37

Towards the QCD equation of state at the physical point using Wilson fermion

WHOT-QCD Collaboration:
T. Umeda (Hiroshima Univ.), S. Ejiri (Niigata Univ.), R. Iwami (Niigata Univ.), K. Kanaya (Univ. of Tsukuba)

Abstract:
We study the N_f=2+1 QCD at nonzero temperatures using nonperturbatively improved Wilson quarks of the physical masses by the fixed scale approach. We perform physical point simulations at finite temperatures with the coupling parameters which were adopted by the PACS-CS collaboration in their studies using the reweighting technique. Zero temperature values are obtained on the PACS-CS configurations which are open to the public on the ILDG. Finite temperature configurations are generated with the RHMC algorithm. The lattice sizes are $32^3 \times N_t$, where $N_t = 14, 13, \cdots, 5, 4$ corresponding to $T \simeq$140 – 500 MeV. We present results of some basic observables at these temperatures and the status of our calculation of the equation of state.

1. Motivation

QCD Thermodynamics with Wilson quarks

It is important to check the results of staggered quarks, whose continuum limit is not guaranteed to be universal to QCD, using theoretically sound lattice quarks like Wilson-type quarks.

WHOT-QCD collab. studies finite T & μ QCD using Wilson-type quarks

2. Strategy to study the QCD equation of states

2-1. formulation of (2+1)-flavor QCD

Iwasaki improved gauge
+ Nonperturbatively improved Wilson quark action

$$S = S_g + S_q$$

$$S_g = -\beta \left\{ \sum_{x,\mu>\nu} c_0 W_{\mu\nu}^{1\times1}(x) + \sum_{x,\mu,\nu} c_1 W_{\mu\nu}^{1\times2}(x) \right\} \qquad \beta = \frac{6}{g^2}$$

$$S_q = \sum_{f=u,d,s} \sum_{x,y} \bar{q}_x^f D_{x,y} q_y^f$$

$$D_{x,y} = \delta_{x,y} - \kappa_f \sum_{\mu} \left\{ (1-\gamma_\mu) U_{x,\mu} \delta_{x+\hat{\mu},y} + (1+\gamma_\mu) U_{x-\hat{\mu},\mu}^{\dagger} \delta_{x-\hat{\mu},y} \right\} - \delta_{x,y} c_{SW} \kappa_f \sum_{\mu>\nu} \sigma_{\mu\nu} F_{\mu\nu}$$

Trace anomaly

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3} \left(a \frac{\partial \beta}{\partial a} \left\langle \frac{\partial S}{\partial \beta} \right\rangle_{sub} + a \frac{\partial \kappa_{ud}}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_{ud}} \right\rangle_{sub} + a \frac{\partial \kappa_s}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_s} \right\rangle_{sub} \right)$$

$$\left\langle \frac{\partial S}{\partial \beta} \right\rangle = N_s^3 N_t \left(- \left\langle \sum_{x,\mu>\nu} c_0 W_{\mu\nu}^{1\times1}(x) + \sum_{x,\mu,\nu} c_1 W_{\mu\nu}^{1\times2}(x) \right\rangle + N_f \frac{a \partial c_{SW}}{\partial \beta} \kappa_f \left\langle \sum_{x,\mu>\nu} \mathrm{Tr}^{(c,s)} \sigma_{\mu\nu} F_{\mu\nu} (D^{-1})_{x,x} \right\rangle \right)$$

$$\left\langle \frac{\partial S}{\partial \kappa_f} \right\rangle = N_f N_s^3 N_t \left(\left\langle \sum_{x,\mu} \mathrm{Tr}^{(c,s)} \left\{ (1-\gamma_\mu) U_{x,\mu} (D^{-1})_{x+\hat{\mu},x} + (1+\gamma_\mu) U_{x-\hat{\mu},x}^{\dagger} (D^{-1})_{x-\hat{\mu},x} \right\} \right\rangle + c_{SW} \left\langle \sum_{x,\mu>\nu} \mathrm{Tr}^{(c,s)} \sigma_{\mu\nu} F_{\mu\nu} (D^{-1})_{x,x} \right\rangle \right)$$

