Two-nucleon scattering in multiple partial waves

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for the CalLat Collaboration

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CalLat

- LBL/UCB: Wick Haxton, Thorsten Kurth, AN, Ken McElvain, Mark Strother
- LLNL: Robert Falgout, Ron Soltz, Pavlos Vranas, Chris Schroeder, Evan Berkowitz, Enrico Rinaldi, Joe Wasem
- SDSU: Calvin Johnson
- nVidia: Michael Clark
- BNL: Sergey Syritsyn
- JLab: Raul Briceno, André Walker-Loud (JLab/W&M)

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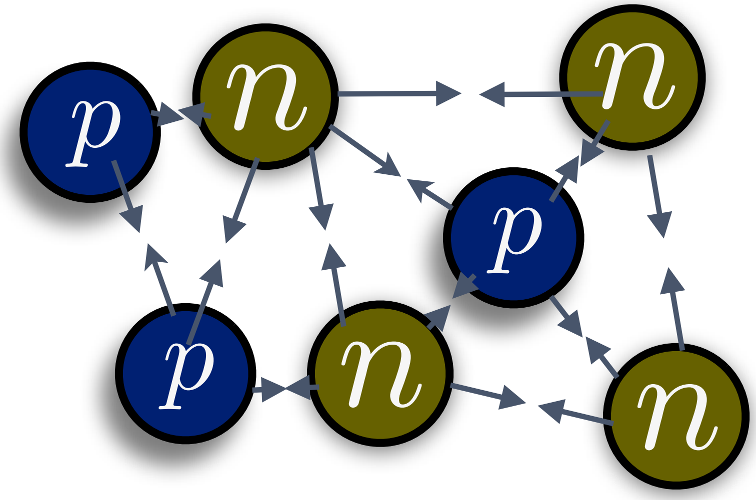
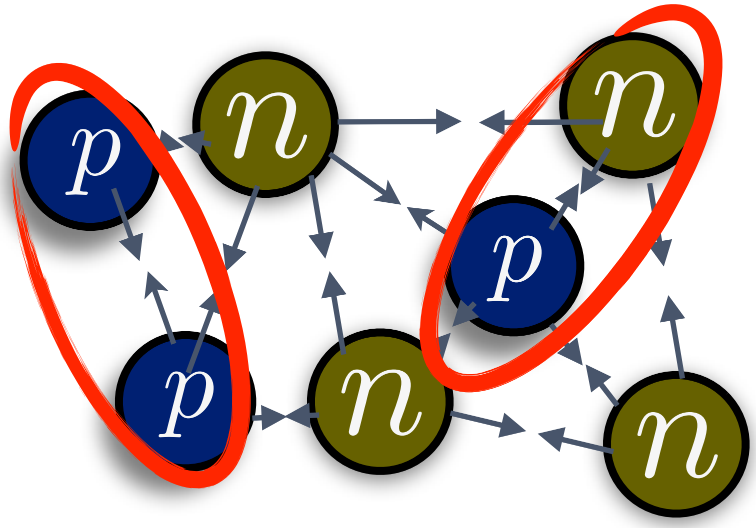
Nucleon-nucleon scattering

- Nuclear physics on the lattice is difficult!

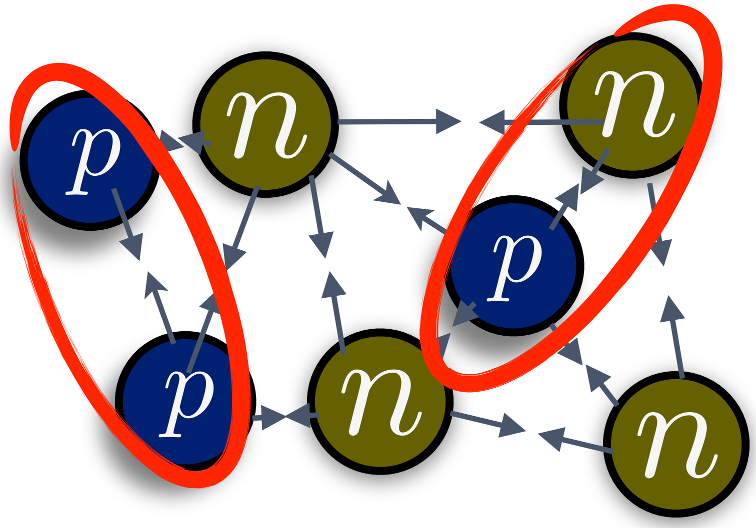


Nucleon-nucleon scattering

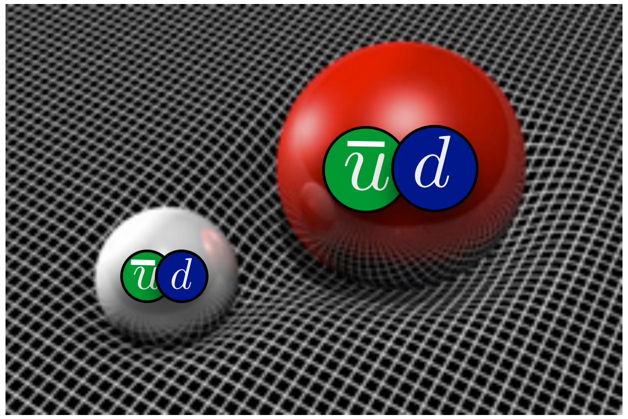


- Nuclear physics on the lattice is difficult!
- Must have full control over 2-body systems
    - How do we project onto desired states?
    - How do we disentangle signals from closely spaced energy levels?
    - How do we interpret finite volume results?

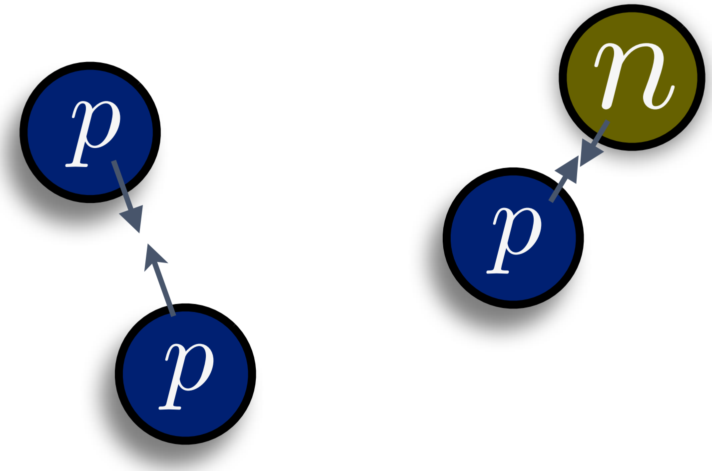
Nucleon-nucleon scattering



- Nuclear physics on the lattice is difficult!
- Must have full control over 2-body systems
  - How do we project onto desired states?
  - How do we disentangle signals from closely spaced energy levels?
  - How do we interpret finite volume results?
- Lattice QCD lets us explore dependence on S.M. parameters - experiment does not
  - Nucleon scattering is finely tuned
  - Useful input for EFTs and models
- Necessary to study scattering to determine 2-body matrix elements

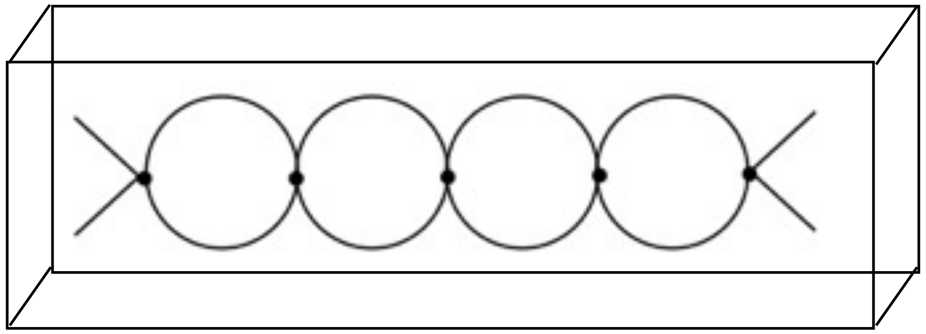


Overview



- Finite volume method
- Correlation functions
- Lattice details
- Spectrum
- Phase shifts
- Conclusions

Finite volume method



- Lüscher's method relates finite volume energies to infinite volume scattering shifts

$$p\cot\delta(p) = \frac{1}{\pi L} S\left(\left(\frac{pL}{2\pi}\right)^2\right)$$

$$S(\eta) = \lim_{\Lambda\to\infty}\left[\sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2-\eta^2} - 4\pi\Lambda\right]$$

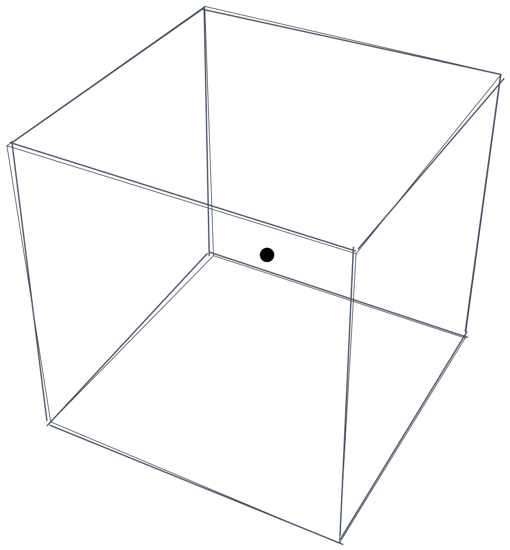
- Partial waves mix in a box (and in real life!), so the relation becomes a matrix eigenvalue equation

$$\det{}_{Jm_J\ell S}\left[\mathcal{M}^{-1}+\delta\mathcal{G}^V\right]=0$$

  - Effective range expansion or modeling necessary to interpolate between discrete points to solve eigenvalue equation

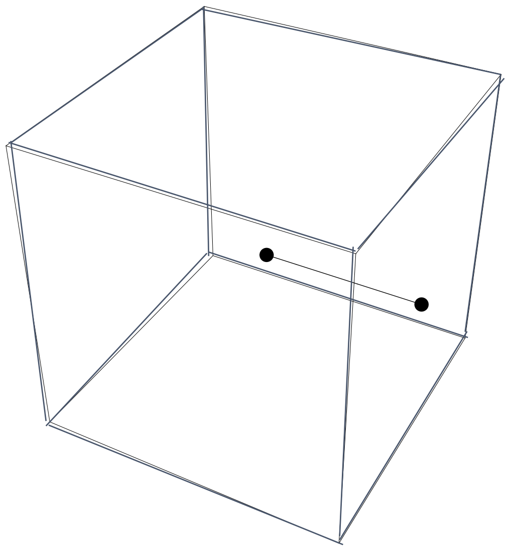
- Complicated!

Correlation functions



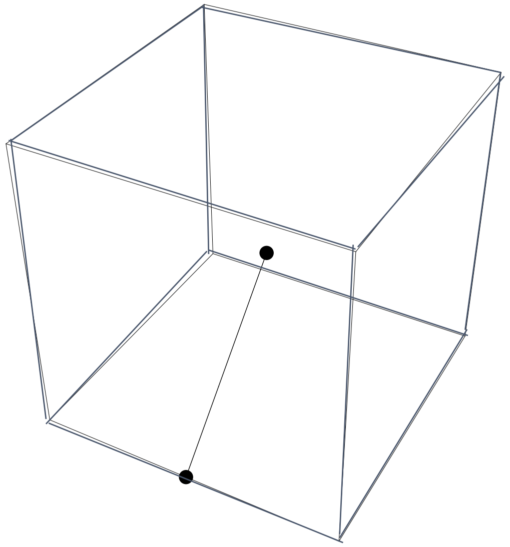
Starting with a good interpolating operator for a single nucleon at x_0....

