n|n\rangle$$

Dirac eigenvalue: $i\lambda_n$





In low-lying Dirac modes region, $\mathrm{Re}(n|\hat{U}_4|n)$ has a large value,
but contribution of low-lying (IR) Dirac modes to Polyakov loop is very small
because of dumping factor $\lambda_n^{N_4-1}$