



Narodowe Centrum Nauki

Gluon density and gluon saturation

Krzysztof Kutak



Supported by NCN with Sonata BIS grant

Based on:

Small-x dynamics in forward-central dijet decorrelations at the LHC

A. van Hameren, P. Kotko, K. Kutak, S. Sapeta Phys.Lett. B737 (2014) 335-340

Saturation effects in forward-forward dijet production in p+Pb collisions

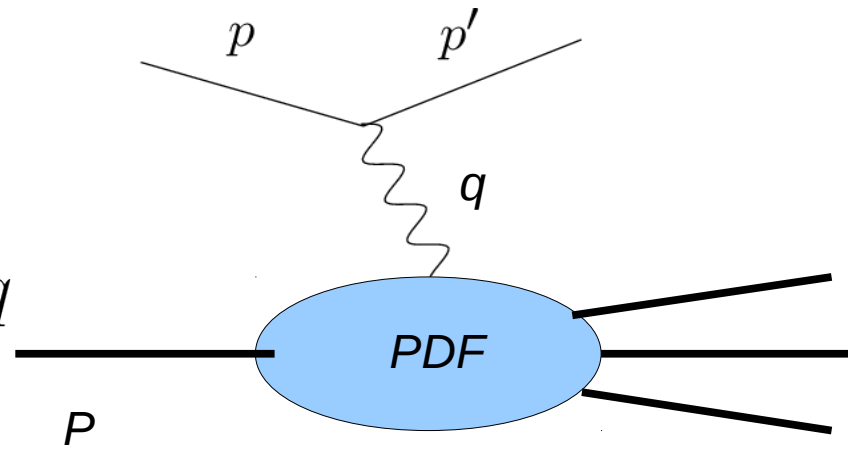
A. van Hameren, P. Kotko, K. Kutak, C. Marquet, S. Sapeta. Phys.Rev. D89 (2014) 9, 094014

Introduction

$$x = \frac{Q^2}{2P \cdot q}$$

$$W^2 = (q + P)^2 = -Q^2 + 2P \cdot q$$

$$q^2 = -Q^2$$



Q is the largest scale, DGLAP evolution of PDF

W is the largest scale x is small i.e. $x < 0.01$, BFKL, BK, CCFM, evolution of PDF

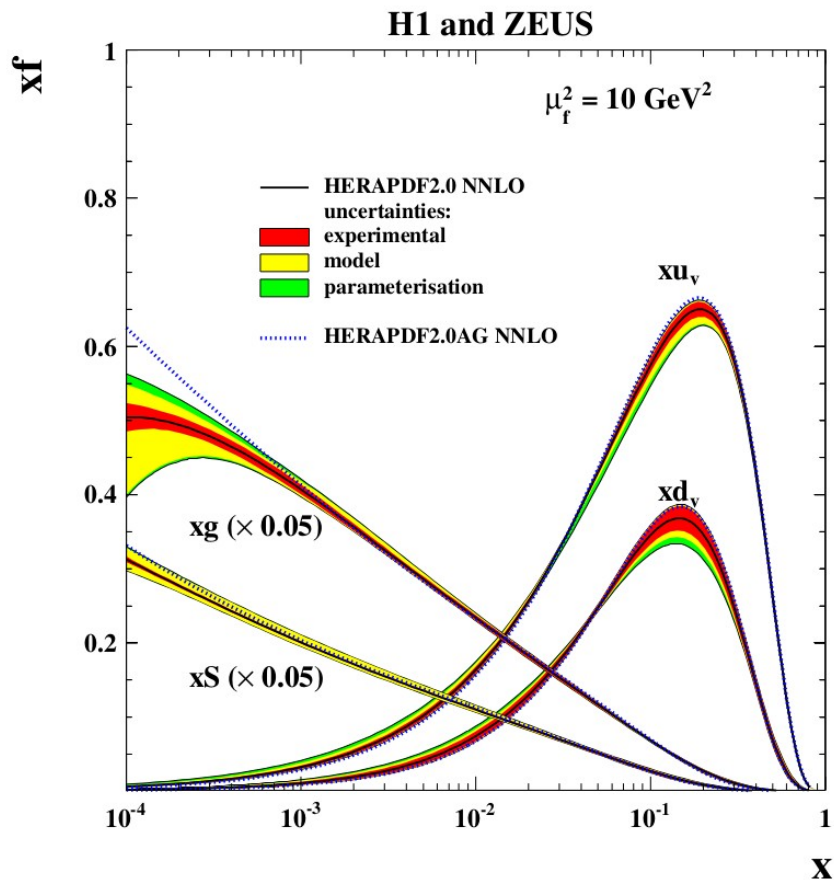
Lipatov, Fadin, Kuraev '77

Ciafaloni '89, Catani, Fiorani, Marchesini,

89Balitsky '96, Kovchegov, 98

Structure of the proton

For example: DIS experiments at DESY



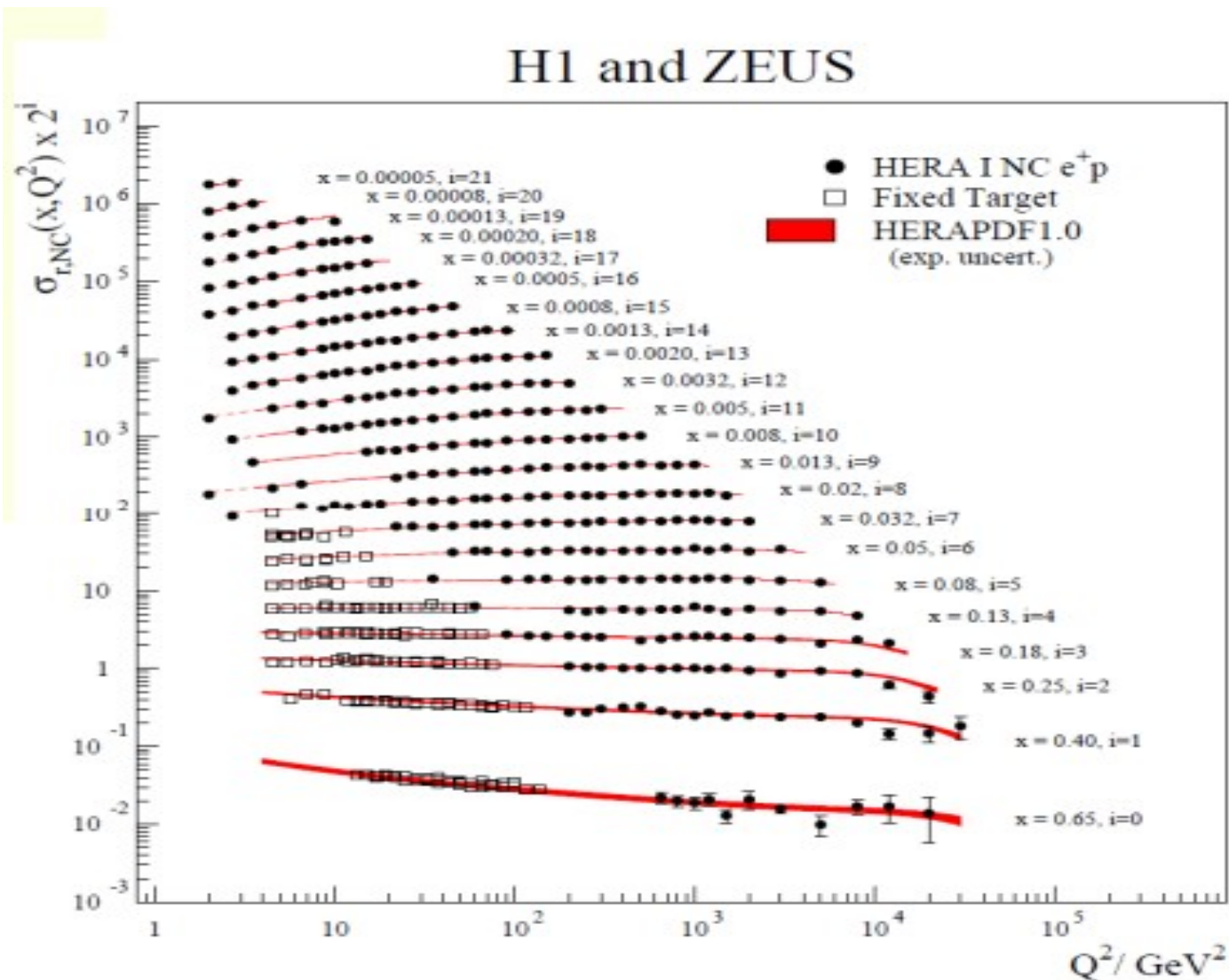
1506.06042

For many processes the accuracy of evaluation of matrix elements is higher than evaluation of pdfs.

