Study of the continuum limit of the Schwinger model using Wilson's lattice formulation

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Introduction 1

We know two kinds of lattice formulations.

1. Path-integral formulation

K. Wilson, Phys. Rev. D **10**, 2445 (1974).

- Both the space and time directions are discretized.
- Free from gauge fixing.
- Many successes in QCD with the HMC algorithm.

2. Hamiltonian formulation

J. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).

- Only the space directions are discretized.
- Temporal gauge fixing.
- No success yet in QCD.

Both the formulations shoud give us the same continuum limit.

Introduction 2

The Schwinger model is a good test bed to check the consistency.

There are reliable numerical results in the KS formulation.

★ Related talks in this conference

- B. Buyens, Wed 15 @ Room 406
- H. Saito, Wed 15 @ room 406

Our target is the critical point of the one-flavor Schwinger model in the θ vaccum.

Byrnes *et al.* already succeeded in estimating it with the KS formulation employing the density matrix renormalization group.

T. Byrnes, P. Sriganesh, R. Bursill and C. Hamer, Phys. Rev. D **66**, 013002 (2002).

$$\left(\frac{m}{g}\right)_c = 0.3335(2) \quad \text{at } \theta = \pi$$

Can we get the same value with Wilson's formulation?

Introduction 3

Monte Carlo simulations fail at large θ because of the numerical sign problem.

→ Tensor Renormalization Group (TRG)

M. Levin and C. P. Nave, Phys. Rev. Lett. **99** 120601 (2007).

★ Related presentations in this conference

- H. Kawauchi, Wed 15 @ Room 406
- Y. Meurice, Wed 15 @ room 406
- Y. Yoshimura, Poster (#42)
- A. Bazavov, Sat 18 @ room 404

- extension to fermionic systems → Grassmann TRG

Z.-C. Gu, F. Verstraete and X.-G. Wen, arXiv:1004.2563 [cond-mat.str-el].

- for gauge theories → Decorated TRG

B. Dittrich, S. Mizera and S. Steinhaus, arXiv:1409.2407 [gr-qc].

We combine the GTRG with the DTRG, and tackle the one-flavor Schwinger model at $\theta = \pi$.

The unimproved Wilson fermion is employed.

Tensor network formulation

We translate the partition function into a tensor network form.

$$Z = \int d\psi\, d\bar{\psi} dU\, e^{-S_f[\psi,\bar{\psi}]-S_g[U]}$$ partition function

e^{-S_f} is decomposed by using properties of Grassmann numbers.

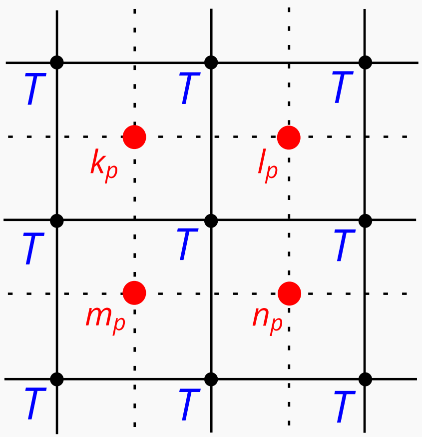
e^{-S_g} is expanded by the character expansion.

$$\exp\left\{\beta \cos\varphi_p + i\frac{\theta}{2\pi}q_p\right\} \simeq \sum_{k_p=-N_{c.e.}}^{N_{c.e.}-1} e^{ik_p\varphi_p} \sum_{l=-\infty}^{\infty} I_l(\beta)\frac{2\sin\frac{\theta+2\pi(k_p-l)}{2}}{\theta+2\pi(k_p-l)}$$

A. S. Hassan, M. Imachi and H. Yoneyama, Prog. Theor. Phys. **93** 161 (1995).

I_k : modified Bessel function, $N_{c.e.}$: truncation number of C.E.