Correlation functions



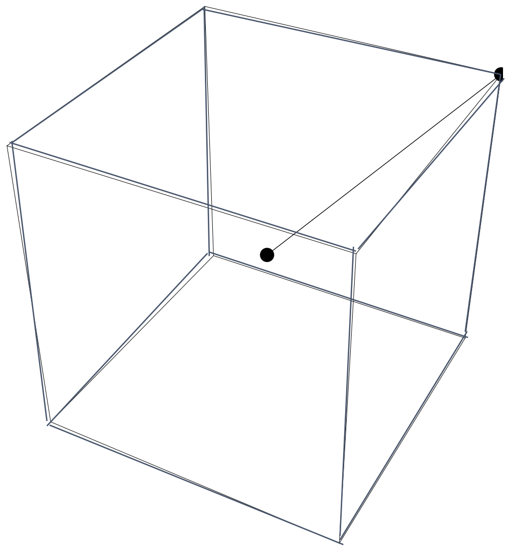
Add displaced nucleon:
"Face"

Correlation functions



Add displaced nucleon:
"Edge"

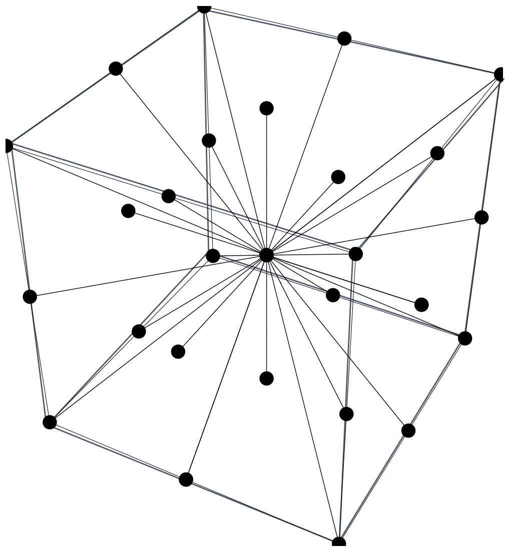
Correlation functions



Add displaced nucleon:
"Corner"

Correlation functions

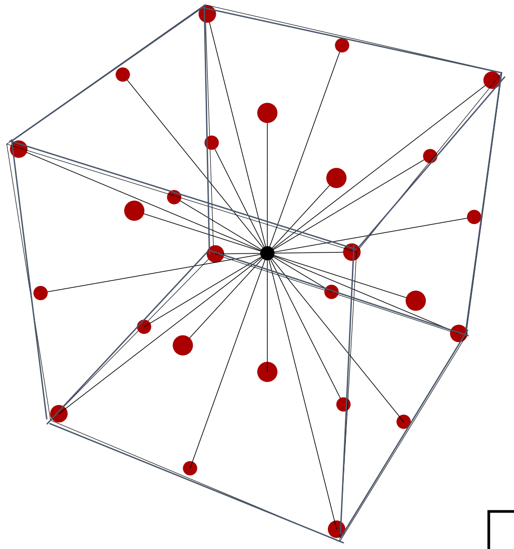
Different source types give us handle
for isolating desired state



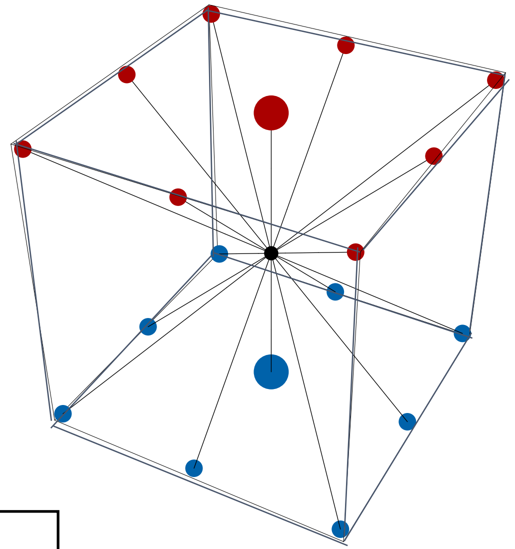
Large displacements necessary for
maximal overlap with low-energy states

Correlation functions

S

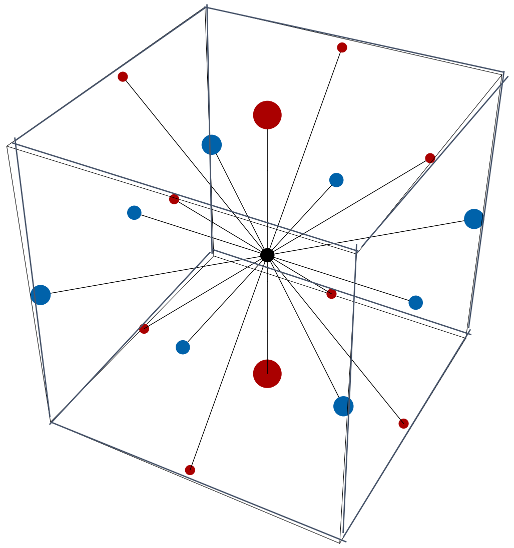


P

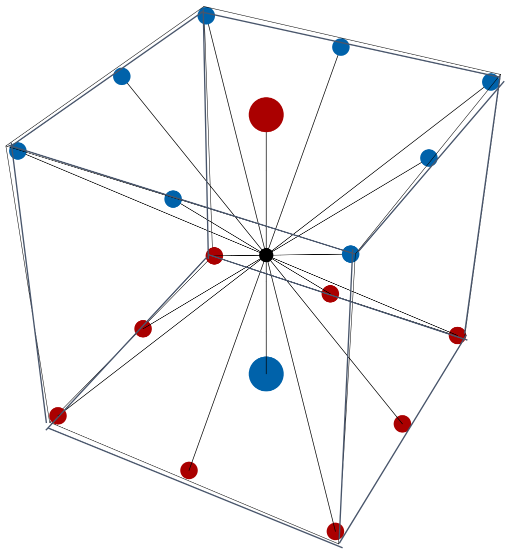


Spherical harmonics

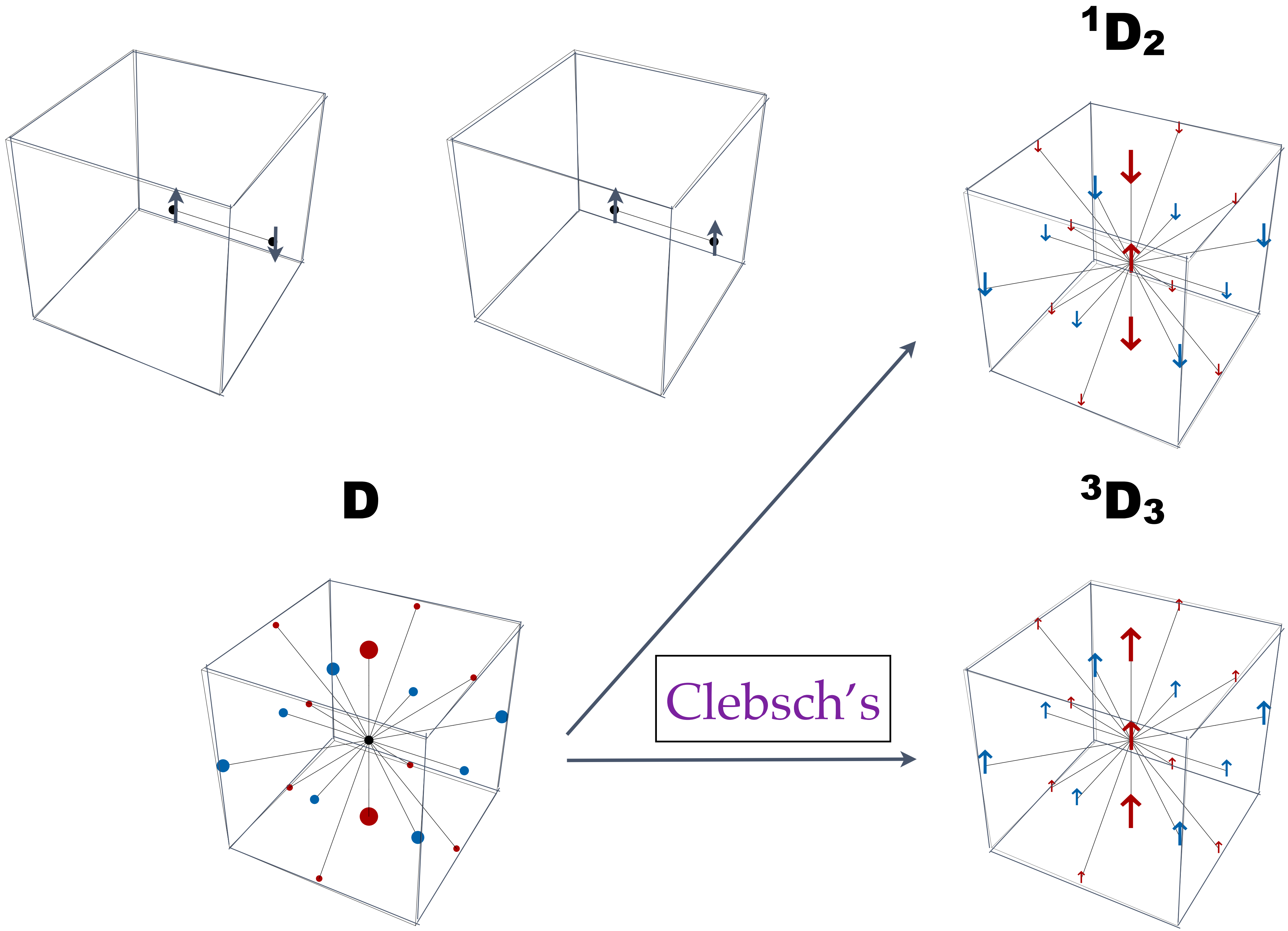
D



F



Correlation functions



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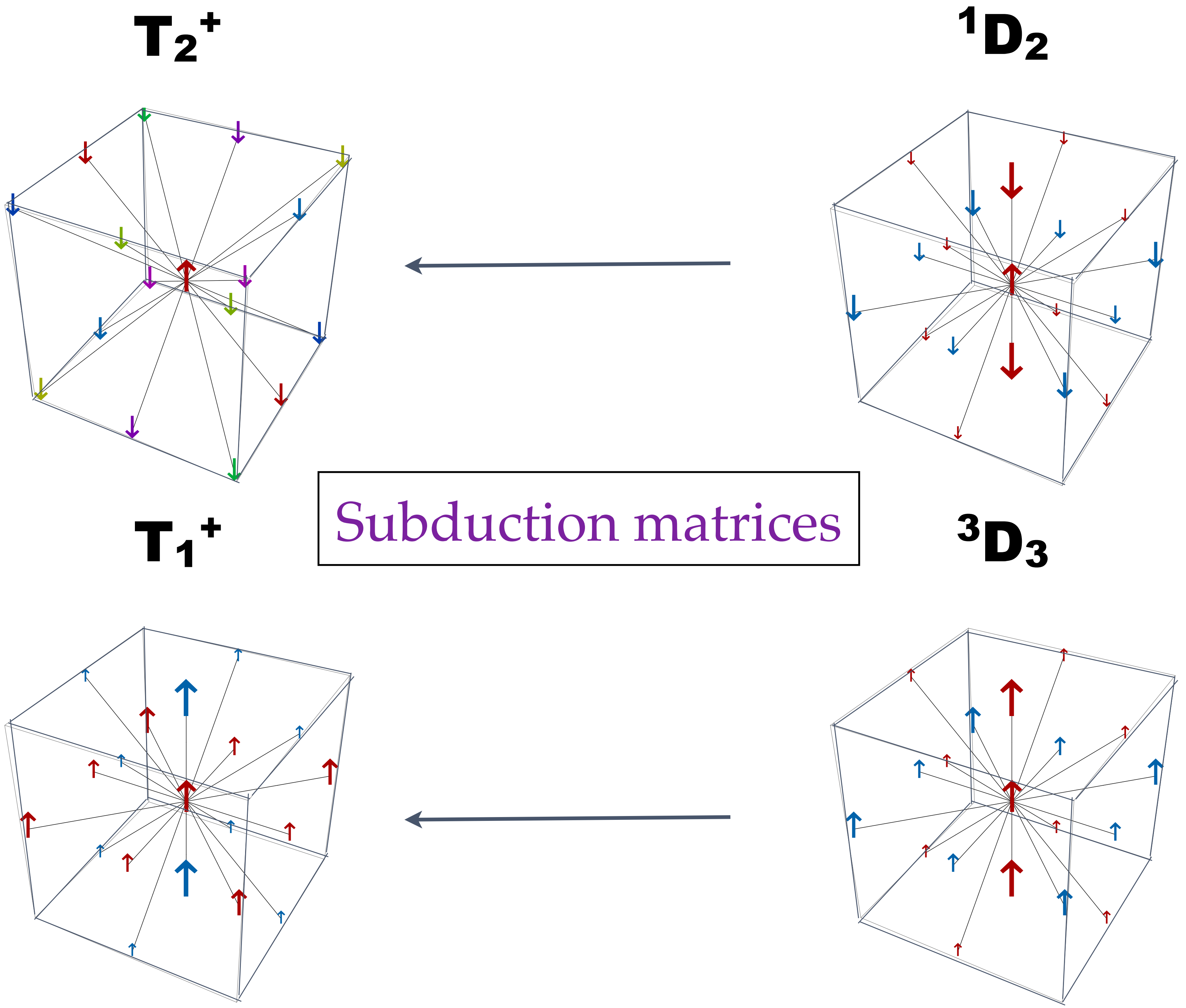