Example is total cross section for Higgs $N^3\text{LO}$ theoretical uncertainty is 4% and **uncertainty due to pdf choice is 10 %** talk at “Parton showers and resummations 2015”. Sven Olaf-Moch

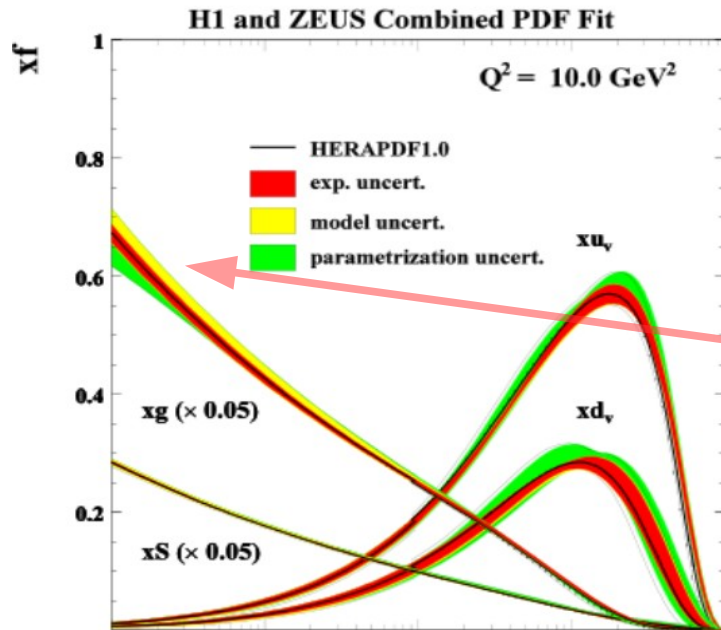
Note the uncertainty of gluon. **Even valence like shape allowed**

Structure function data



Good fit but strongly depends on the form of assumed input

Unitarity problem arises



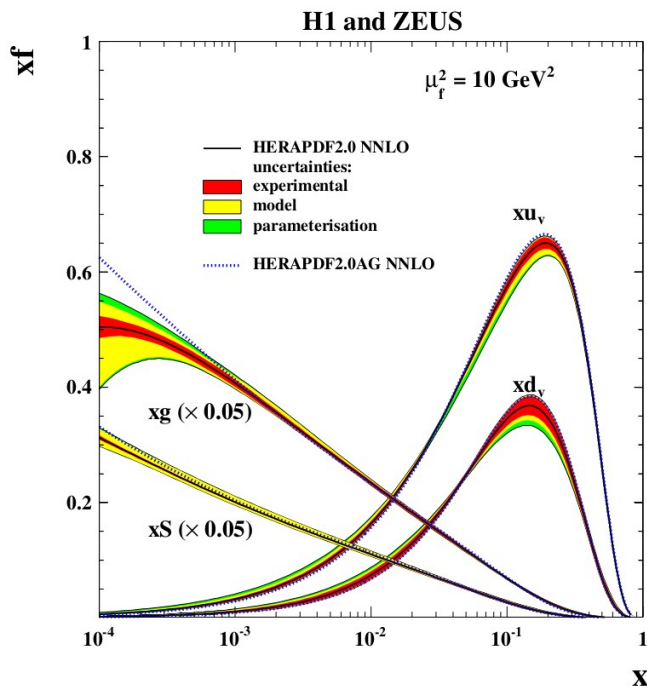
0911.0884

Strong power like growth of gluon density may lead to violation of unitarity bound.

$$\sigma_{\text{tot}} \stackrel{E \rightarrow \infty}{\leq} \ln^2 E$$

Limitations in going to low values of Q^2

NNLO has large effect on gluon

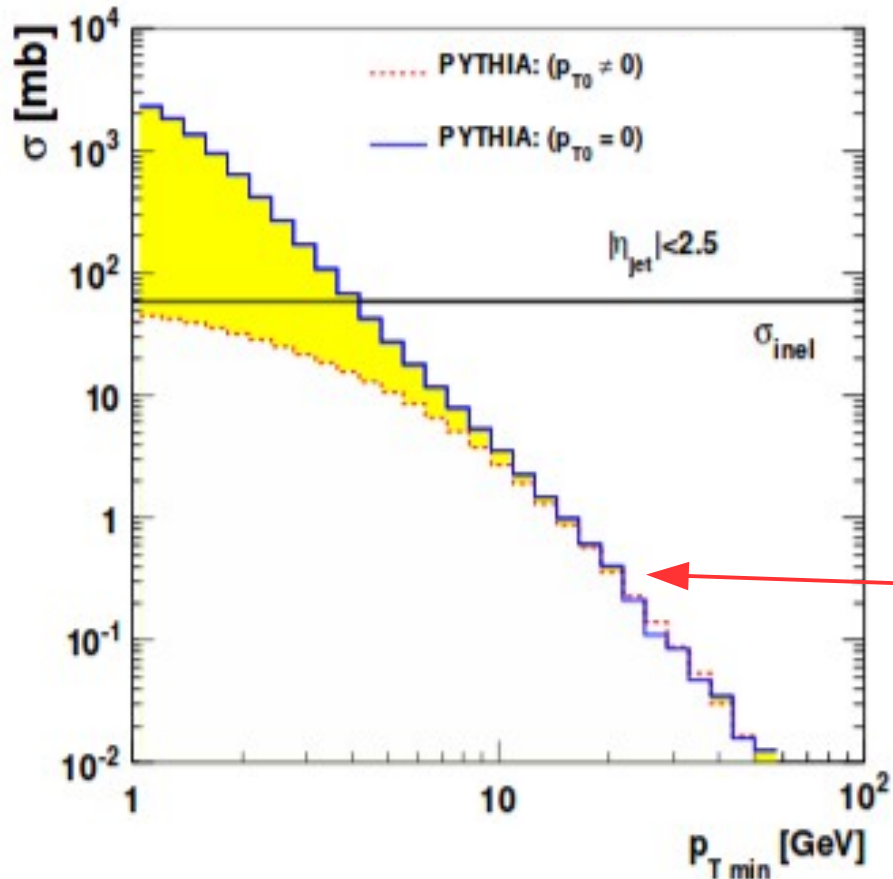


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Example of onset of unitarity problems

Theoretical relevance for onset of low x effects

Grebenyuk, Hautmann, Jung '12.



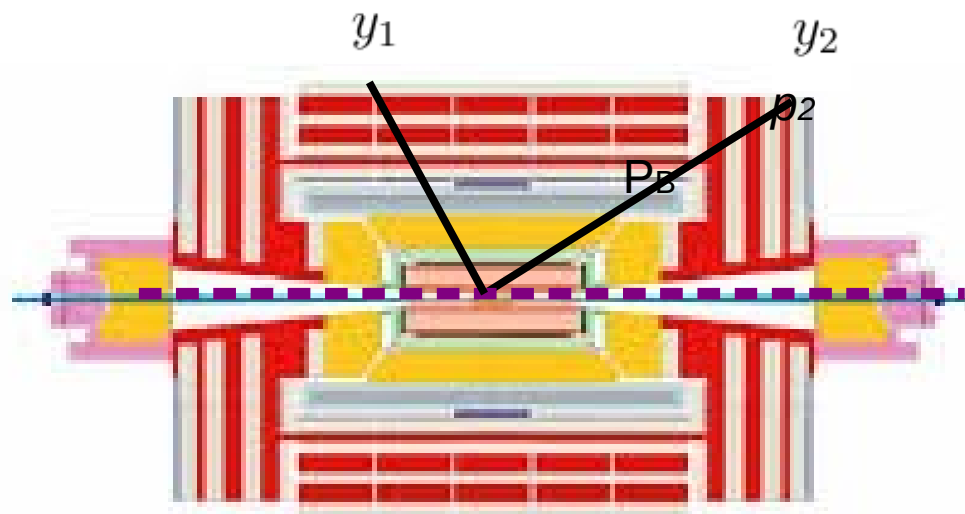
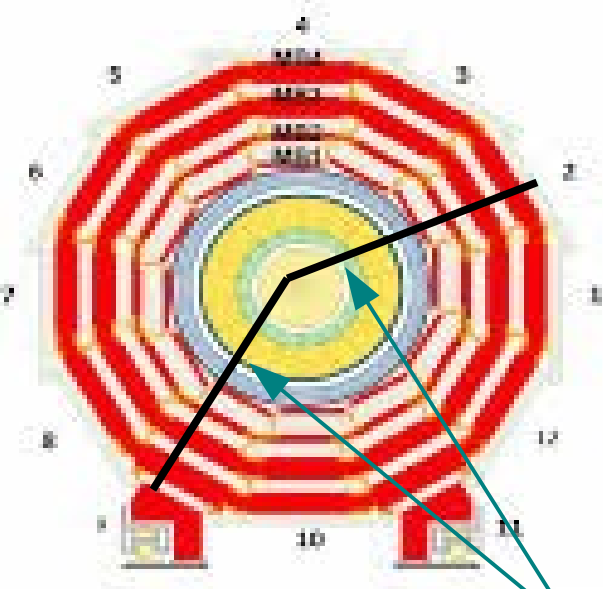
Various approaches show that suppression of gluon density at low p_t is relevant for jet observables and for unitarization.