decorated tensor network



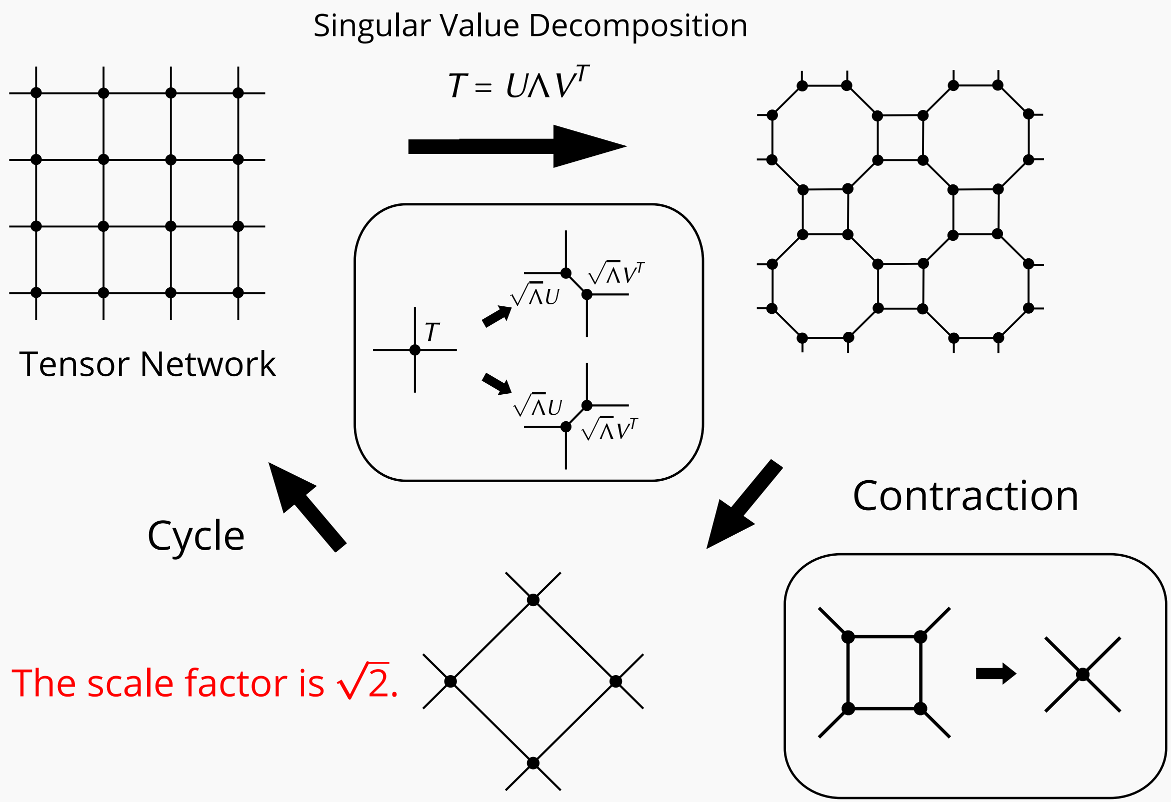
After integrating out all link variables,

$$Z = \int \sum_{i,j,k,l,\cdots} T_{i,m,n,l} T_{s,t,i,j} T_{r,j,k,q} T_{k,l,o,p} \cdots$$

$T_{i,j,k,l}$ includes Grassmann numbers ψ, $\bar{\psi}$, $d\psi$, $d\bar{\psi}$.

Each $T_{i,j,k,l}$ is decorated by plaquette indices $\{k_p, l_p, \cdots\}$.

Tensor renormalization group 1



The scale factor is $\sqrt{2}$.

The key idea is low-rank approximation by the SVD.

$$T_{i,j,k,l} \simeq \sum_{m=1}^{D_{cut}} U_{(i,j),m} \Lambda_m V_{m,(k,l)}$$

truncated by a number D_{cut}!

We keep only large singular values.

Tensor renormalization group 2

The TRG fails to integrate out some short-range correlations.

Z.-C. Gu and X.-G. Wen, Phys. Rev. B **80**, 155131 (2009).