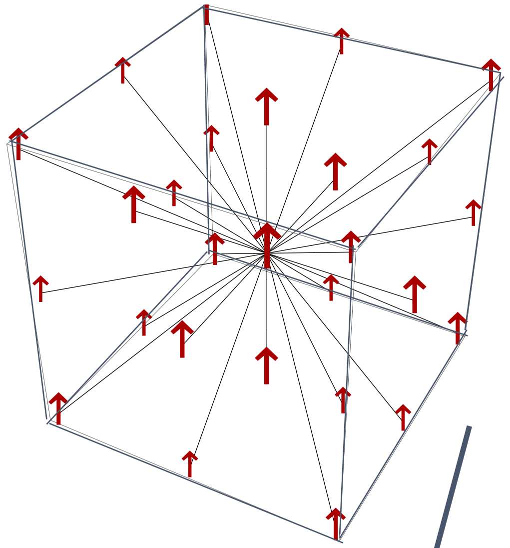
T_2^+ **$^1D_2$**

Subduction matrices

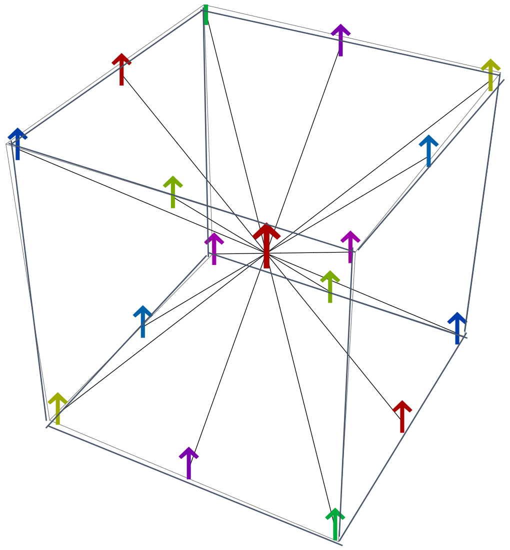
T_1^+ **$^3D_3$**

Correlation functions

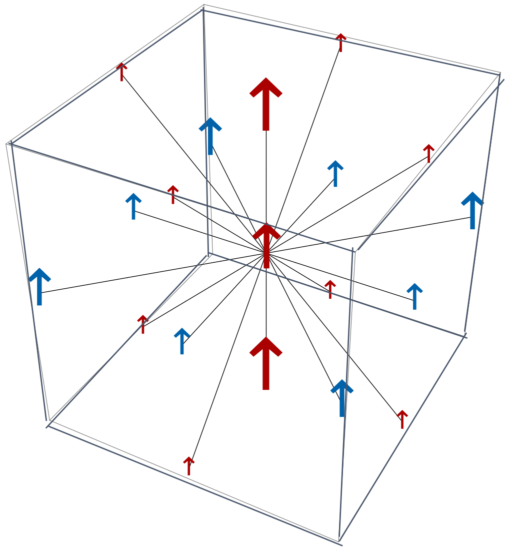
$^3S_1$



$^3D_1$



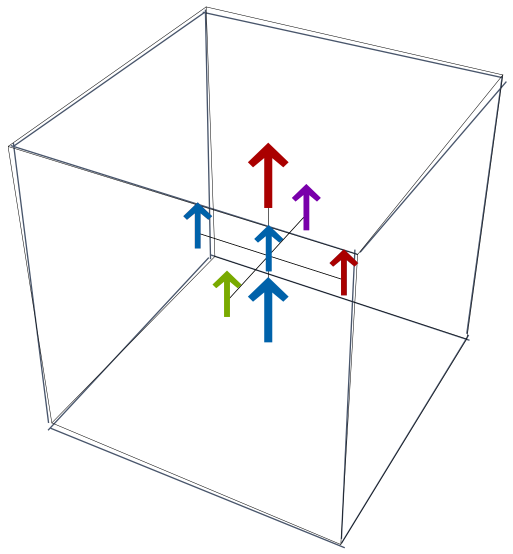
$^3D_3$

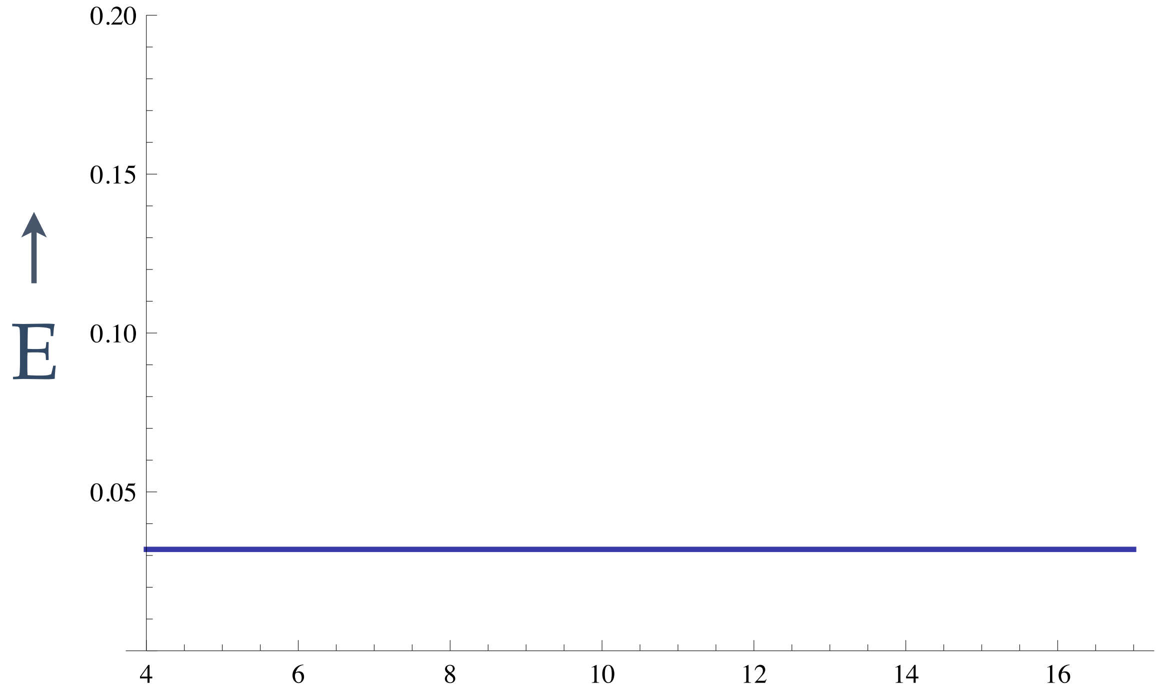


T_1^+

Set of multiple sources
coupling to same cubic irrep

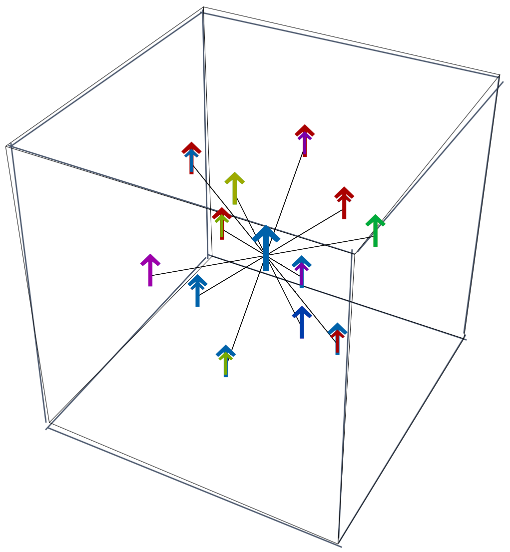
Correlation functions





Sink: project onto non-interacting momentum shells

Correlation functions



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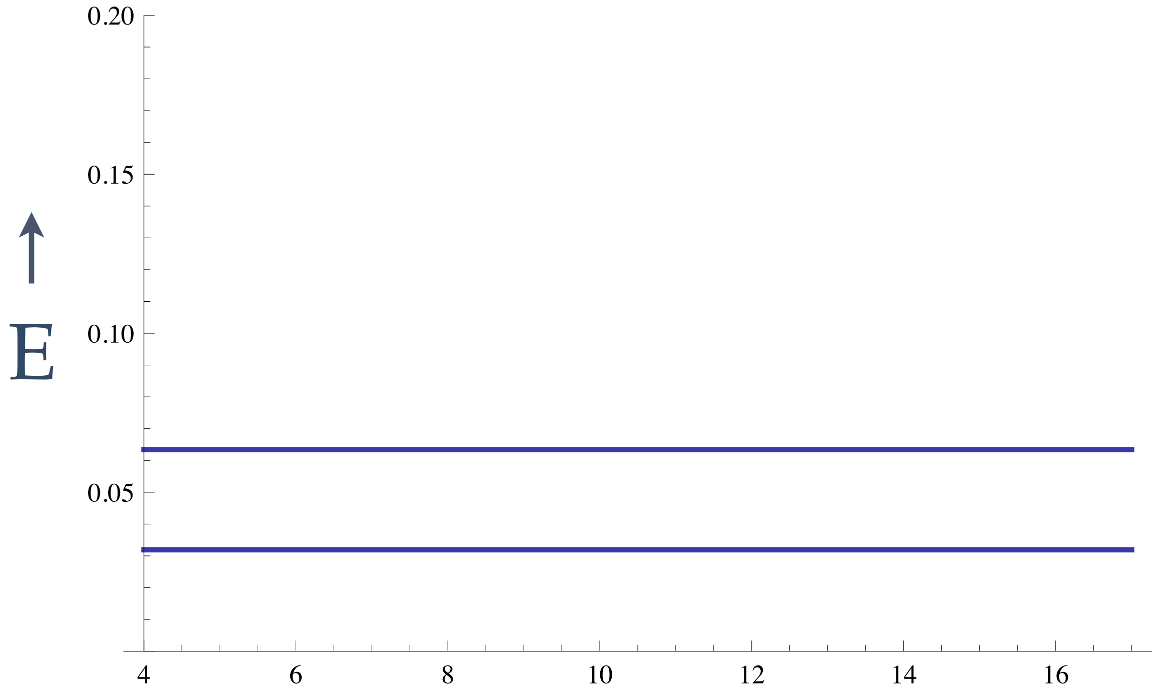

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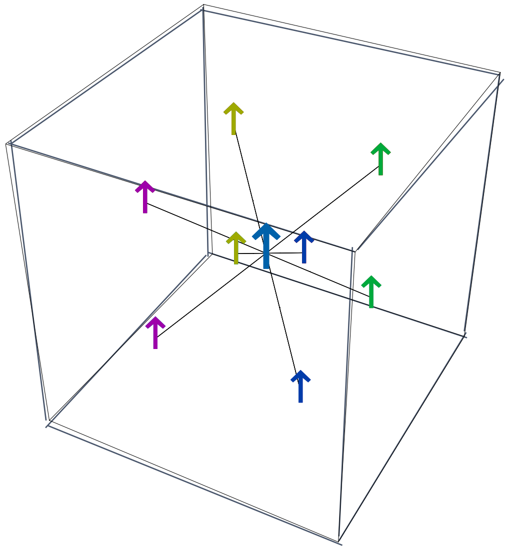

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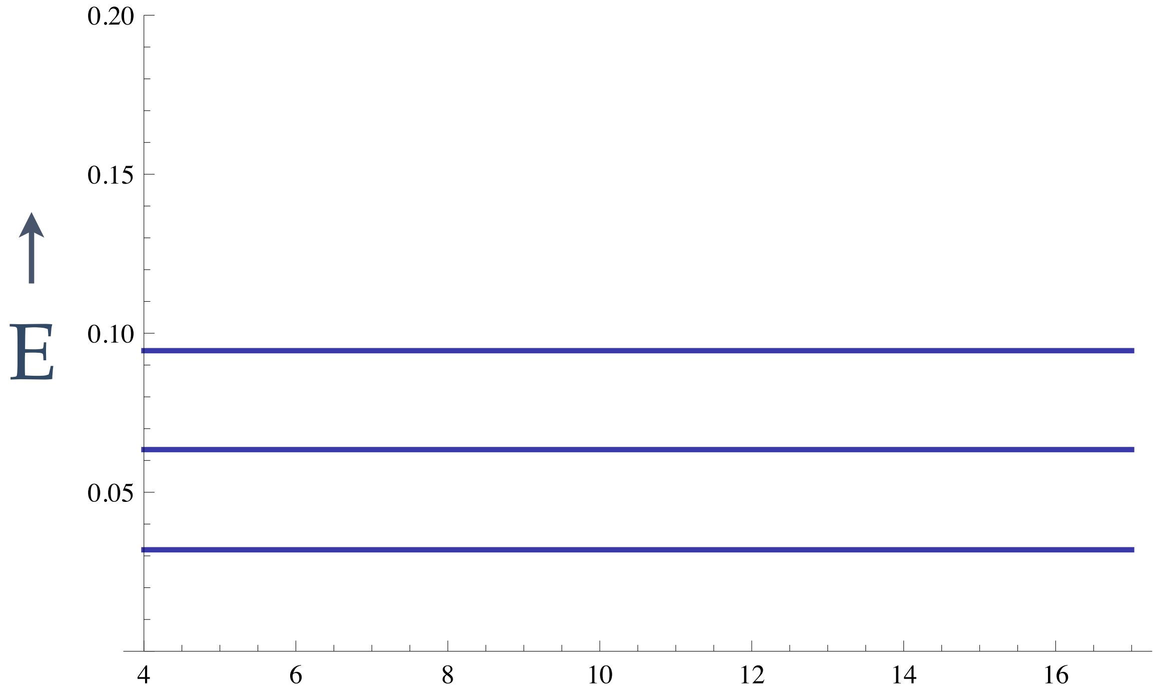

