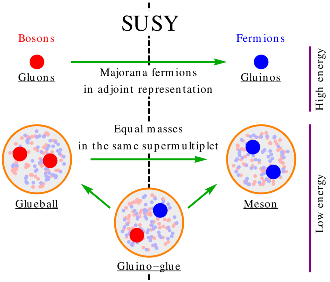
Witten index and phase diagram of compactified $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the lattice



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Münster (WWU)

Presentation based on:

G. Bergner, S. Piemonte: *Compactified N=1 supersymmetric Yang-Mills theory on the lattice: Continuity and the disappearance of the deconfinement transition*, JHEP12(2014)133, arXiv:1410.3668

18 July, 2015

The project

DESY-Münster collaboration:

G. Bergner (Bern), I. Montvay (DESY), G. Münster, P. Giudice, U. D. Özugurel,

S. Piemonte, D. Sandbrink (WWU)

We are interested in gauge models with adjoint fermions, i.e. Minimal Walking Technicolor (MWT) and the $\mathcal{N} = 1$ Supersymmetric Yang-Mills (SYM) theory.

- Masses and bound spectrum of $\mathcal{N} = 1$ SYM $\rightarrow$ Talk of P. Giudice
- Computation of the critical exponent γ for $N_f = 4$ MWT $\rightarrow$ Talk of G. Bergner
- Confinement in $\mathcal{N} = 1$ SYM with one space-time dimension compactified $\rightarrow$ this talk

The action

We study on the lattice Yang-Mills theories with fermions in the adjoint representation of the gauge group $SU(2)$, the action reads in the continuum

$$S = \int d^4x \left\{ \frac{1}{4}(F^a_{\mu\nu}F^a_{\mu\nu}) + \frac{1}{2}\sum_{i=1}^{N_f} \bar{\lambda}^i_a \gamma^\mu D^{ab}_\mu \lambda^i_b \right\}$$

where λ is a Majorana fermion:

$$\begin{aligned} \bar{\lambda}_a &= \lambda^T_a C \\ D^{ab}_\mu \lambda_b &= \partial_\mu \lambda_a + igA^c_\mu (T^A_c)^{ab}\lambda_b \end{aligned}$$

The properties of the theory depend on the number of adjoint Majorana fermions

1. $N_f = 0 \rightarrow$ Pure gauge theory
2. $N_f = 1 \rightarrow \mathcal{N} = 1$ supersymmetric Yang-Mills theory
3. $N_f > 1 \rightarrow$ AdjQCD - technicolor models

Polyakov loop effective potential

Adjoint fermions give a relevant contribution in the large N_c limit, in particular the effective potential for the Polyakov loop is attractive

$$V_{\mathrm{eff}} = (N_f - 1)\,\frac{2}{\pi^2 L^4}\sum_{n=1}^{\infty} |\mathrm{Tr}(\Omega^n)|^2\,, \qquad (1)$$

at one-loop order of perturbation theory for $N_f > 1$. It is zero for $N_f = 1$, i.e. for $\mathcal{N} = 1$ Supersymmetric Yang-Mills theory: confinement is in this last case a result of non-perturbative effects.[1]

The interest is to study confinement when one space-time dimension is compactified.[2]

[1] [N. M. Davies, T. J. Hollowood, V. V. Khoze, M. P. Mattis: Nucl. Phys. B 559 (1999) 123, hep-th/9905015]

[2] [M. Ünsal and L. G. Yaffe: Phys. Rev. D **78** (2008) 065035, hep-th/0803.0344. G. Cossu, M. D'Elia: JHEP, 0907:048 (2009) hep-lat/0904.1353. J. C. Myers, M. C. Ogilvie: JHEP 0907:095 (2009) hep-th/0903.4638.]

Supersymmetry and the Witten index

If the time dimension is compactified, fermion fields could satisfy different boundary conditions (BC):

- Antiperiodic BC, $\lambda(x+L) = -\lambda(x)$. The corresponding path integral is the partition function $Z(T)$

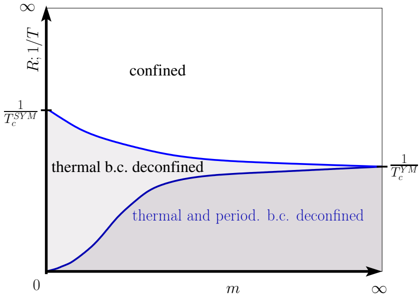
$$Z(T) = \mathrm{Tr}\left\{\exp\left(-\frac{H}{T}\right)\right\} = \sum_{\text{all states}} \exp\left(-\frac{E_i}{T}\right) \tag{2}$$

  at a physical temperature $T = 1/L$.
- Periodic BC, $\lambda(x+L) = \lambda(x)$. The corresponding path integral is the Witten index $W(T)$

$$\begin{aligned} W(T) &= \mathrm{Tr}\left\{(-1)^F \exp\left(-\frac{H}{T}\right)\right\} \\ &= \sum_{\text{bosons}} \exp\left(-\frac{E_i^B}{T}\right) - \sum_{\text{fermions}} \exp\left(-\frac{E_i^F}{T}\right) \end{aligned}$$