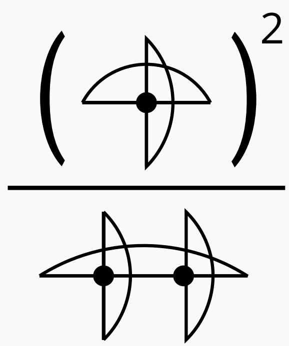
Recently, Evenbly and Vidal developed an improvement which can integrate out all short-range correlations, but its computational cost is rather expensive.

G. Evenbly and G. Vidal, arXiv:1412.0732 [cond-mat.str-el].

We can still derive some information about long-distance physics from the coarse-grained tensor obtained by the unimproved TRG.

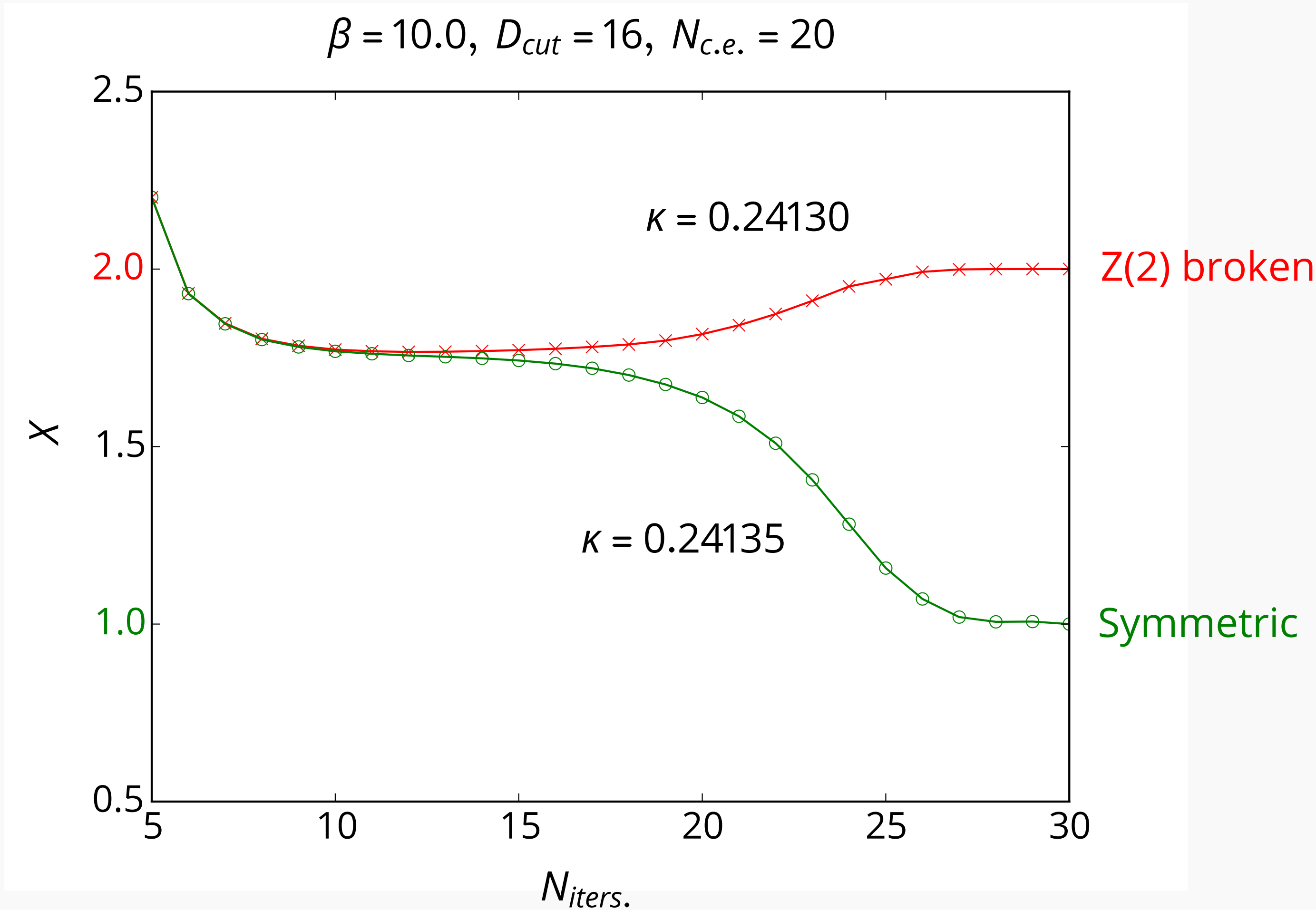
Gu and Wen proposed the following quantity for such purpose.