By hand suppression of growth of gluon density applied

Total cross section for dijet as a function of minimal p_t carried by jets

Dijets at LHC

$$\frac{d\sigma}{d^2q_{1\perp}d^2q_{2\perp}dy_1dy_2} = \sum_{ij} \int dx_1x_2 f_{i/1}(x_1, \mu^2) f_{j/2}(x_2, \mu^2) \frac{d\hat{\sigma}_{ij}}{d^2q_{1\perp}d^2q_{2\perp}dy_1dy_2}$$



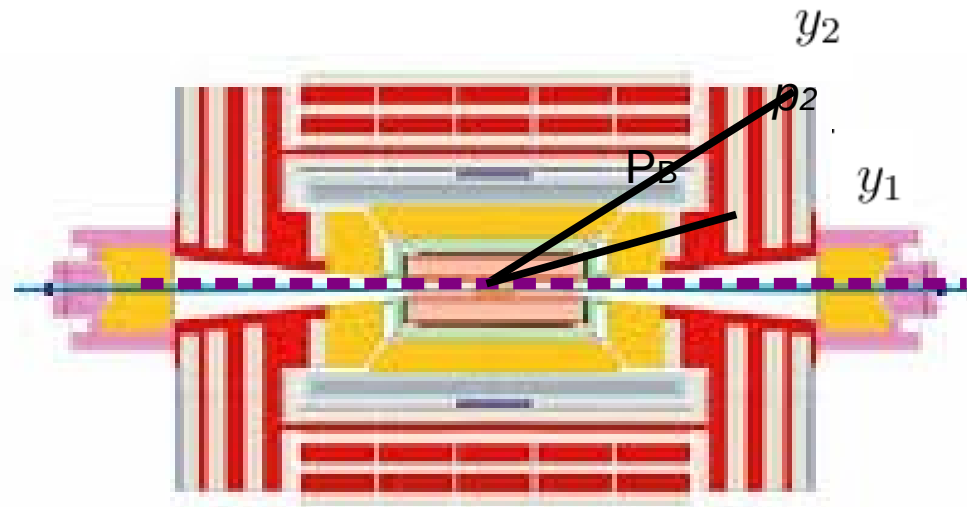
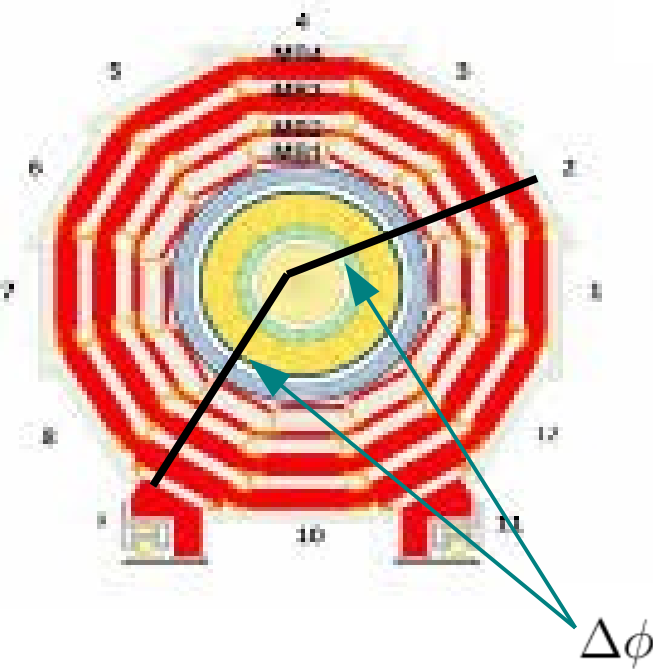
$\Delta\phi$

$$x_1 = \frac{q_{\perp}}{\sqrt{s}} e^{y_1} + \frac{q_{\perp}}{\sqrt{s}} e^{y_2} \quad \sim 1$$

$$x_2 = \frac{q_{\perp}}{\sqrt{s}} e^{-y_1} + \frac{q_{\perp}}{\sqrt{s}} e^{-y_2} \quad \ll 10^{-3}$$

Dijets at LHC

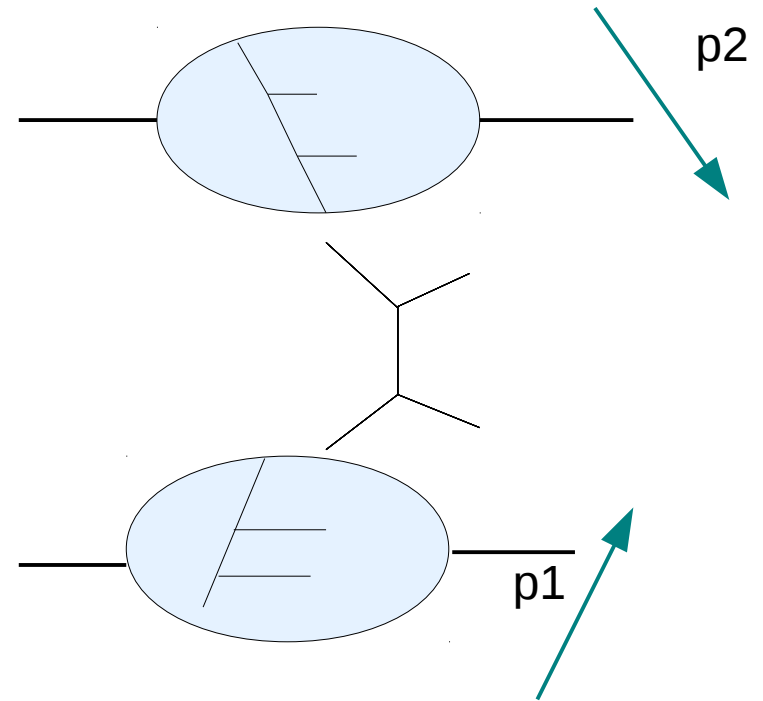
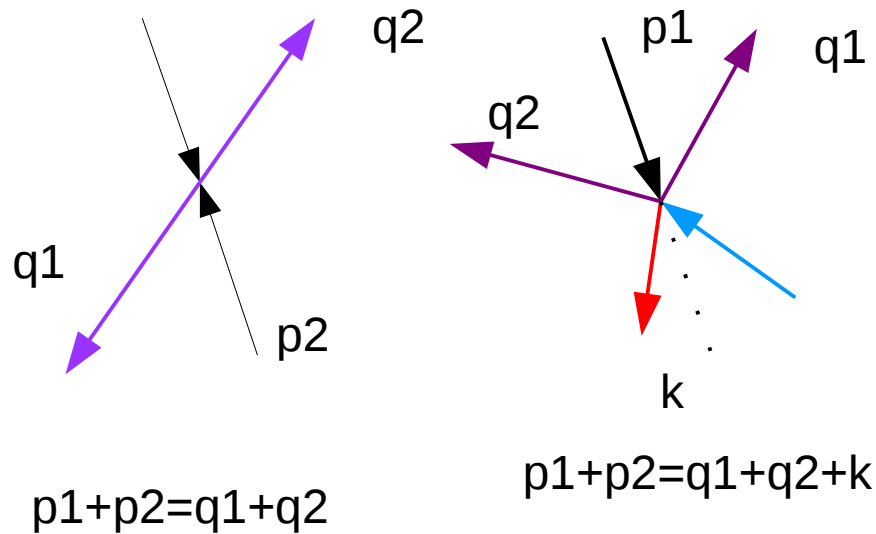
$$\frac{d\sigma}{d^2q_{1\perp}d^2q_{2\perp}dy_1dy_2} = \sum_{ij} \int dx_1x_2 f_{i/1}(x_1, \mu^2) f_{j/2}(x_2, \mu^2) \frac{d\hat{\sigma}_{ij}}{d^2q_{1\perp}d^2q_{2\perp}dy_1dy_2}$$



$$x_1 = \frac{q_{\perp}}{\sqrt{s}} e^{y_1} + \frac{q_{\perp}}{\sqrt{s}} e^{y_2} \sim 1$$

$$x_2 = \frac{q_{\perp}}{\sqrt{s}} e^{-y_1} + \frac{q_{\perp}}{\sqrt{s}} e^{-y_2} \ll 10^{-4}$$

QCD at high energies – hybrid high energy factorization



Decreasing longitudinal momentum fractions of off-shell partons

Deak, Jung, K.K, Hautmann '09

New helicity based methods for ME

Kutak, van Hameren, Kotko, '12

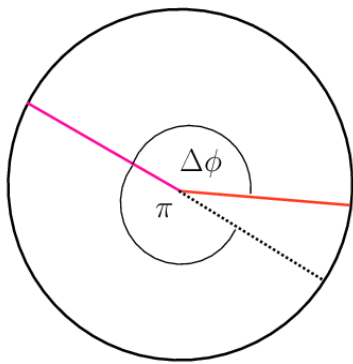
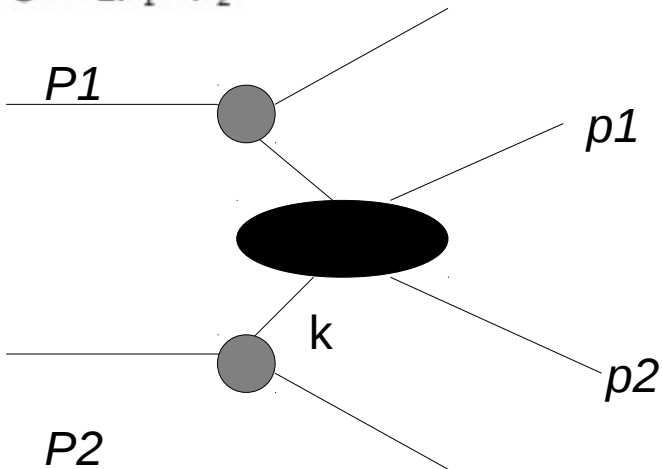
Hybrid high energy factorization

Deak, Jung, K.K, Hautmann '09

$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} |\mathcal{M}_{ag \rightarrow cd}|^2 x_1 f_{a/A}(x_1, \mu^2) \mathcal{F}_{g/B}(x_2, k^2) \frac{1}{1 + \delta_{cd}}$$

In general description of dijets requires much more complicated formula. However, the formula above can give us estimates of Saturation effects. The formula is strictly valid in linear regime