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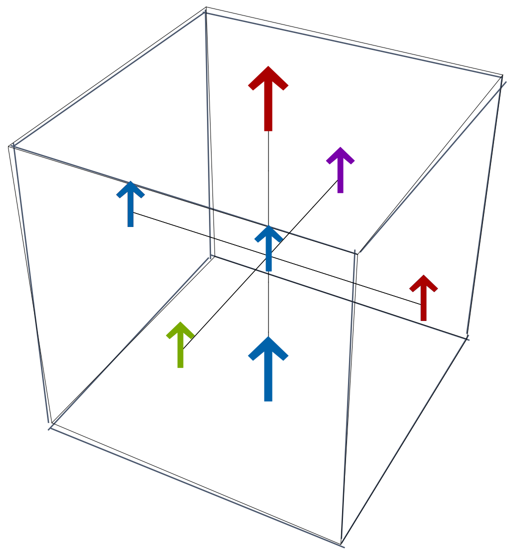
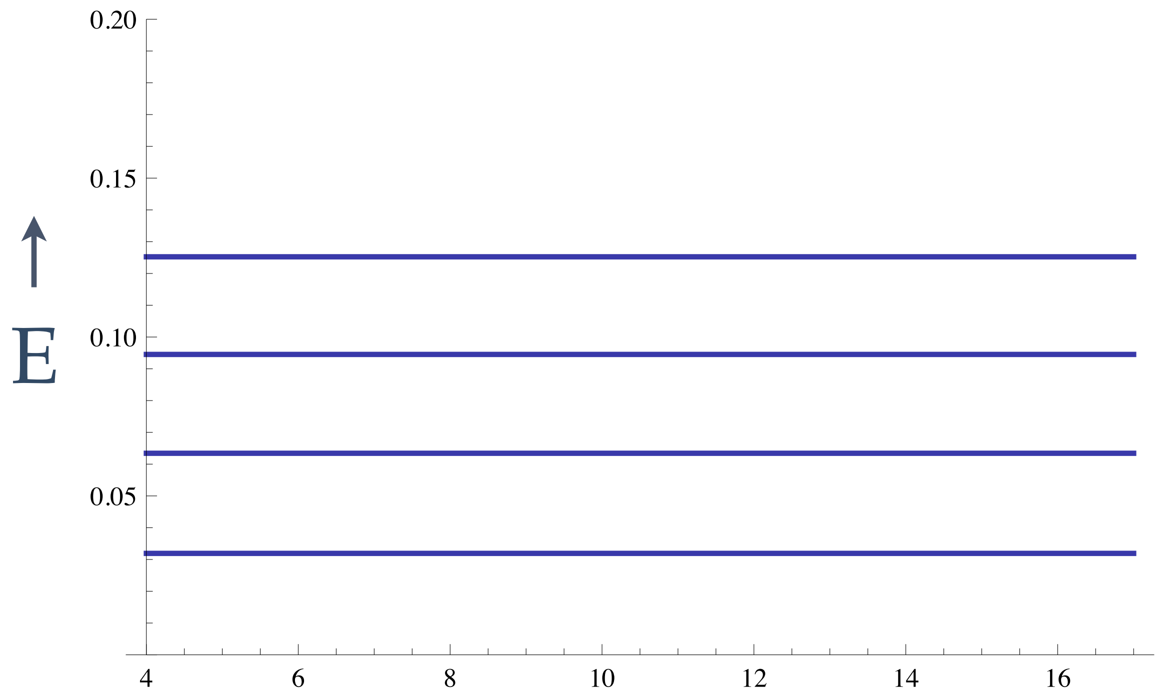
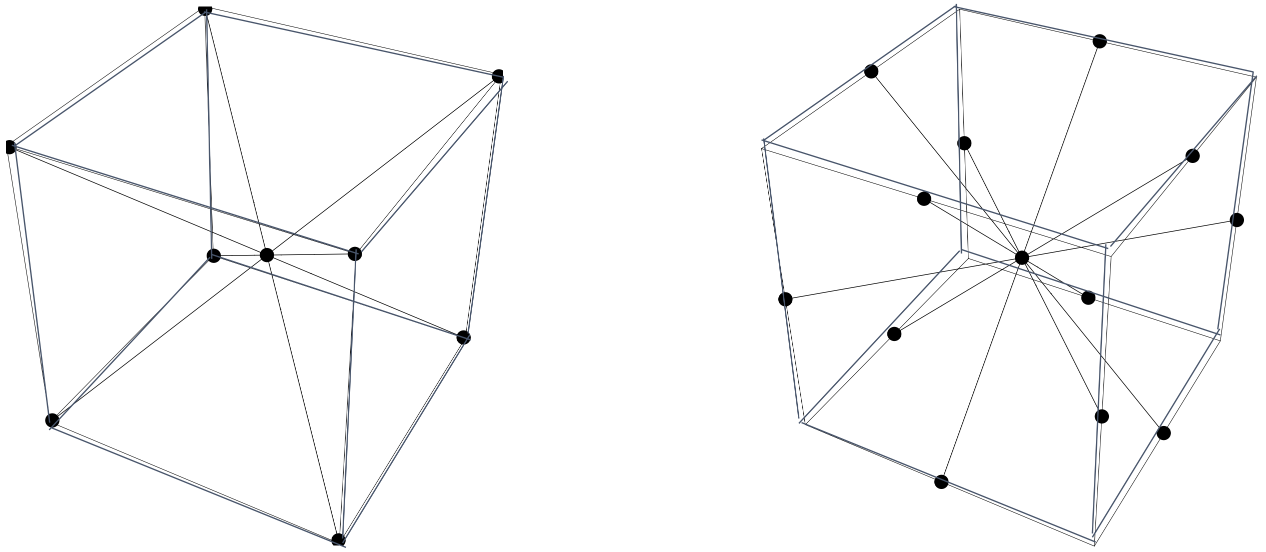




Sink: project onto non-interacting momentum shells

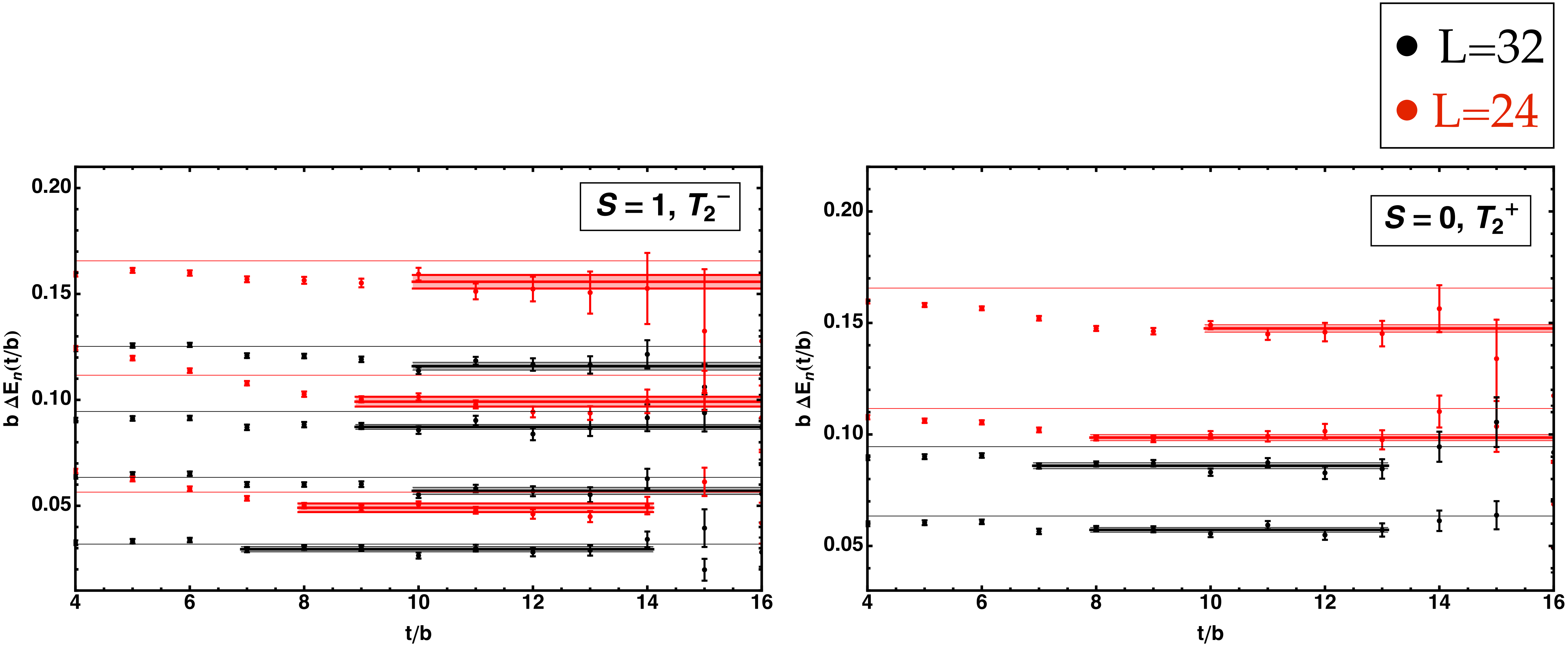
Lattice details

- HadSpec isotropic clover
- a ~ 0.145 fm
- V = $24^3(32^3)$x 48
- $m_\pi = m_K \sim 800$ MeV
- ~ 20(75) sources on each of 4000(1000) configs
  - x 8 (“corner” sources) or x 12 (“edge” sources)
- Same configs used by NPLQCD for S-wave scattering



NPLQCD, Phys.Rev. C88 (2013) 2, 024003

Spectrum

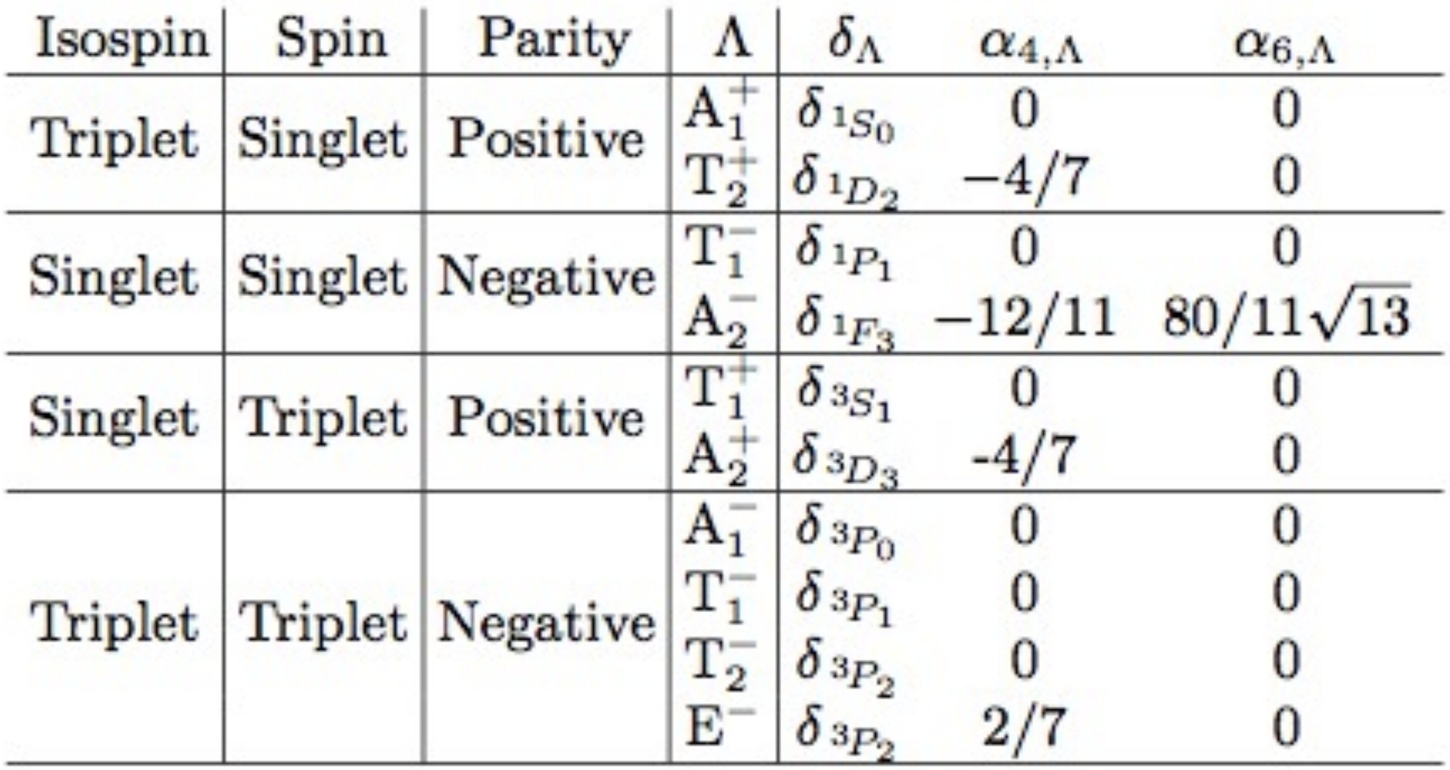


- ❖ Clean separation between energy levels
- ❖ Many signals several sigma away from non-interacting

Phase shifts

- Neglect partial wave mixing for the moment
- Simple Luscher relations: $q\cot\delta_\Lambda(q) = 4\pi\left(c_{00}(q^2) + \alpha_{4,\Lambda}\frac{c_{40}(q^2)}{q^4} + \alpha_{6,\Lambda}\frac{c_{60}(q^2)}{q^6}\right)$

kinematic factors: $c_{\ell m_\ell}(q^2) \ = \ \frac{\sqrt{4\pi}}{L^3}\left(\frac{2\pi}{L}\right)^{\ell-2} \sum_{\mathbf{r}\in Z^3} \frac{|\mathbf{r}|^\ell Y_{\ell m_\ell}(\mathbf{r})}{(r^2-q^2)}.$