$X \equiv$

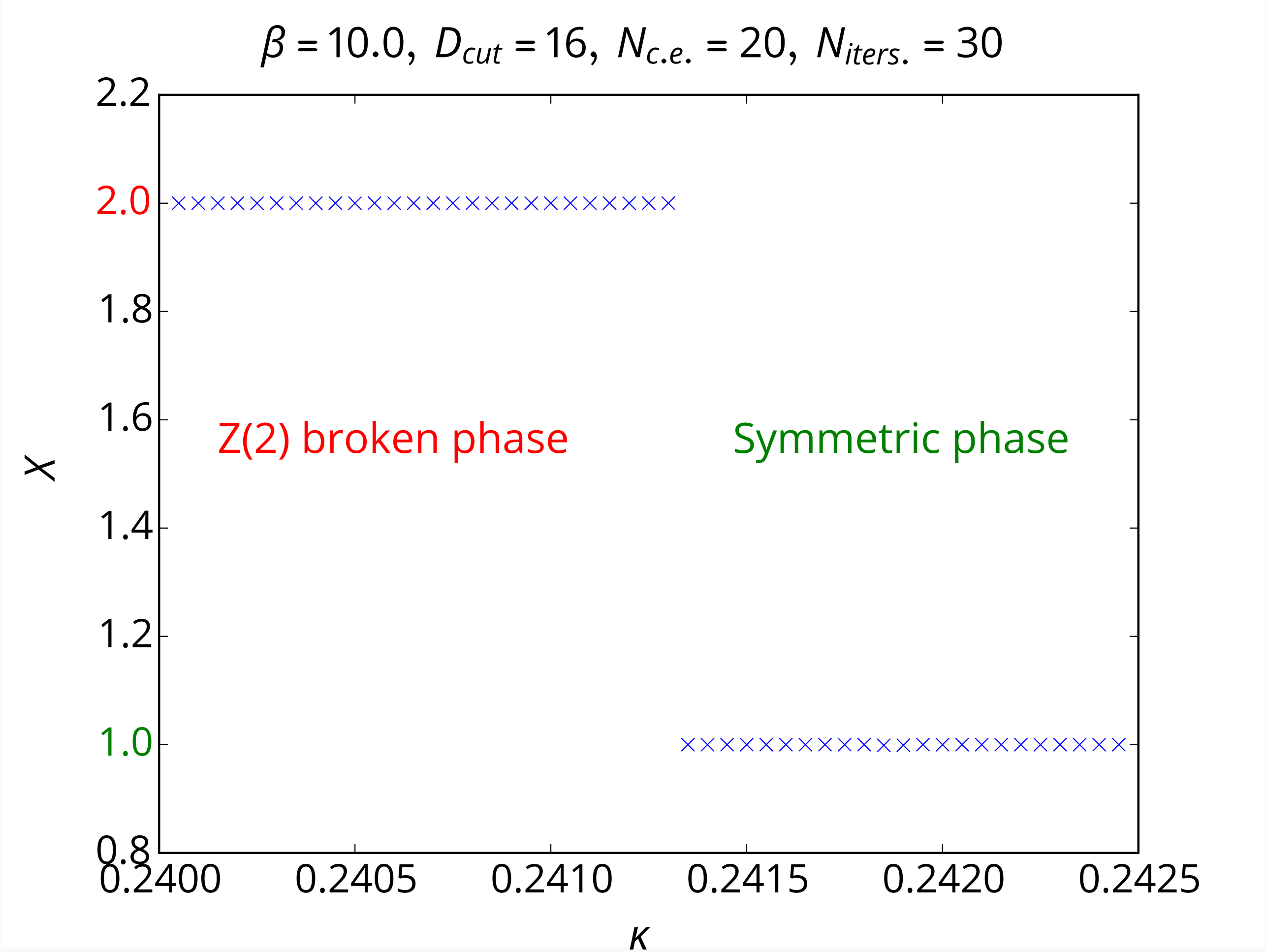


$=$ # of degeneracy of the vacuum in the infrared limit