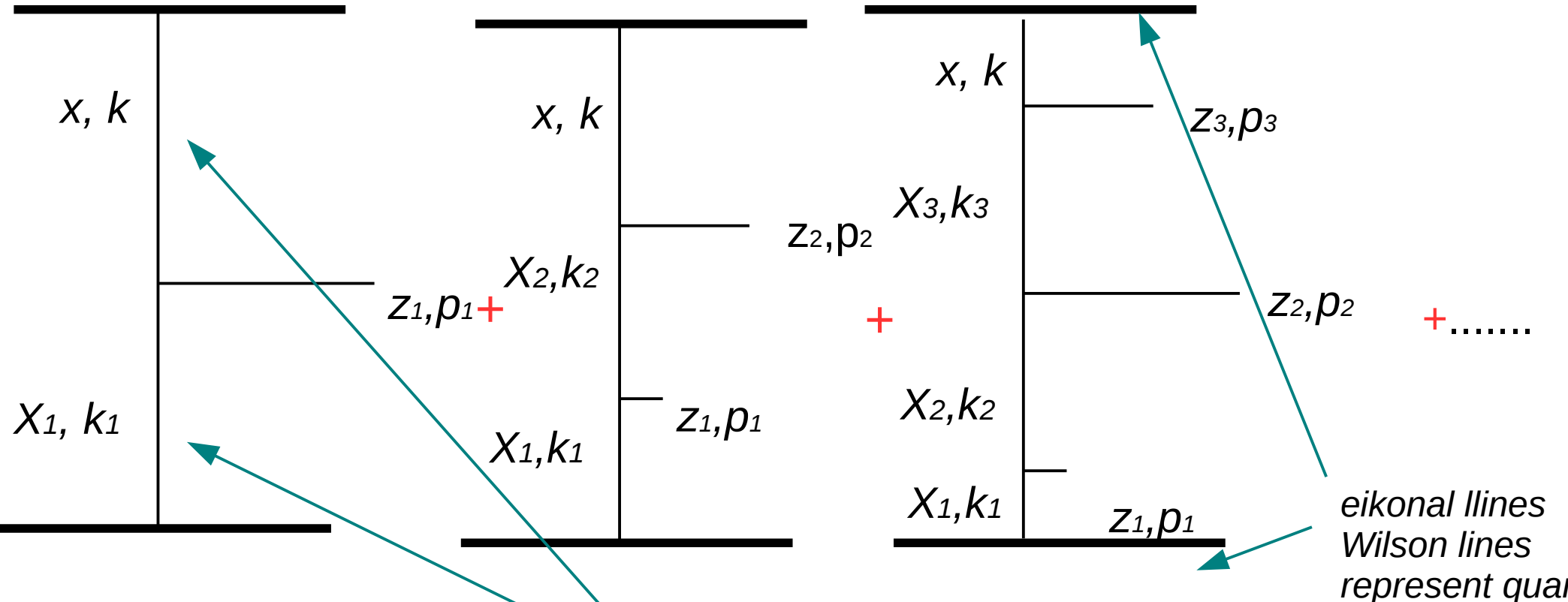
$$S = 2P_1 \cdot P_2$$



- Resummation of logs of x and logs of hard scale
- Knowing well parton densities at large x one can get information about low x physics
- Framework goes recently under name “hybrid framework”

BFKL amplitude

Balitsky, Fadin, Kuraev, Lipatov '77



Linear equation
no saturation
 strong ordering in „z”
no ordering in p

gluons

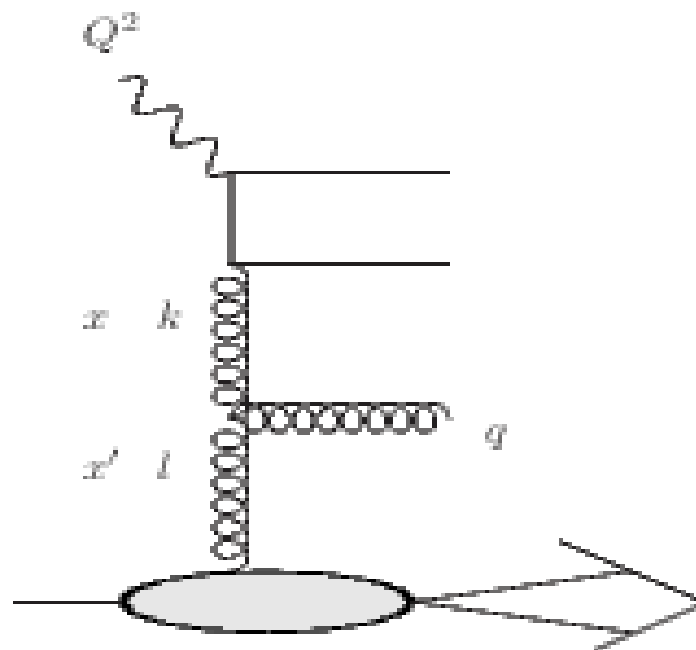
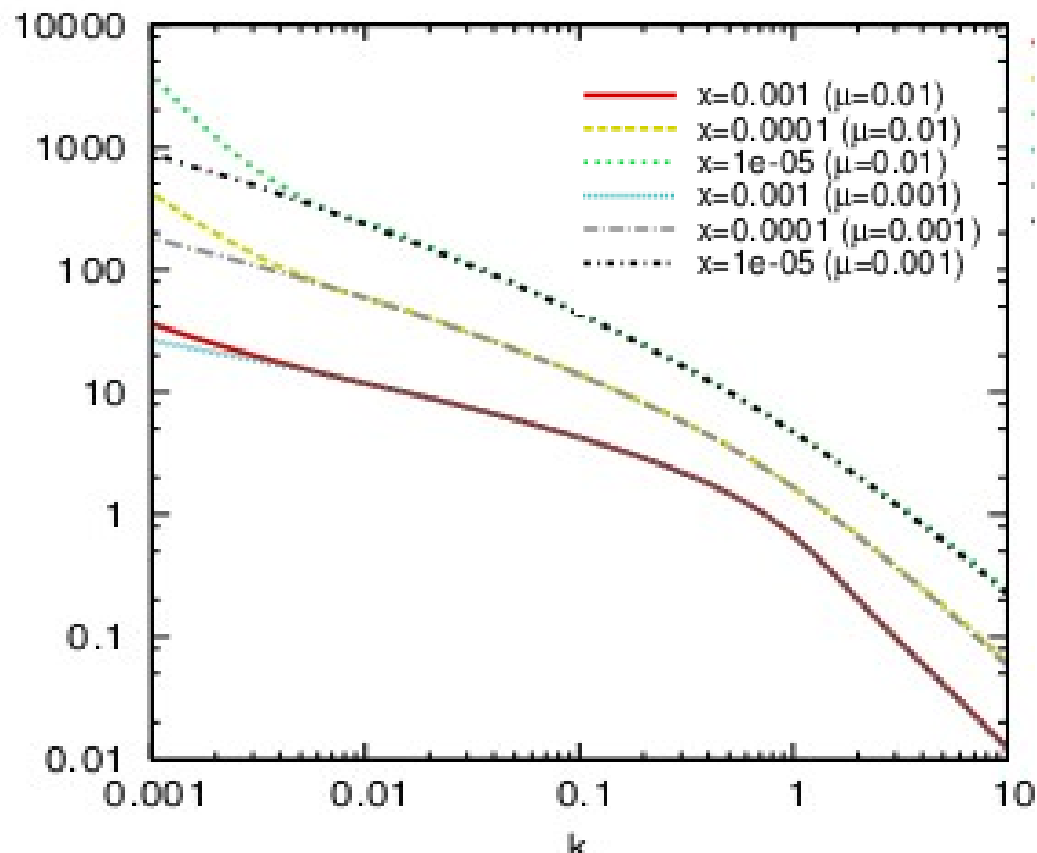
$x \ll 1$

$z_1 \gg z_2 \gg z_3 \gg \dots \gg z_n$

$p_1 \sim p_2 \sim p_3 \sim \dots \sim p_n$

eikonal lines
 Wilson lines
 represent quarks

The LO BFKL equation



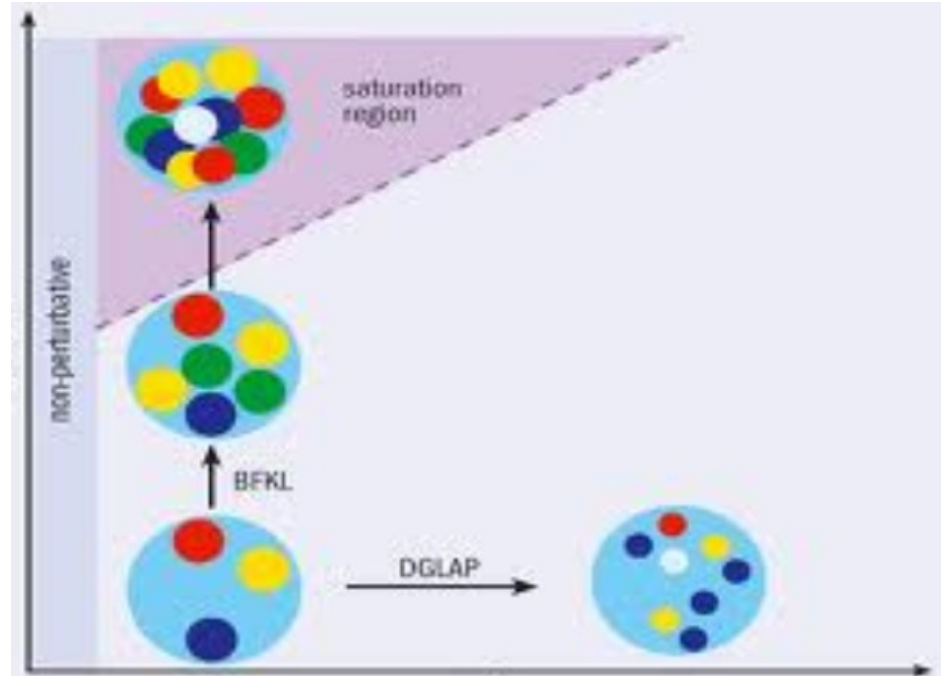
$$\mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$

One can give definition in terms of field strengths and Wilson lines

High energy factorization and saturation

Saturation – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD. More generally saturation is an example of **percolation** which has chance to happen since partons have size $1/k_t$ and hadron has finite size. Cross sections (e.g. F_2) change their behavior from power like to **logarithmic like**.