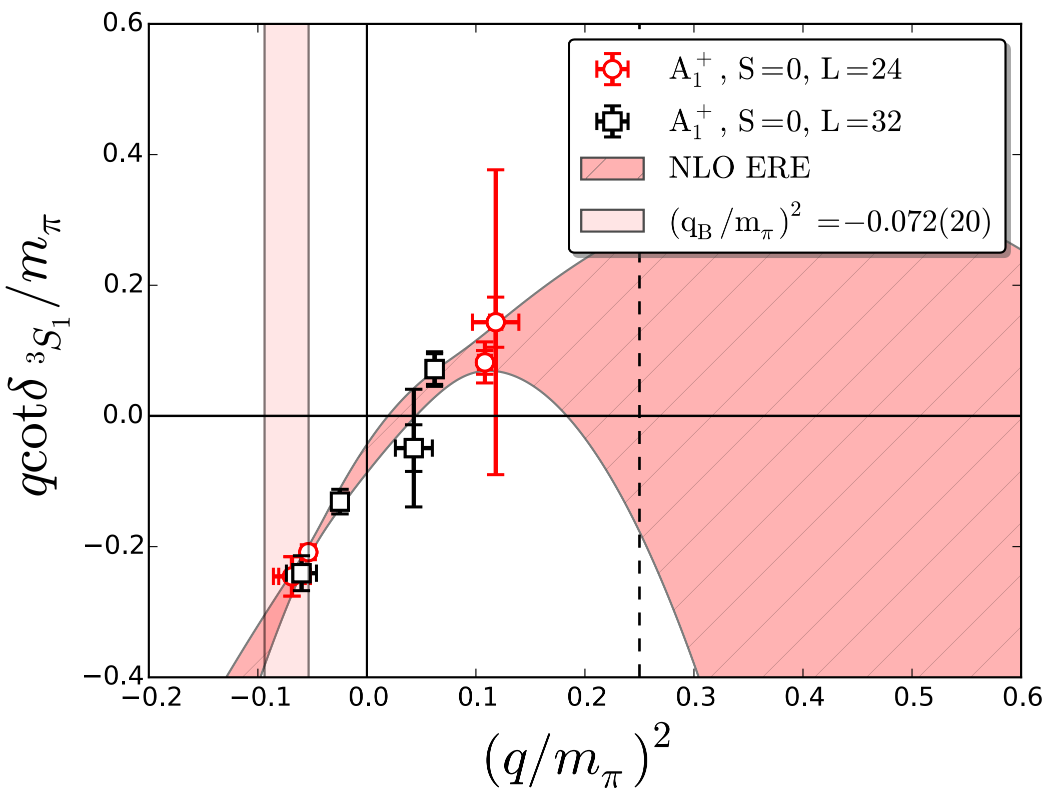
Isospin	Spin	Parity	Λ	δ_Λ	$\alpha_{4,\Lambda}$	$\alpha_{6,\Lambda}$
Triplet	Singlet	Positive	A_1^+	$\delta_{^1S_0}$	0	0
			T_2^+	$\delta_{^1D_2}$	$-4/7$	0
Singlet	Singlet	Negative	T_1^-	$\delta_{^1P_1}$	0	0
			A_2^-	$\delta_{^1F_3}$	$-12/11$	$80/11\sqrt{13}$
Singlet	Triplet	Positive	T_1^+	$\delta_{^3S_1}$	0	0
			A_2^+	$\delta_{^3D_3}$	-4/7	0
Triplet	Triplet	Negative	A_1^-	$\delta_{^3P_0}$	0	0
			T_1^-	$\delta_{^3P_1}$	0	0
			T_2^-	$\delta_{^3P_2}$	0	0
			E^-	$\delta_{^3P_2}$	2/7	0

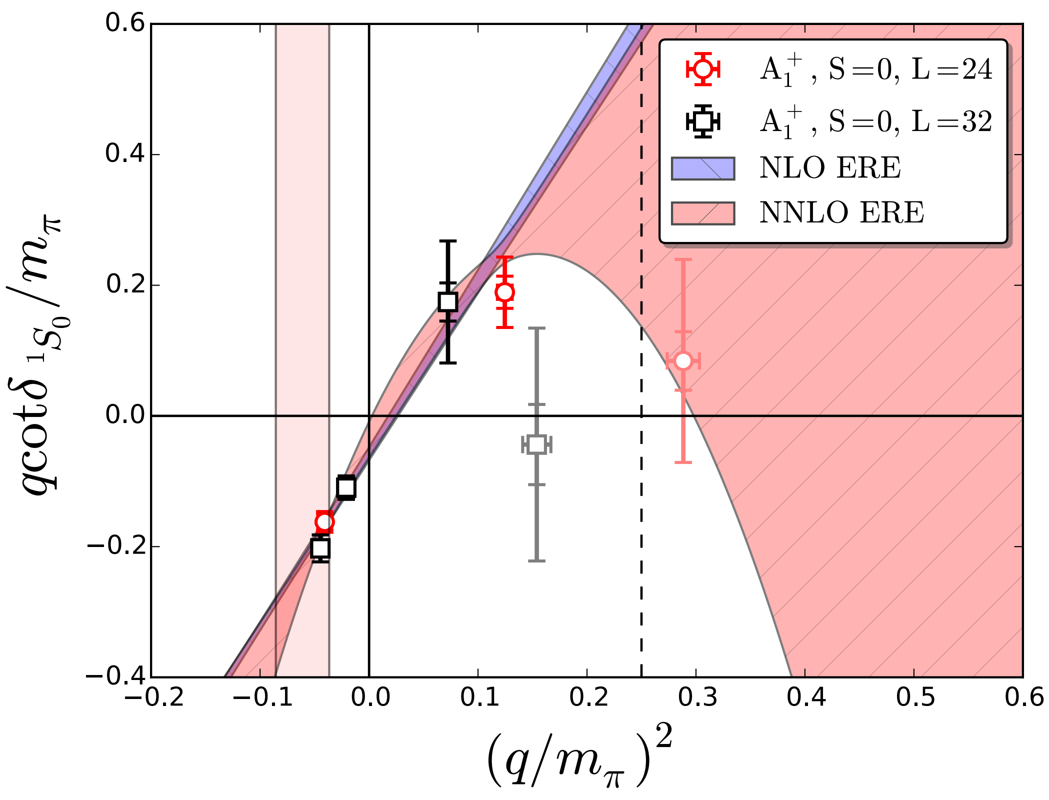
Briceno, Davoudi, Luu,
Phys.Rev. D 88 034502
(2013)

Phase shifts

Preliminary

- S-wave





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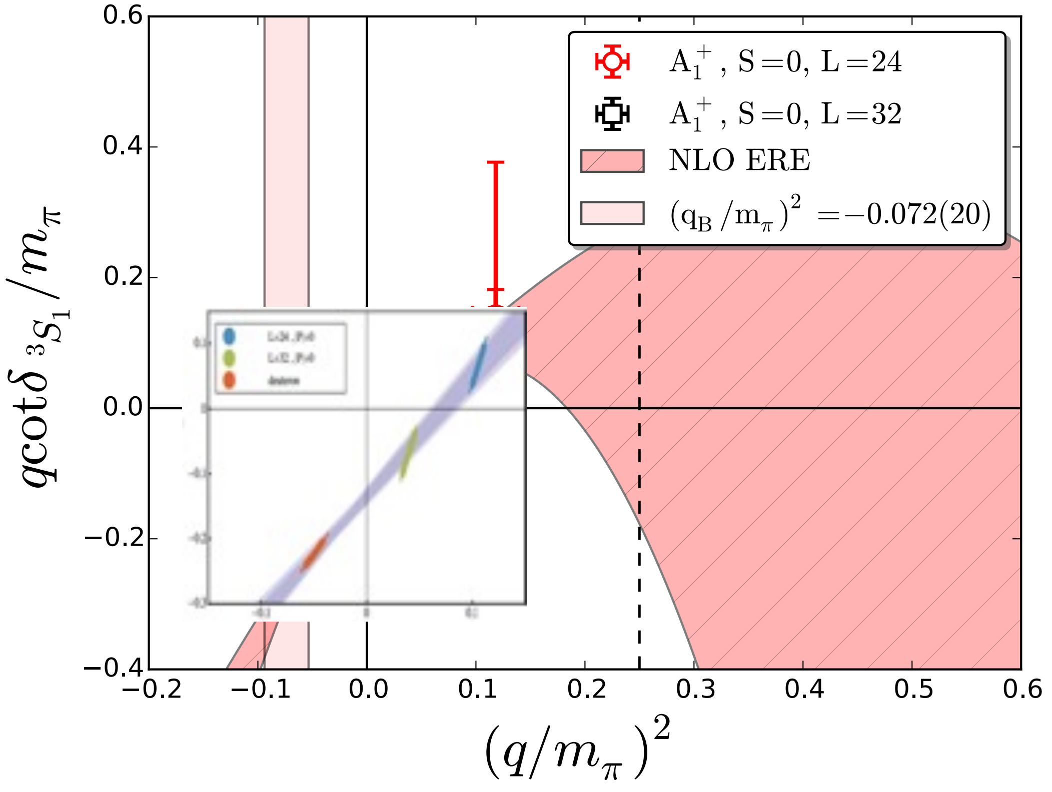