Numerical results 1

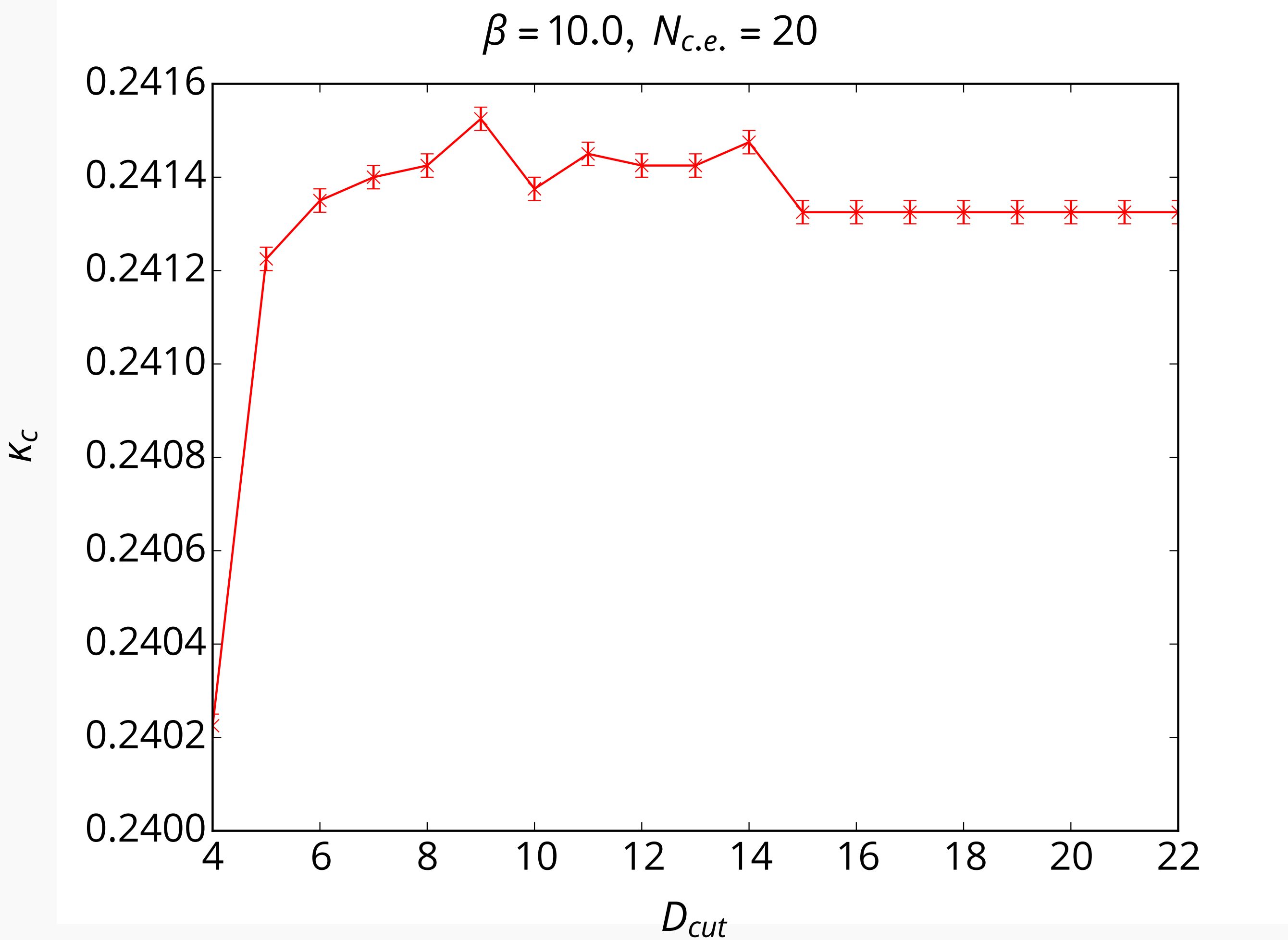


After many iterations, each flow reaches the different fixed point.