$\ln x$

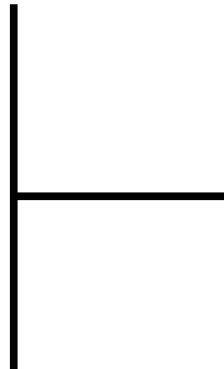


On microscopic level it means that gluon apart splitting recombine

$\ln k$

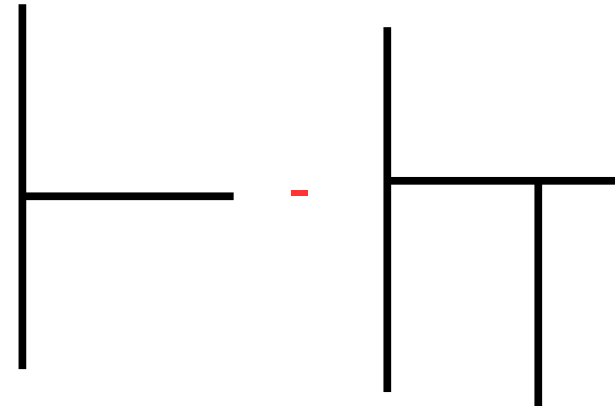
splitting

Linear evolution equation



Nonlinear evolution equations
BK, JIMWLK
CGC framework
DIPSY

recombination



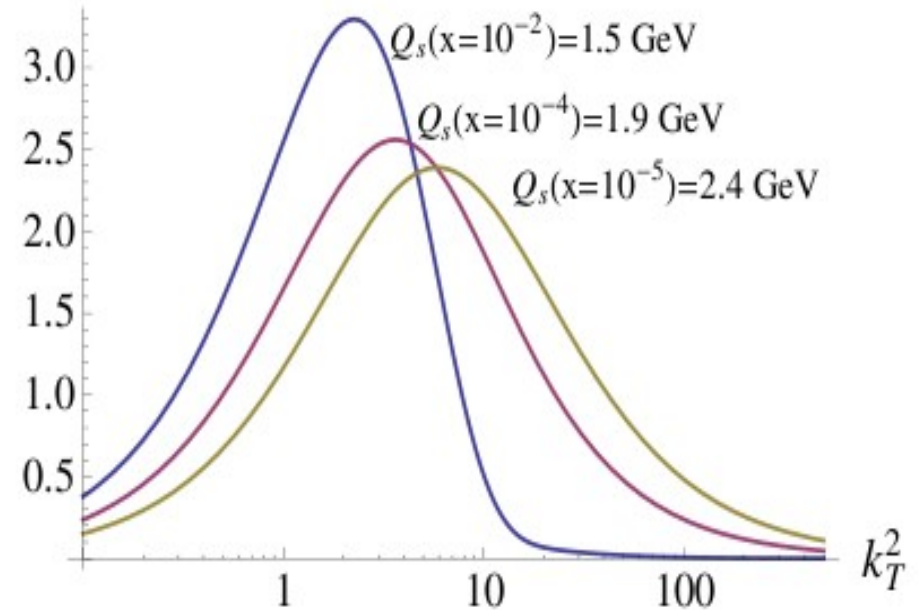
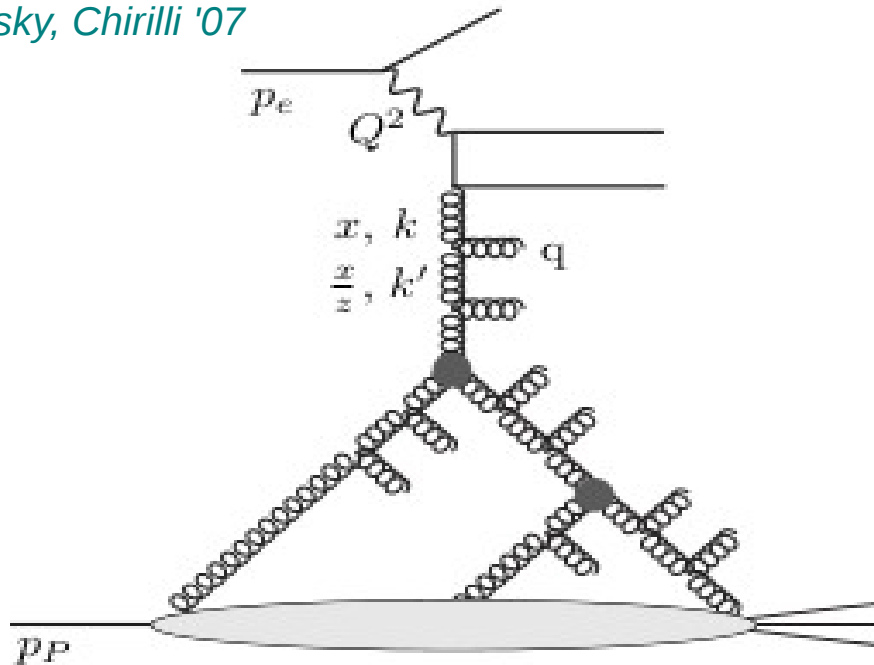
The BK equation for unintegrated gluon density

Originally formulated in coordinate space

Balitsky '96, Kovchegov'99

Now at NLO accuracy

Balitsky, Chirilli '07



$$\mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$

$$- \frac{2\alpha_s^2 \pi}{N_c R^2} \int_{x/x_0}^1 \frac{dz}{z} \left\{ \left[\int_{k^2}^\infty \frac{dl^2}{l^2} \mathcal{F}(x/z, l^2) \right]^2 + \mathcal{F}(x/z, k^2) \int_{k^2}^\infty \frac{dl^2}{l^2} \ln \left(\frac{l^2}{k^2} \right) \mathcal{F}(x/z, l^2) \right\}$$

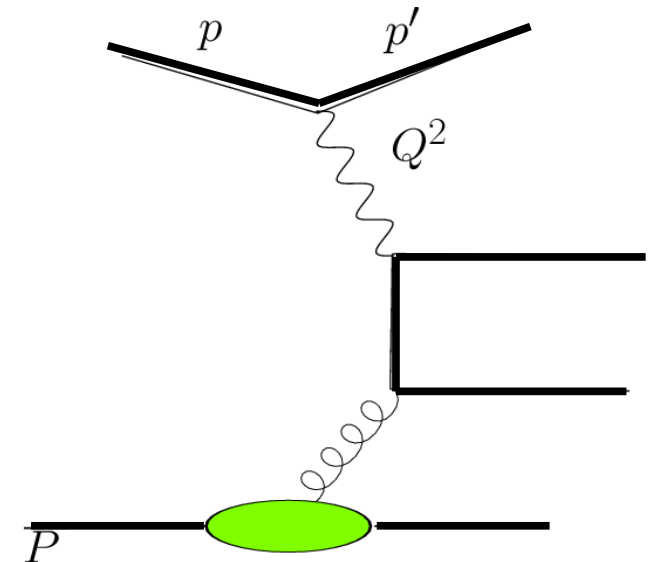
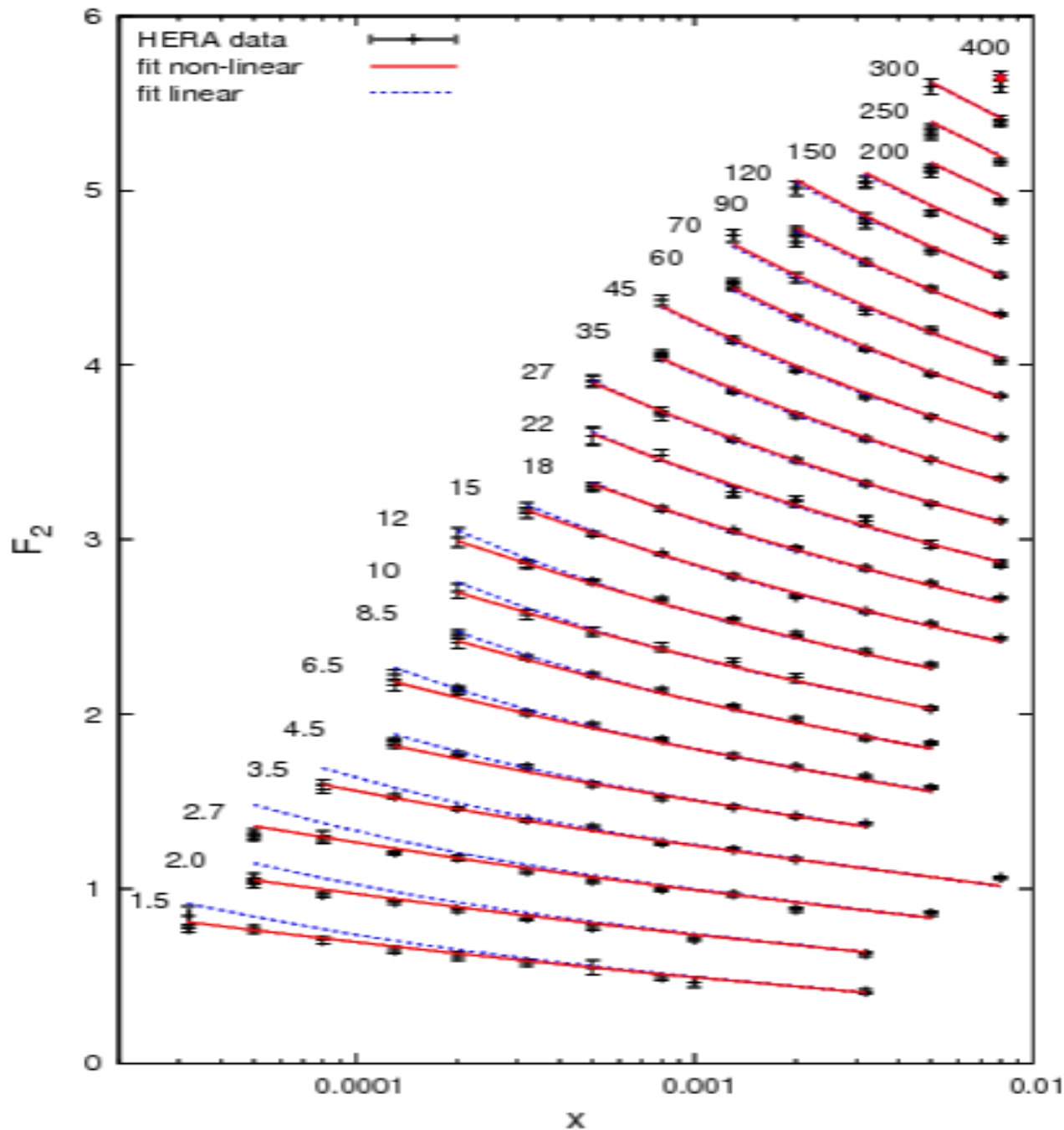
Note dependence
on hadron size. Nonperturbative