Expected phase diagram

Different number of flavors and fermion boundary conditions lead to different phase diagrams:

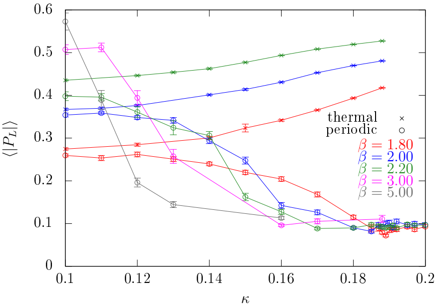


(a) $N_f = 1$

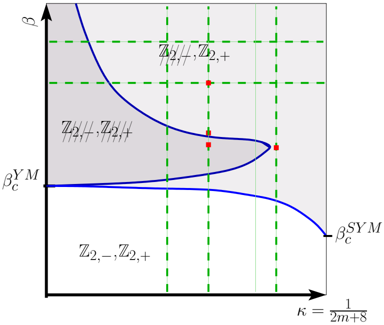
(b) $N_f > 1$

Observed phase diagram

Lattice simulations done with Symanzik improved gauge action, while plain and tree level improved Wilson fermions are used to discretize the gluino action. Different lattice volumes have been important to distinguish between different phases.



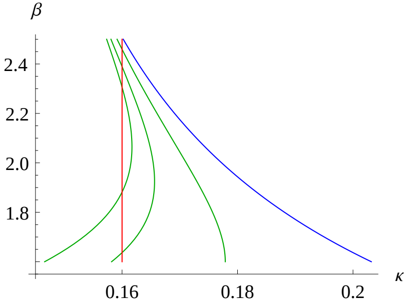
(c) Volume $8^3 \times 4$, $c_{SW} = 0$



(d) Observed phase diagram in the bare phase space

Line of constant physics

Line of constant physics (LCP) $m_g = (w_0 m_\pi)^2 = \mathrm{const}$ for unimproved action:



The expected phase diagram of $N_f > 1$ seems similar to that observed for $N_f = 1$!

Fixed scale approach

The fixed scale approach avoids the systematic uncertainties in the determination of the LCP. Tree level clover improved fermions could reduce lattice artefacts.

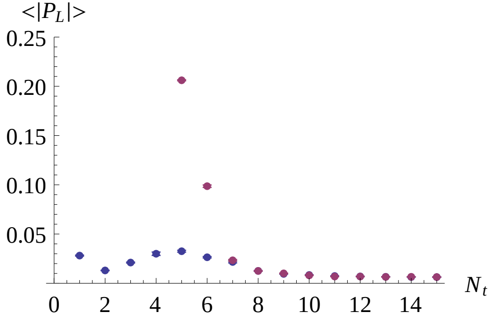
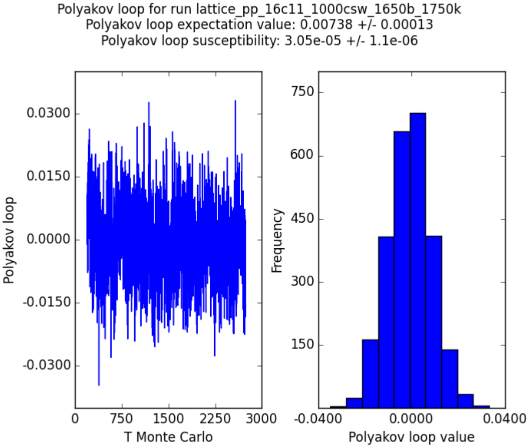


Figure: Polyakov loop expectation value, purple antiperiodic BC, blue periodic BC, lattice $16^3 \times N_t$, $\beta = 1.65$, $c_{SW} = 1.0$, $(am_\pi)^2 = 0.41(1)$.