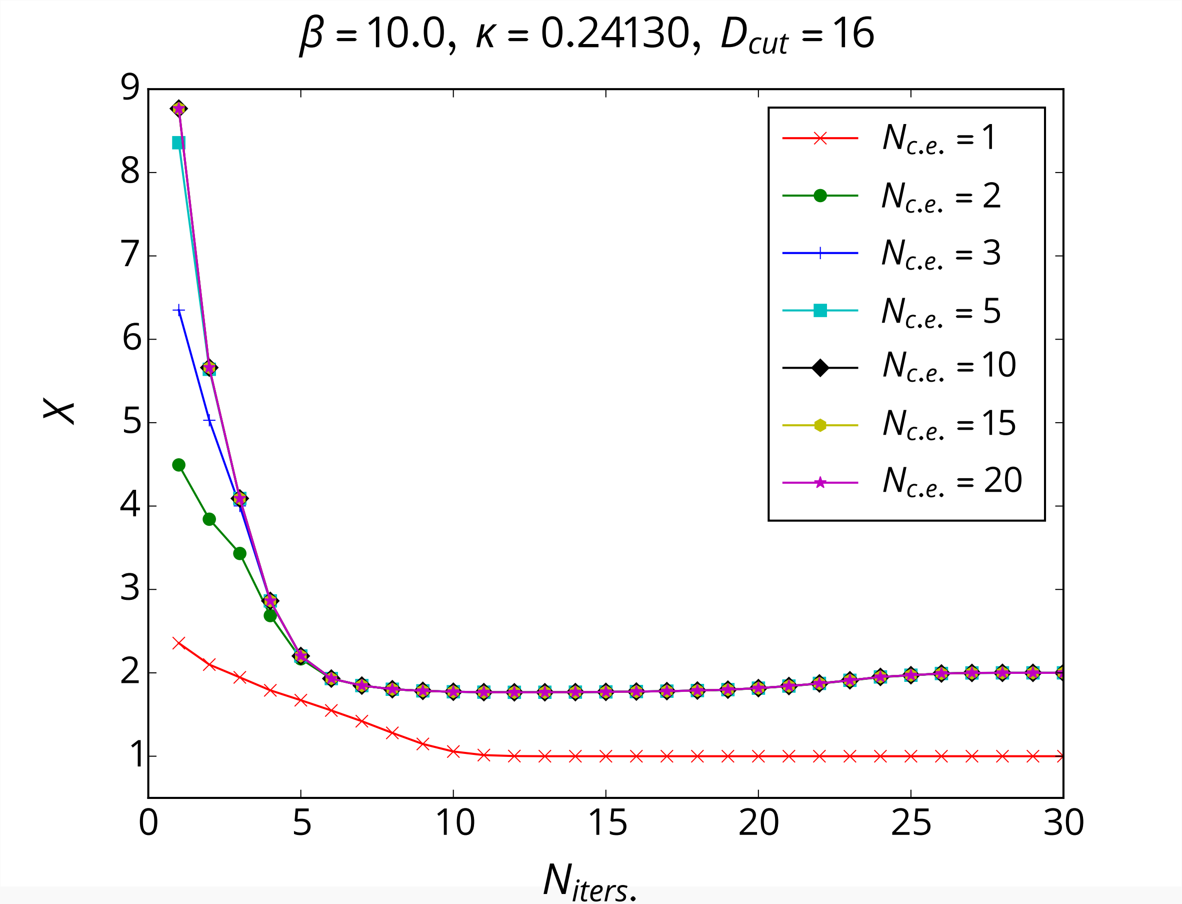


It clearly shows a Z(2)-breaking phase transition.