Kwiecinski, KK '02

Stasto, KK '05

Nikolaev, Schafer '06

BFKL and BK applied to DIS - some recent results

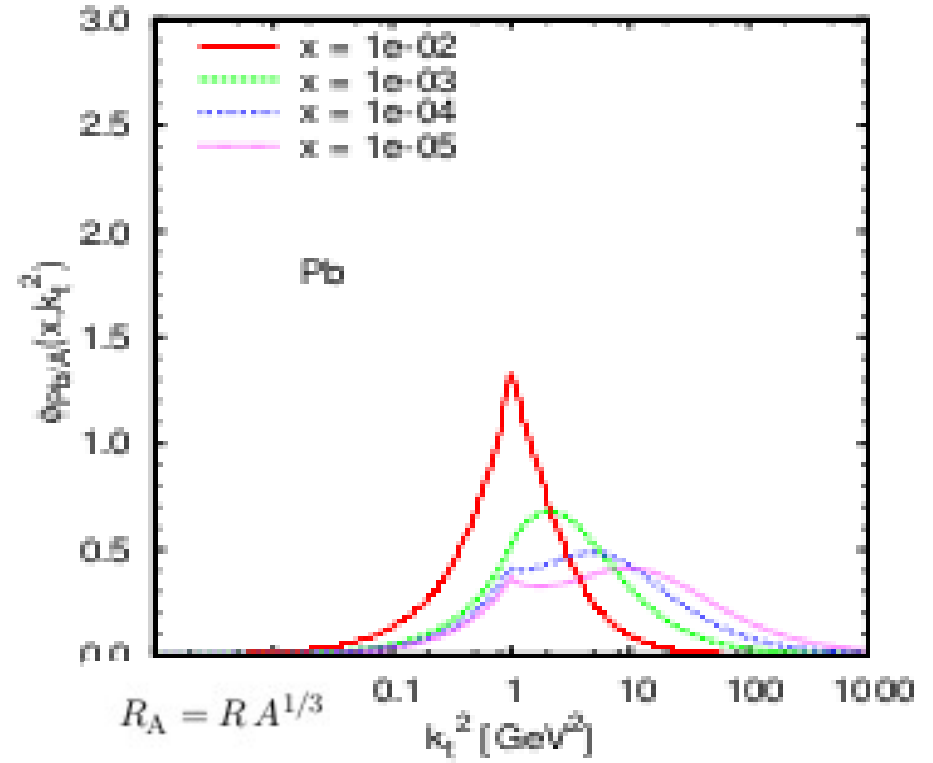
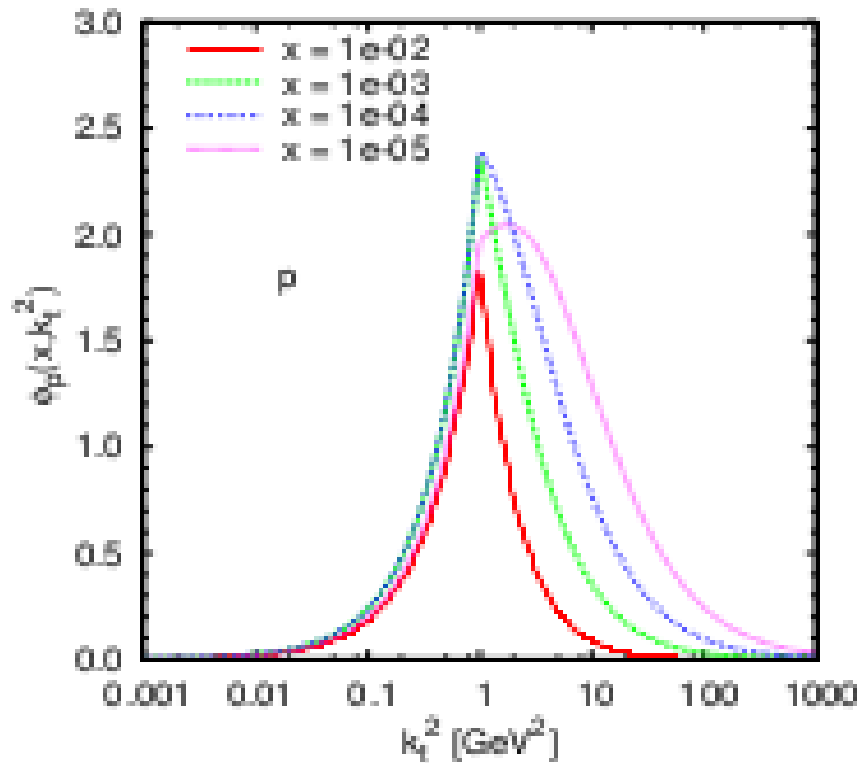


From BK equation with corrections of higher order

K.K, Sapeta '12

Used BK + resummation of Higher orders K.K Kwiecinski

Glue in p vs. glue in Pb



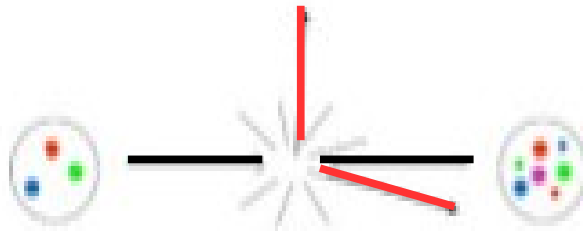
Nonlinear equation for unintegrated gluon density.

Related to BK via Fourier transform

Includes corrections of higher order Kutak, Kwiecinski '02

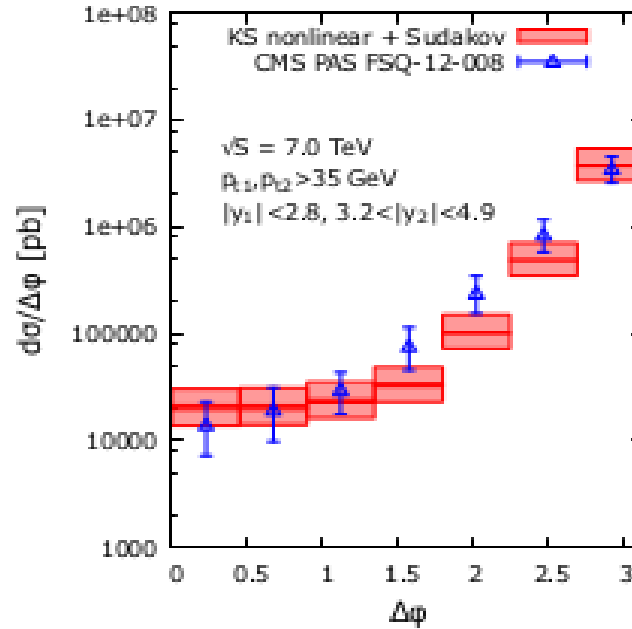
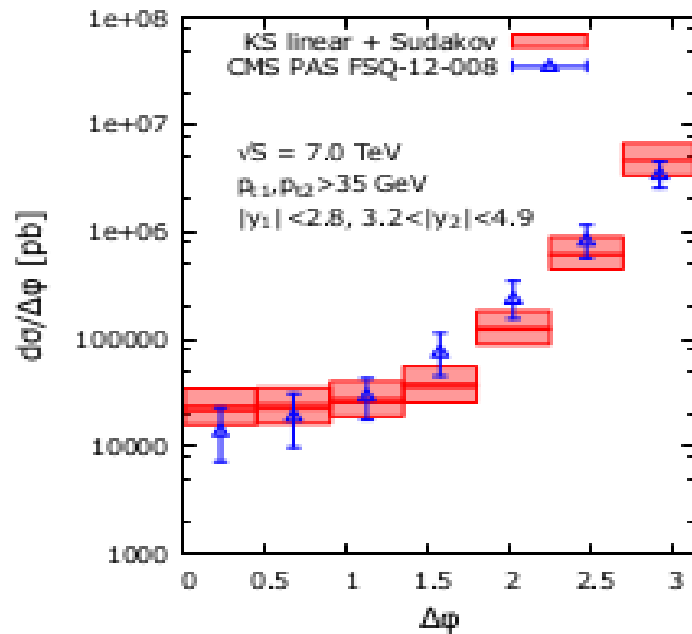
Fitted to latest HERA data Kutak, Sapeta '11