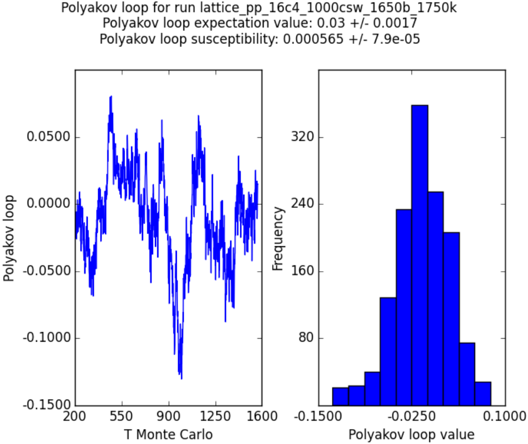
Observed phase diagram

Confinement is clear for large compactification radius, i.e. at $N_t = 11$.



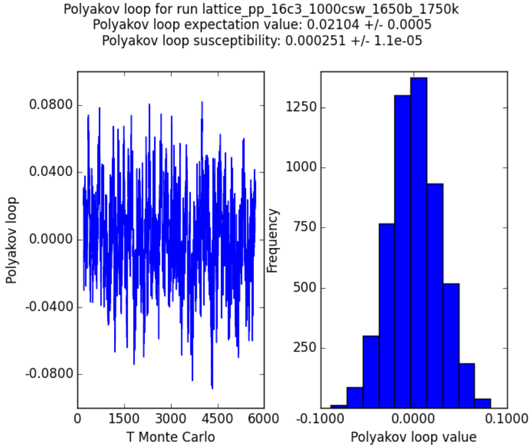
Observed phase diagram

Deconfinement phase transition appears for $N_t = 6 - 4$!



Observed phase diagram

Confinement appears again at $N_t \leq 3$!



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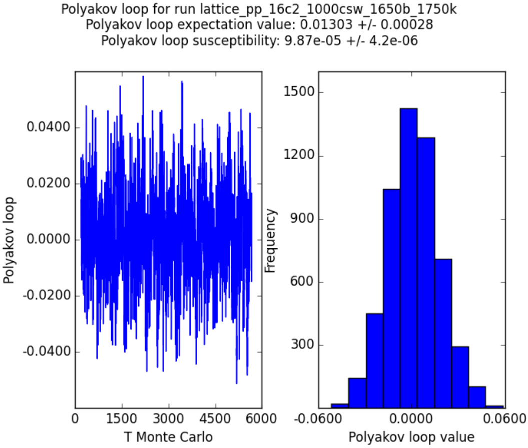

Is supersymmetry restored on the lattice?

The Witten index $W(T)$

$$W(T) = \sum_{\text{bosons}} \exp\left(-\frac{E_i^B}{T}\right) - \sum_{\text{fermions}} \exp\left(-\frac{E_i^F}{T}\right)$$

is a constant for a supersymmetric theory ($= N_c$ in $\mathcal{N} = 1$ SYM).
Supersymmetry is broken on the lattice, since $a \neq 0$ and $m_g \neq 0$.
Derive the expression above

$$E_G(T) = \frac{\partial W(T)}{\partial(1/T)} = \sum_{\text{fermions}} E_i^F \exp\left(-\frac{E_i^F}{T}\right) - \sum_{\text{bosons}} E_i^B \exp\left(-\frac{E_i^B}{T}\right) \doteq 0$$

and subtract a zero temperature value, $E_G(T) - E_G(0)$, to get a "graded energy density" that can be easily computed on the lattice!

Is supersymmetry restored on the lattice?

If supersymmetry is restored, the "graded energy" has to vanish!

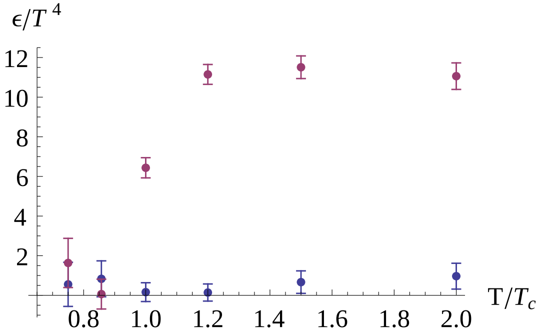

Figure: Energy density for (blue) periodic - (purple) antiperiodic BC.

ϵ is compatible with zero even at $L = 1/(2T_c) \simeq 0.5$ fm. SUSY is restored in the infrared regime for distances $\gtrsim 0.5$ fm!.

Witten index and phase diagram of compactified $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the lattice

Lattice simulations of compactified $\mathcal{N} = 1$ SU(2) SYM have shown:

- First evidence for continuity of confinement in the supersymmetric limit
- Phase diagram of $\mathcal{N} = 1$ SYM similar to the predictions for $N_f > 1$ AdjQCD!

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What next?

1. Smaller lattice spacing simulations
2. Different N_f and N_c phase diagram

Thank you!