Numerical results 3



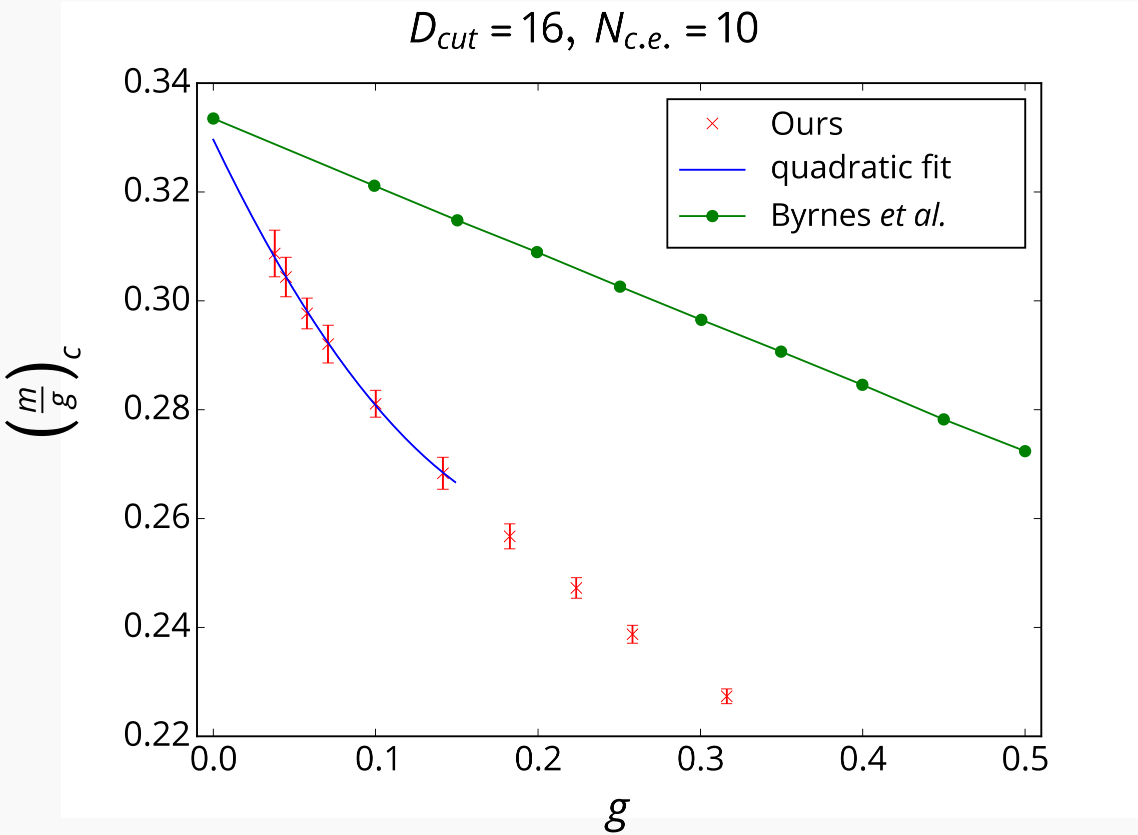
κ_c is well converged where $D_{cut} \geq 15$.



$N_{c.e.} = 2$ seems to be sufficient to reach the correct fixed point.

The fact encourages us to go to higher β with rather small $N_{c.e.}$.

Numerical results 5



$$m = \frac{1}{2\kappa} - 2$$

$$g = \frac{1}{\sqrt{\beta}}$$

Continuum extrapolation:

$$\left(\frac{m}{g}\right)_c = 0.330(9), \quad \text{fit range is } \beta \in [50, 700].$$

Our result is consistent with that of Byrnes *et al.* but less accurate.

Summary and Outlook

We have combined the GTRG with the DTRG and applied it to the one-flavor Schwinger model in Wilson's lattice formulation.

The critical point we estimated agrees with that of Byrnes *et al.*, which was derived from the KS lattice formulation.

Our estimate is less accurate because we employed the unimproved Wilson fermion while they employed the KS fermion.

The DTRG allows us to introduce the clover-improved Wilson fermion, but naive implementation requires much more computational resources.

We should try it in case of $N_{c.e.} = 2$ first.