Central-forward di-jets

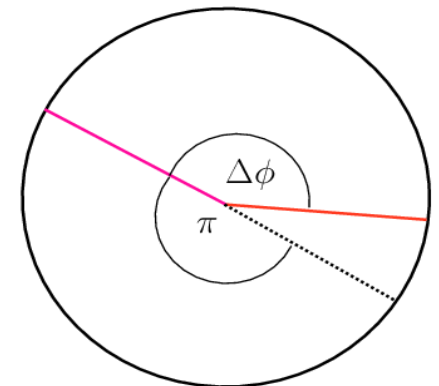


Decorelations inclusive scenario forward-central

van Hameren,, Kotko, K.K, Sapeta '14



$p_{T1}, p_{T2} > 35$, leading jets
 $|y_1| < 2.8, 3.2 < |y_2| < 4.7$
No further requirement on jets



$$\Delta\phi = \pi$$

In DGLAP approach
i.e $2 \rightarrow 2 + pdf$ one would
Get delta function at

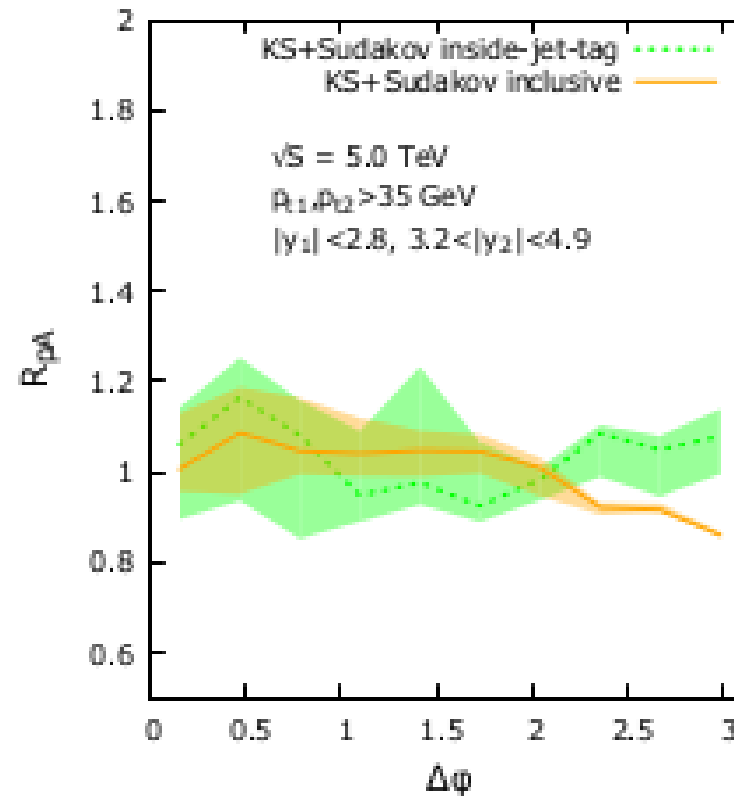
Sudakov effects by reweighing
implemented in LxJet Monte Carlo
P. Kotko

Observable suggested to
study BFKL effects
Sabio-Vera, Schwensen '06

Studied also context of RHIC
Albacete, Marquet '10

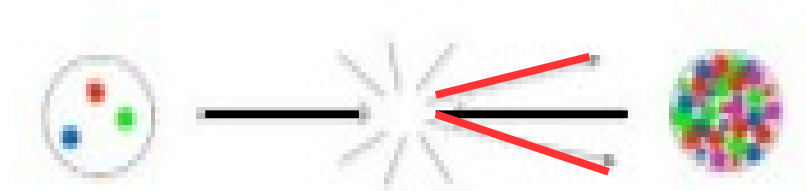
Predictions for p-Pb

Kutak A. van Hameren, Kotko, Sapeta '14



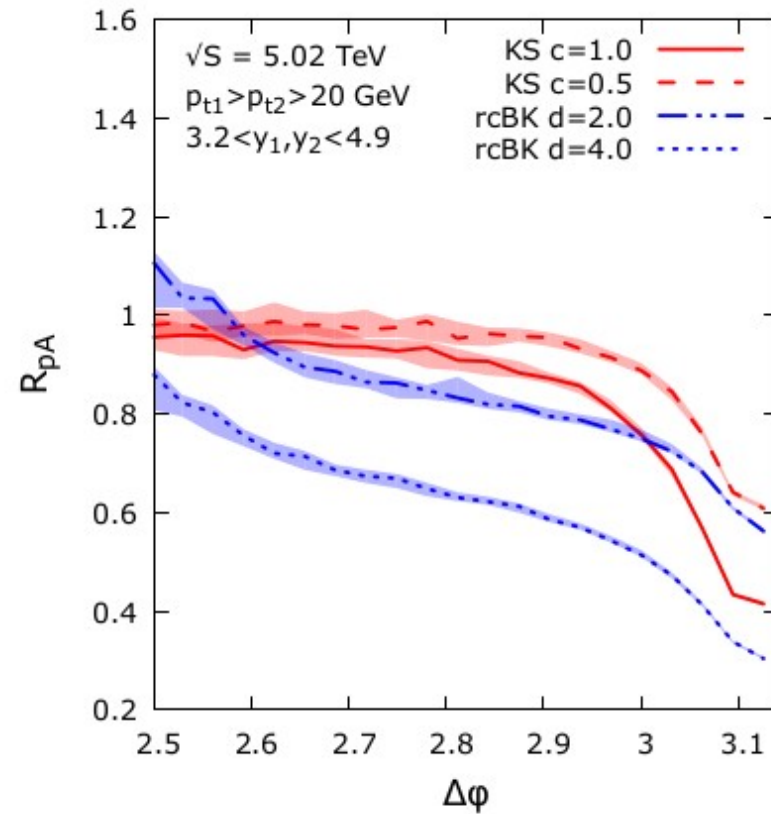
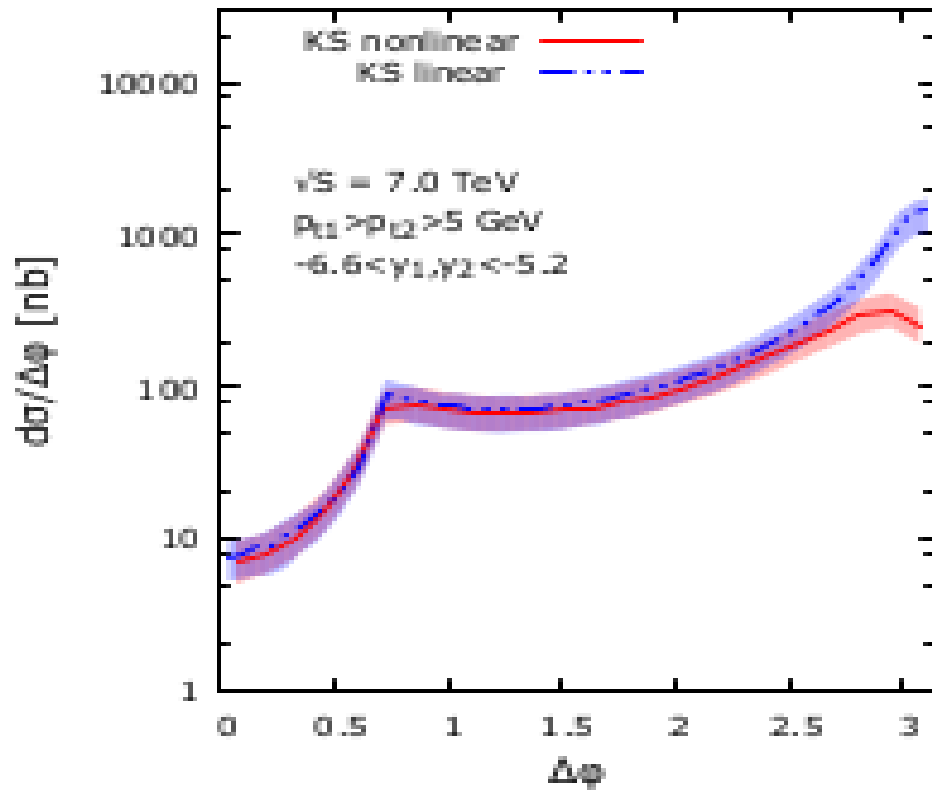
- *Plot of ratio of p-p/p-Pb, so called “nuclear modification ratio”*
- *However, saturation effects are rather weak*

Forward-forward di-jets



Results for decorrelations

Kutak A. van Hameren, Kotko, Marquet, Sapeta '14



Noticable differences already for p - p case and even stronger for p - Pb

Used BK with corrections of higher orders

Conclusions and comments

- *Achieved good description of forward-central jet measurement*
- *We provide prediction for saturation in p-Pb in forward-forward dijets*
- *There are hints for saturation and therefore for shape of gluon density*
- *Recently an extension of factorization for dijets has been obtained*

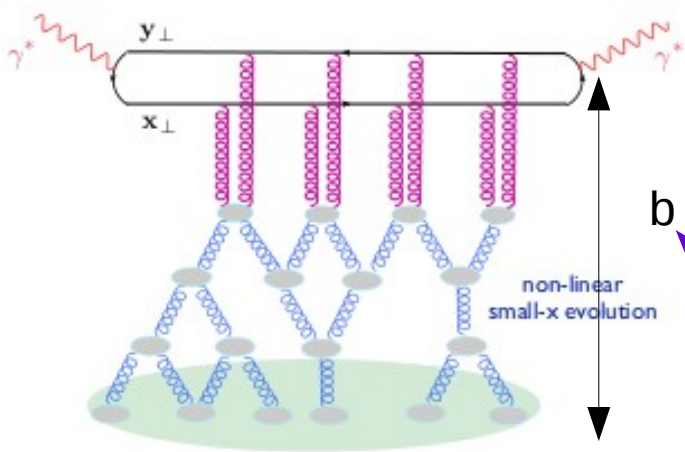
*Improved TMD factorization for forward dijet production in dilute-dense hadronic collisions
1503.03421, P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren*