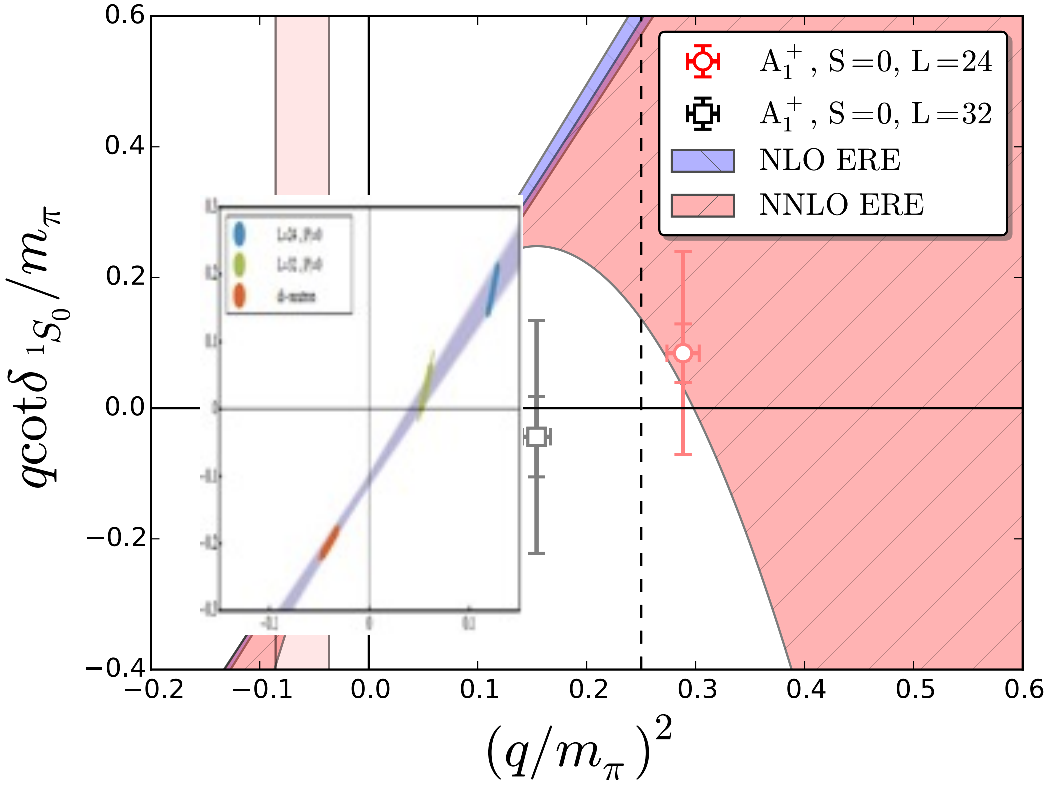

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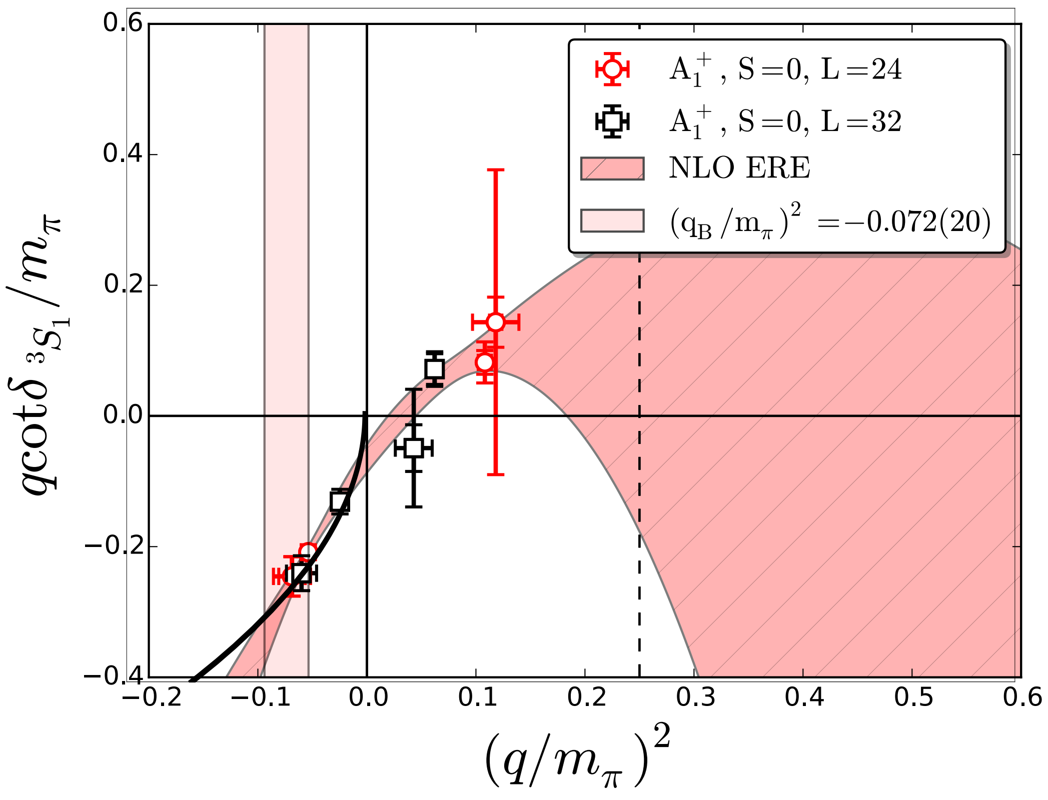
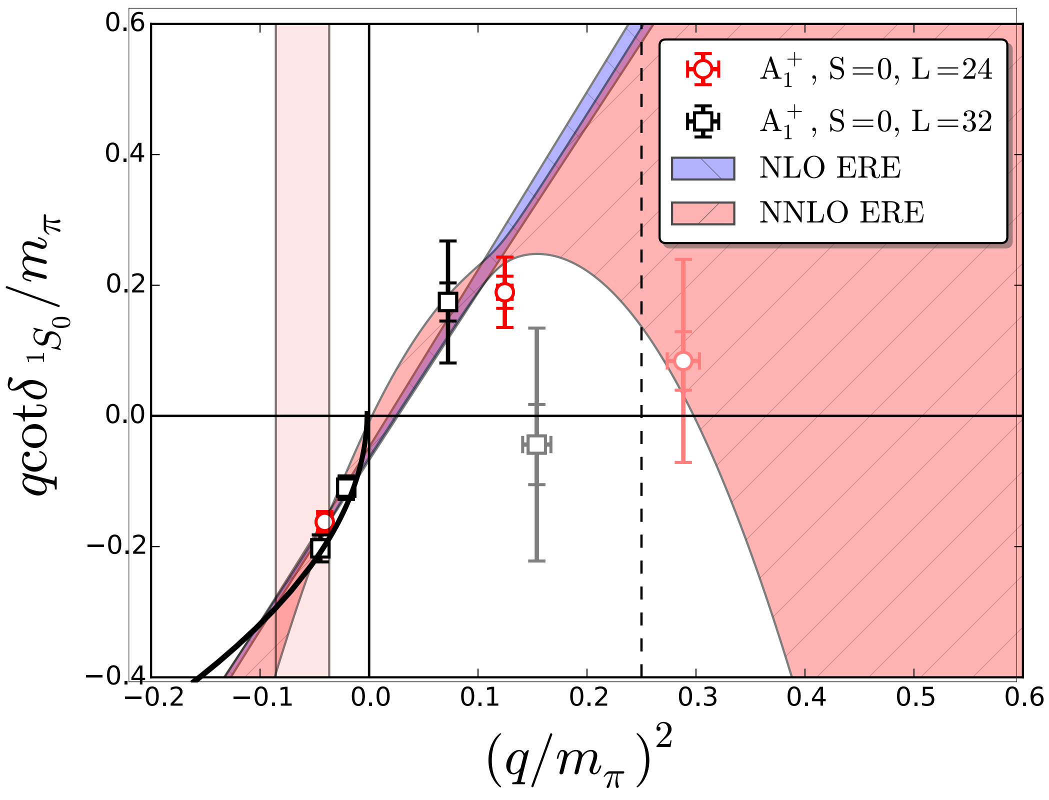
Preliminary

Phase shifts

- Bound States

$\mathrm{p}\cot\delta = \mathrm{ip}$



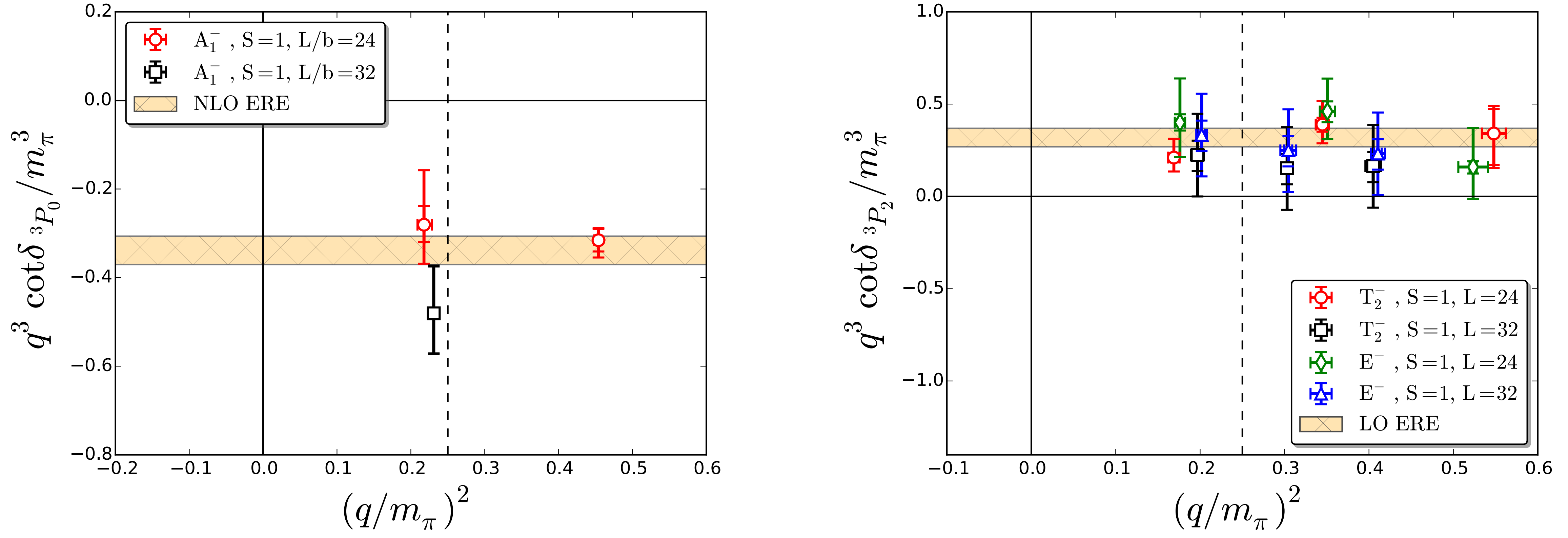


both NPLQCD & Yamazaki, et. al. found relatively deeply bound states

Phase shifts

Preliminary

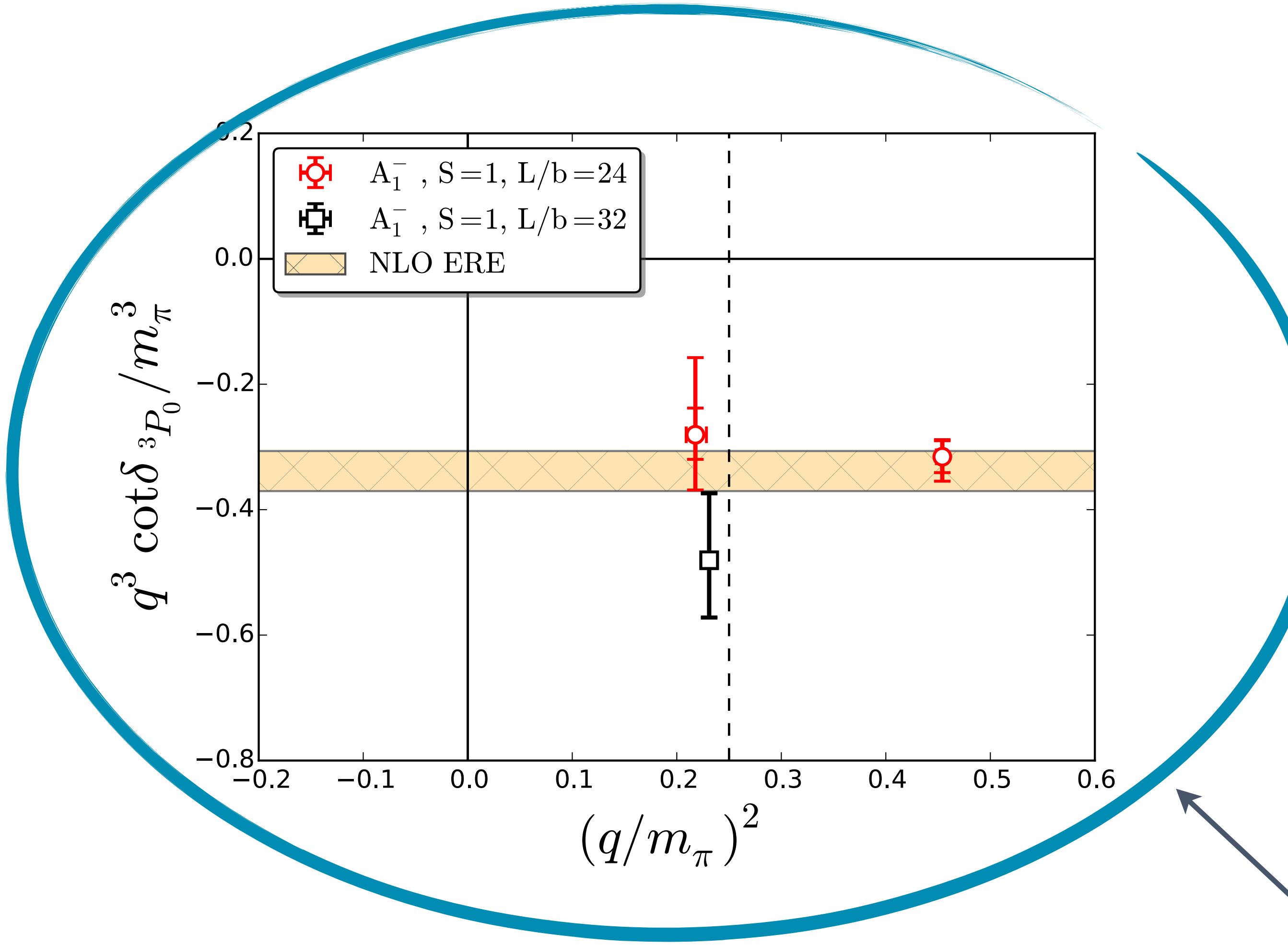
- P-wave

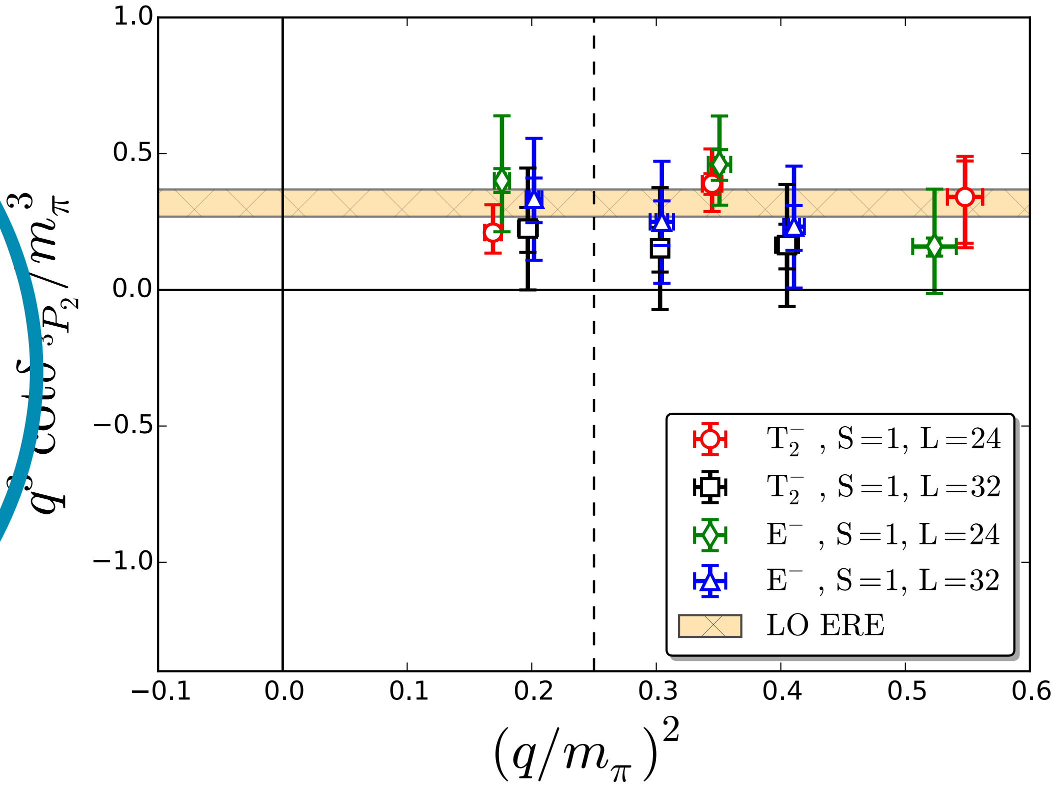


Phase shifts

Preliminary

- P-wave





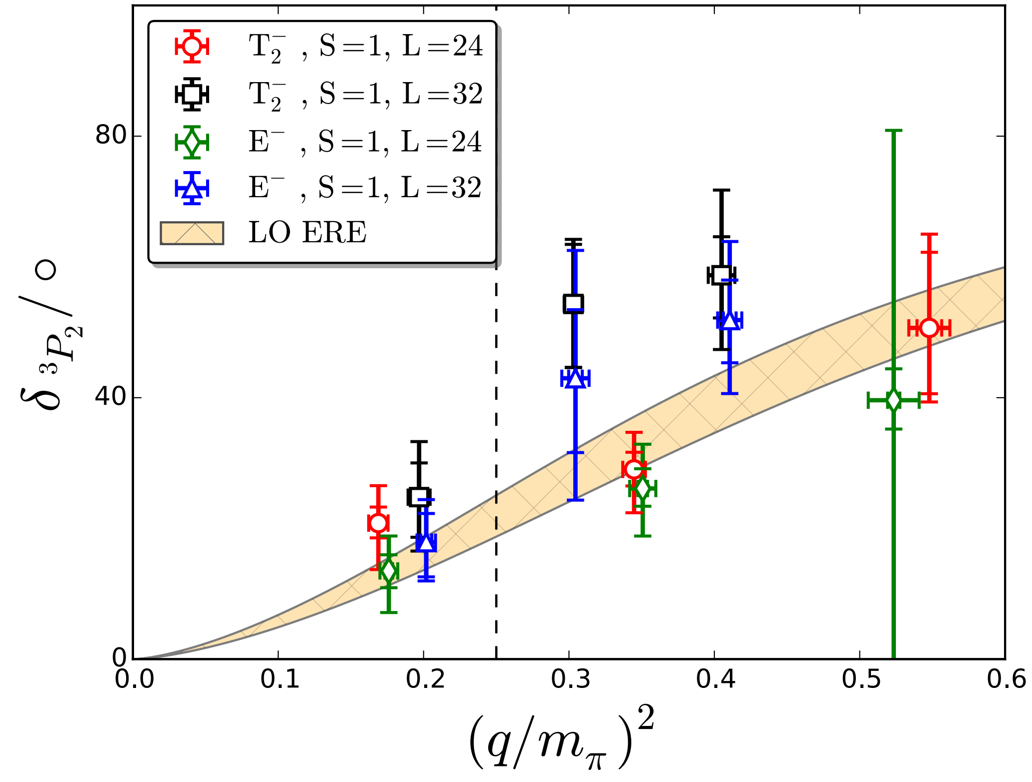
Necessary for nuclear PV calculation (T. Kurth, Tues. 17:50)

Preliminary

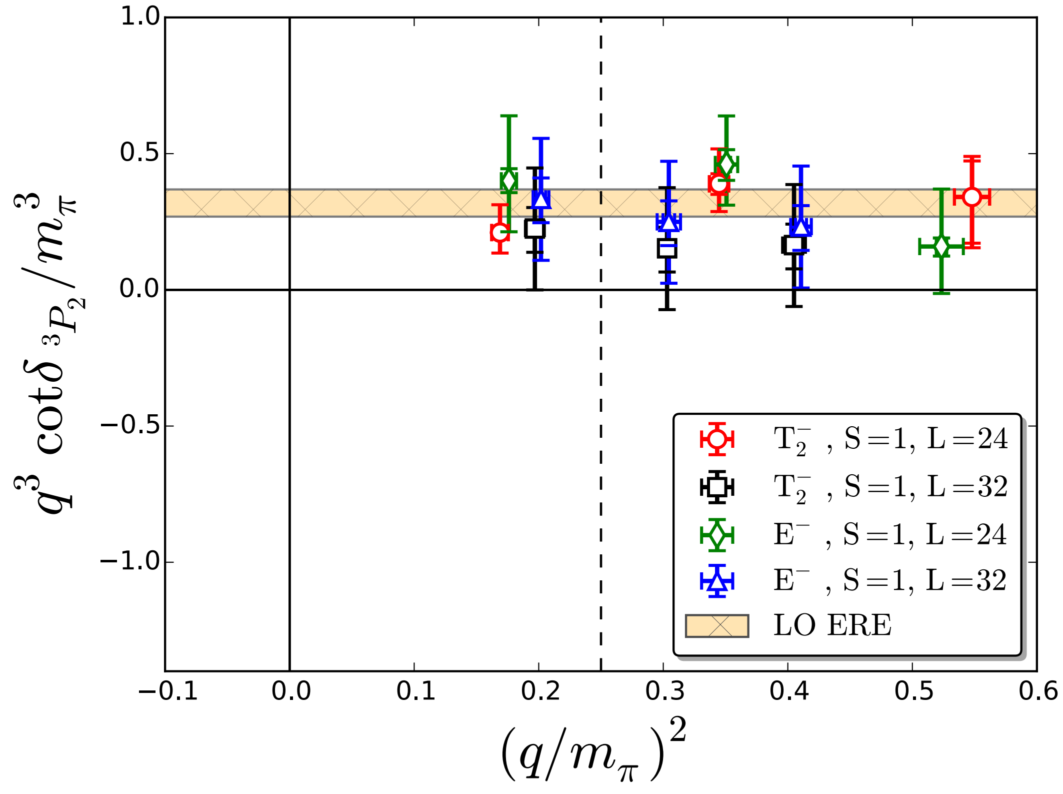
Phase shifts

- P-wave

No evidence for breakdown of
ERE above t-channel cut



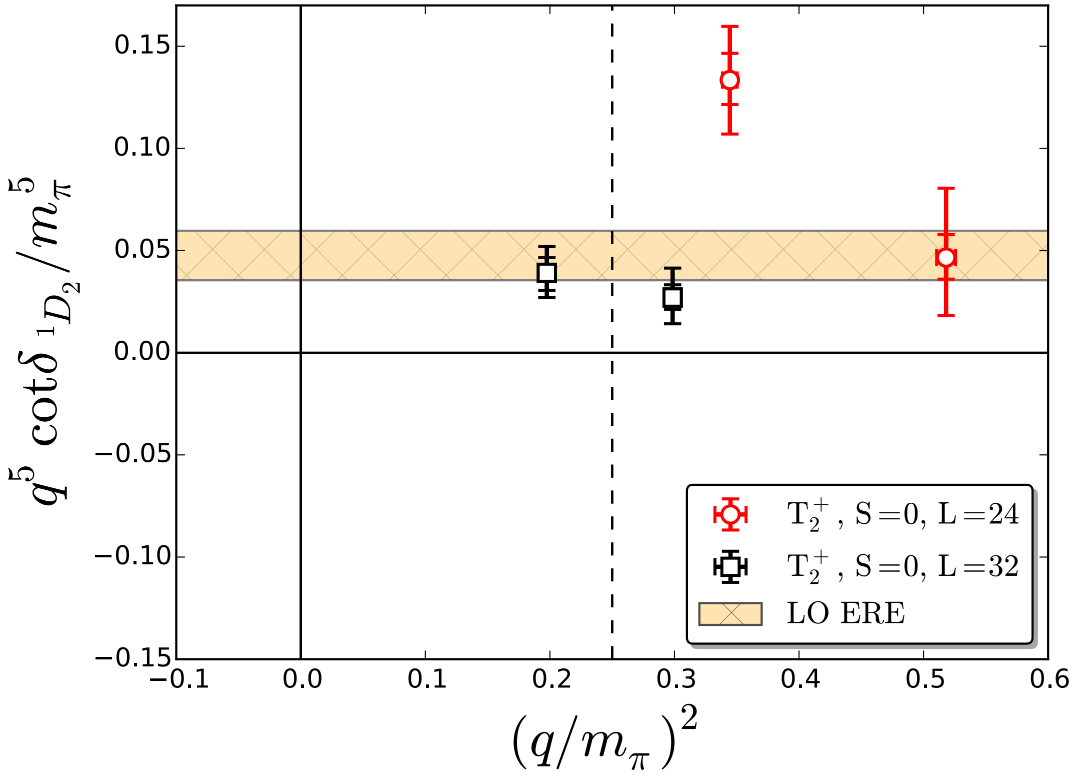
Agrees qualitatively with experiment
(even with 800 MeV pion!) and HalQCD