Backup slides

Low x physics – formal structures

In the dipole formalism (coordinate space)

Mueller, Patel '95

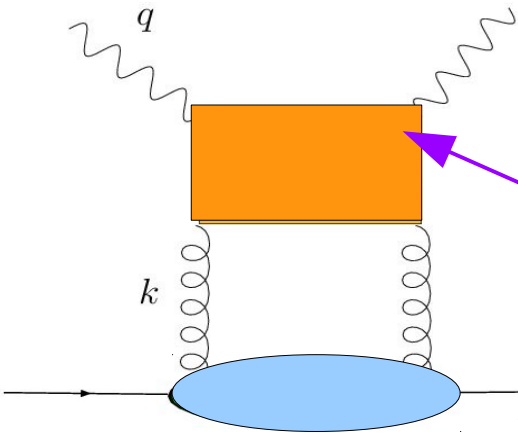


plot from Albacete, Marquet '14

- Quark and antiquark are represented by Wilson lines interacting with color field of the target.
- In general leads to system of equations known as JIMWLK or equivalent formulation known as Balitsky hierarchy. Solution by Rummukainen, Wigert 03
- One can get simpler equation for dipole amplitude N in large N_c known as Balitsky-Kovchegov equation. Now known at NLO. Numerical solutions of LO (Stasto, Golec-Biernat'03), and NLO (Lappi, Mantysari'15)

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2b \int_0^1 dz \int d^2r (|\psi_L(z, r)|^2 + |\psi_T(z, r)|^2) N(x, r, b)$$

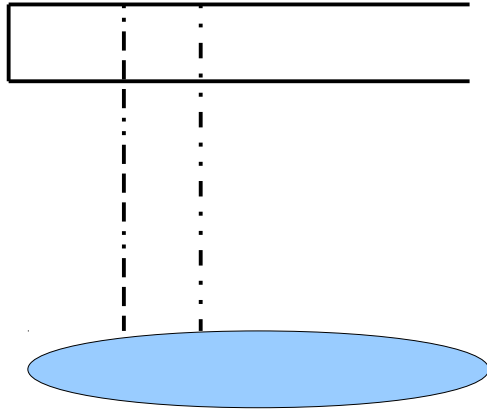
One needs to model large impact parameter behavior "b" of BK since the equation is missing confinement effects. Long range Coulomb like interactions of gluons



$$F_2 \sim \phi(k_{\perp}, Q^2) \otimes \mathcal{F}(x, k_{\perp})$$

$$\mathcal{F}(x, k_{\perp}) \equiv 2 \int \frac{d\xi^- d\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_{\perp} \cdot \xi_{\perp}} \langle P | \text{Tr} [F^{+i}(\xi^-, \xi_{\perp}) \mathcal{U}^{[-]\dagger} F^{+i}(0) \mathcal{U}^{[+]}] | P \rangle$$

BK in coordinate space



Dipole propagating in an external field of target

$$U(\mathbf{x}_\perp) = \text{P exp} \left\{ ig \int dx^+ A_a^-(x^+, \mathbf{x}) t^a \right\}$$

$$\begin{aligned} \frac{d}{dY} \langle \text{tr} \{ U(x_\perp) U^\dagger(y_\perp) \} \rangle_Y &= \frac{1}{\pi^2} \int d^2 \mathbf{z} \mathcal{K}_{\mathbf{xyz}} \left(\langle [\tilde{U}(z_\perp)]^{ab} \text{tr} \{ t^a U(x_\perp) t^b U^\dagger(y_\perp) \} \rangle_Y \right. \\ &\quad \left. - C_F \langle \text{tr} \{ U(x_\perp) U^\dagger(y_\perp) \} \rangle_Y \right) \end{aligned}$$

Fiertz identity

$$\tilde{U}(z_\perp)^{ab} \text{tr} \{ t^a U(x_\perp) t^b U^\dagger(y_\perp) \} = \text{tr} \{ U(x_\perp) U^\dagger(z_\perp) \} \text{tr} \{ U(z_\perp) U^\dagger(y_\perp) \} - \frac{1}{N_c} \text{tr} \{ U(x_\perp) U^\dagger(y_\perp) \}$$

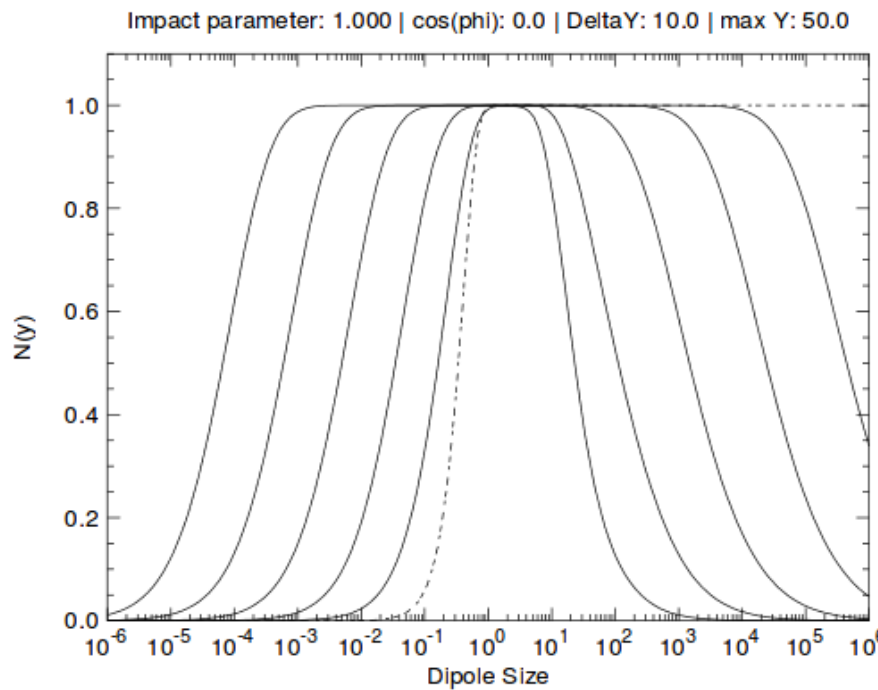
$$\langle S_{xz} S_{zy} \rangle_Y \rightarrow \langle S_{xz} \rangle_Y \langle S_{zy} \rangle_Y$$

$$\mathcal{N}_{\mathbf{xy}}(Y) \equiv 1 - \langle S(\mathbf{x}, \mathbf{y}) \rangle_Y$$

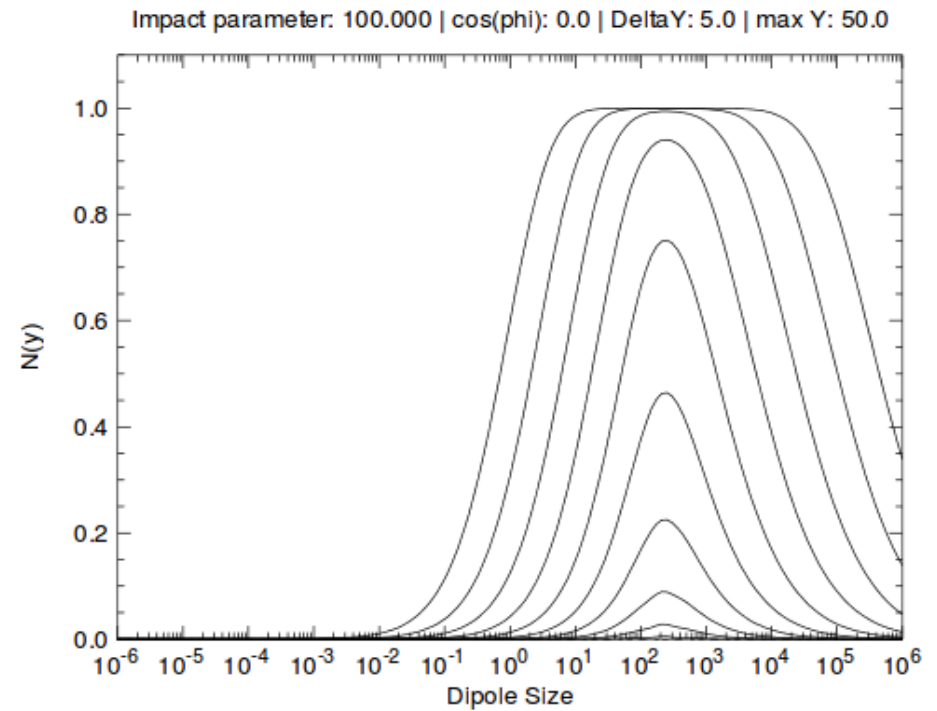
Dipole scattering amplitude

BK in coordinate space

$$\frac{\partial \mathcal{N}_{xy;Y}}{\partial Y} = \frac{N_c}{2\pi^2} \int d^2\mathbf{z} \mathcal{K}_{xyz} [\mathcal{N}_{xz;Y} + \mathcal{N}_{zy;Y} - \mathcal{N}_{xy;Y} - \mathcal{N}_{xz;Y} \mathcal{N}_{zy;Y}]$$



(a) Small impact parameter: $b = 1$.



(b) Large impact parameter: $b = 100$.