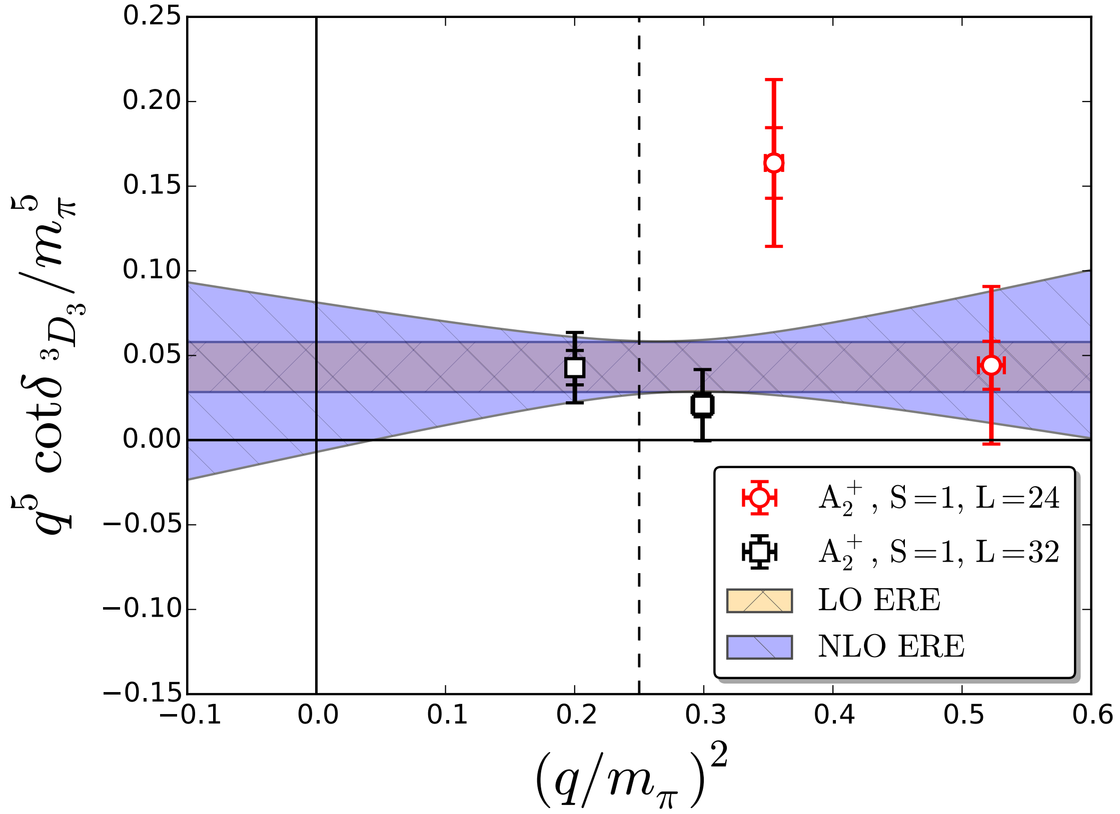


Phase shifts

Preliminary

- D-wave

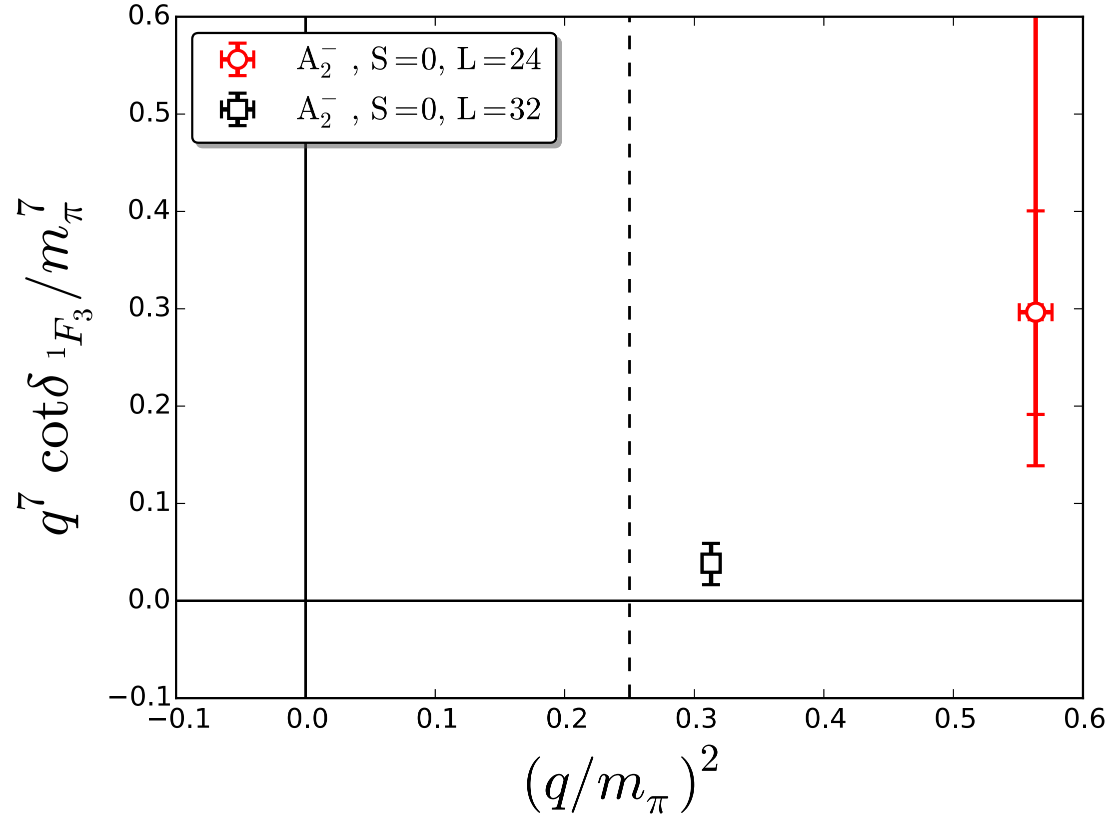




Phase shifts

Preliminary

- F-wave

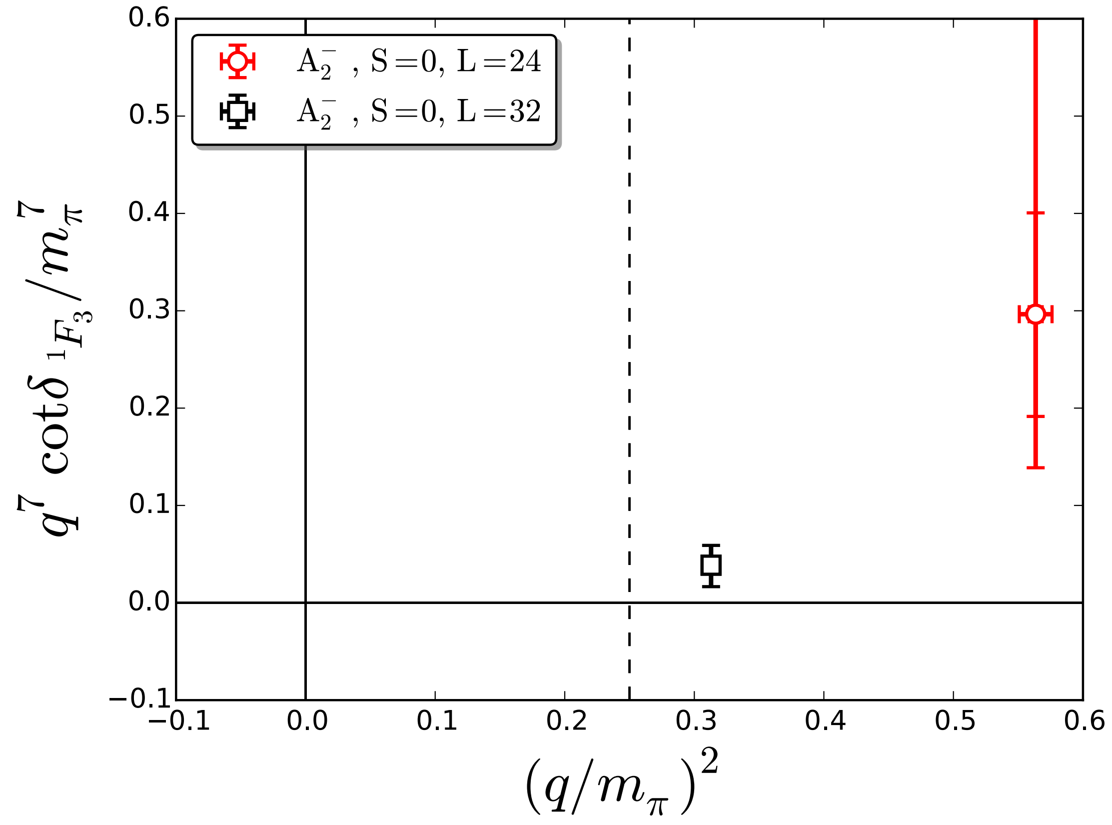


Phase shifts

Preliminary

- F-wave

Phase shift seems to be small
- ok to neglect unphysical mixing in p-wave channels?



Conclusions

- Used Lüscher method to determine nucleon-nucleon scattering phase shifts in S, P, D, and F partial wave channels

- Sophisticated sources/sinks give multiple clearly separated levels in most channels

- Find deeply bound states in ${}^1S_0$ and ${}^3S_1$ channels, in agreement with past works using Lüscher method - possible second bound state not previously found in ${}^3S_1$ channel

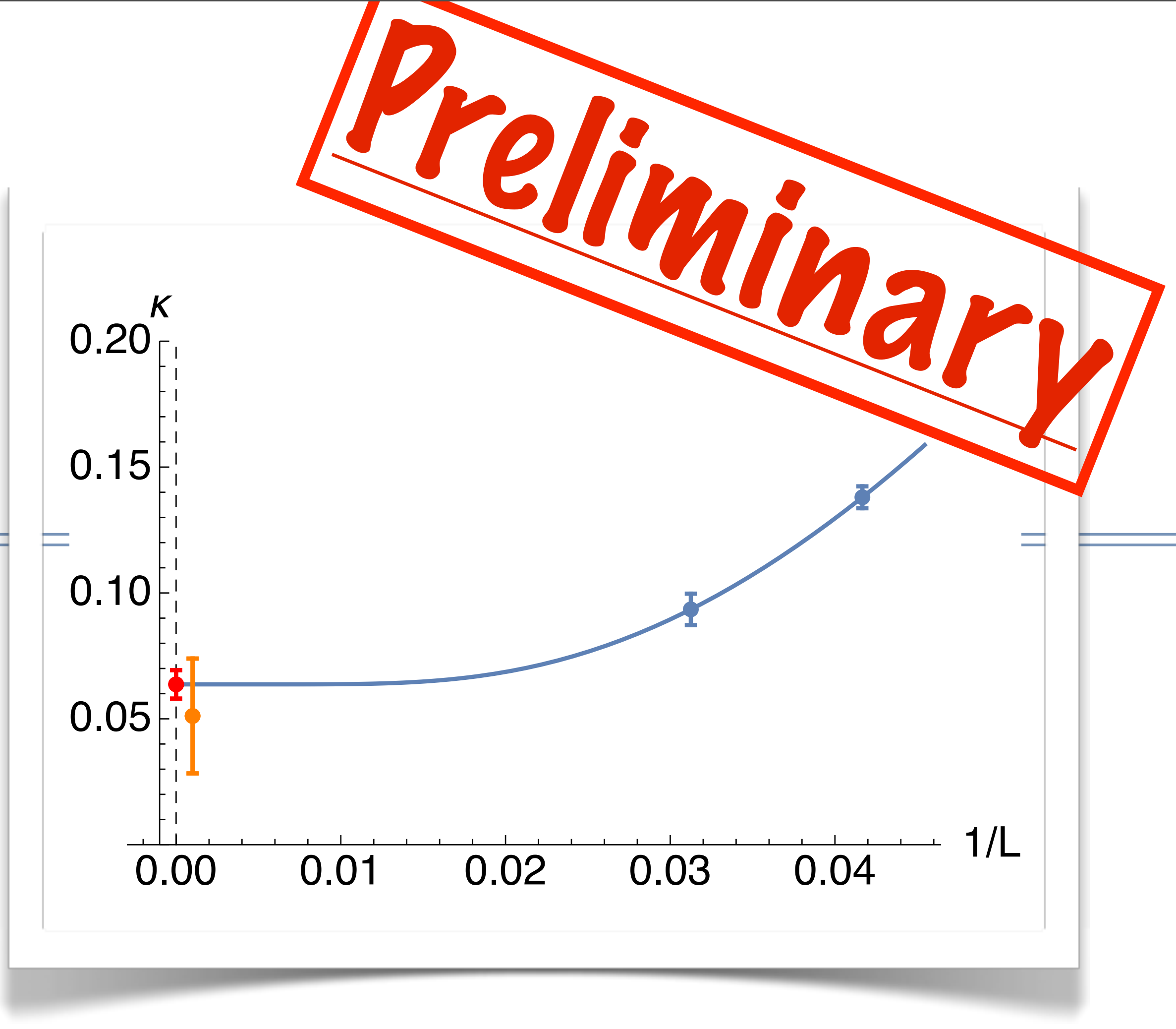
- Success with ${}^1S_0$ and ${}^3P_0$ channels allows us to explore hadronic parity violation (talk by T. Kurth)

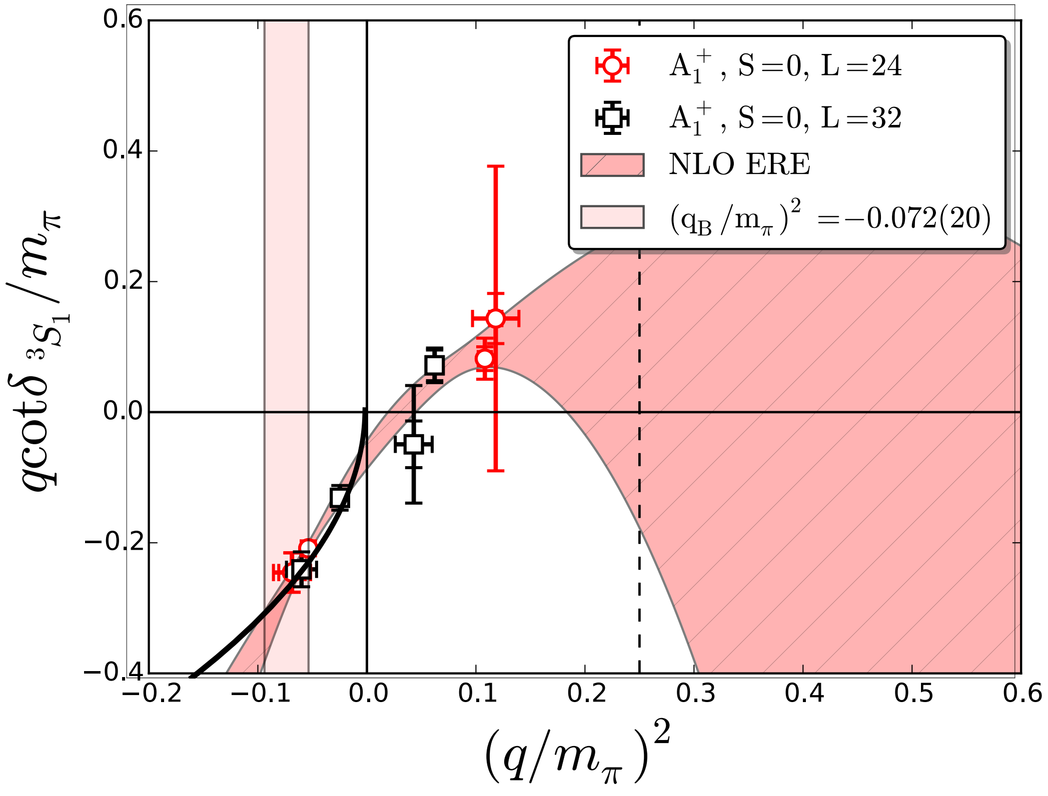
- ${}^3P_2$ channel displays remarkable consistency for different cubic irreps and over a large range of energies

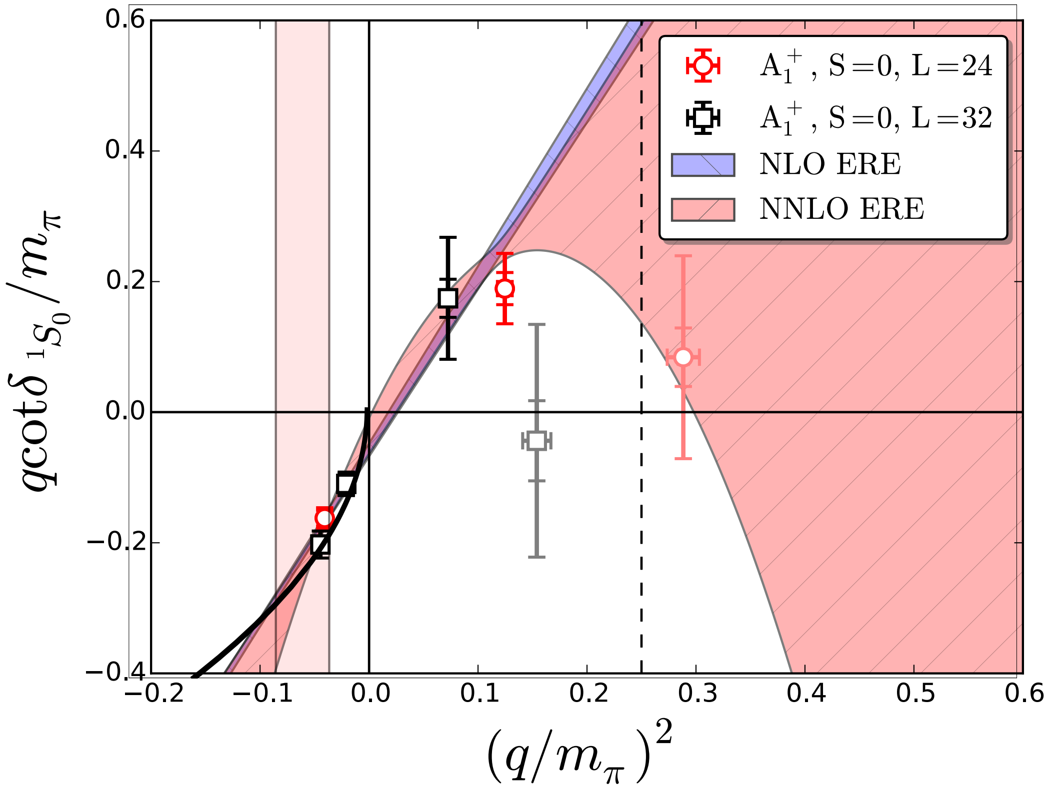
- For the moment, we neglect partial wave mixing - both physical and due to the cubic volume - will include mixing in the future

Phase shifts

- Bound States







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