$$c_{SW}(\beta) = 1 + 0.113 g^2 + 0.0209 g^4 + 0.0049 g^6$$

Phys. Rev. D73, 034501, CP-PACS/JLQCD

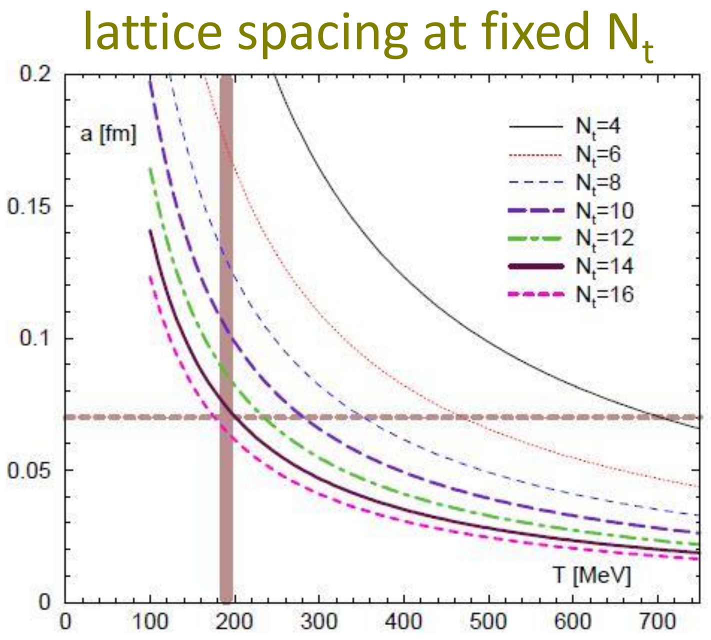
2-2. Fixed scale approach

Temperature $T = \frac{1}{N_t a}$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in temporal direction

Phys. Rev. D79, 051501 (2009), WHOT-QCD

lattice spacing at fixed N_t

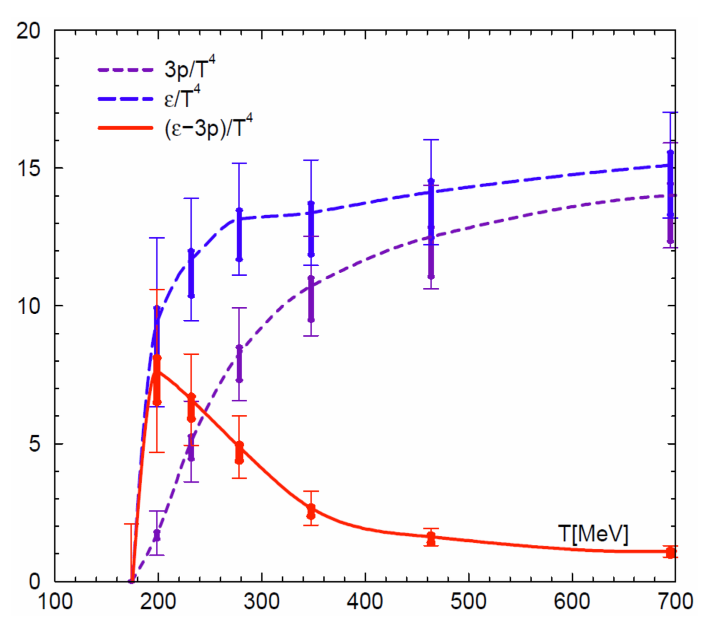


- Advantages
  - Line of Constant Physics
  - Common T=0 subtraction (spectrum study at T=0)
  - Lattice spacing at lower T
  - Finite volume effects
- Disadvantages
  - T resolution due to integer N_t
  - UV cutoff eff. at high T's

2-3. T-integration method at the fixed scale

$$\frac{p}{T^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3p}{T'^5}$$

with T_0 chosen such that $p(T_0) \approx 0$



Phys. Rev. D85, 094508 (2012), WHOT-QCD

- (2+1)-flavor QCD Equation of state using Wilson quarks heavier ud-quarks ($m_\pi/m_\rho \approx 0.63$)
- T=0 data from CP-PACS/JLQCD We have generated only T>0 configs.
- A systematic error for beta-functions

3. Physical point simulation

3-1. Zero-temperature

PHYSICAL POINT SIMULATION IN 2 + 1 FLAVOR ...

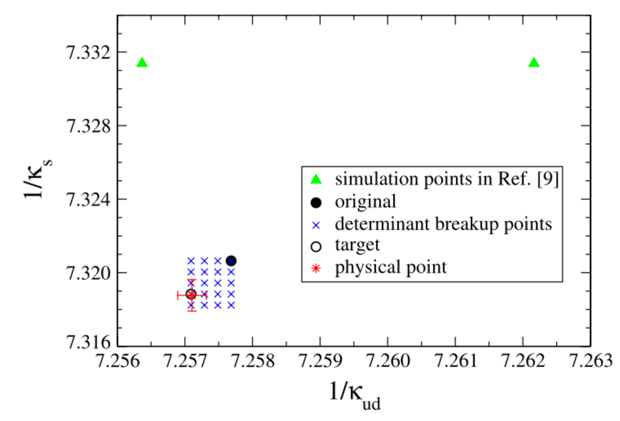
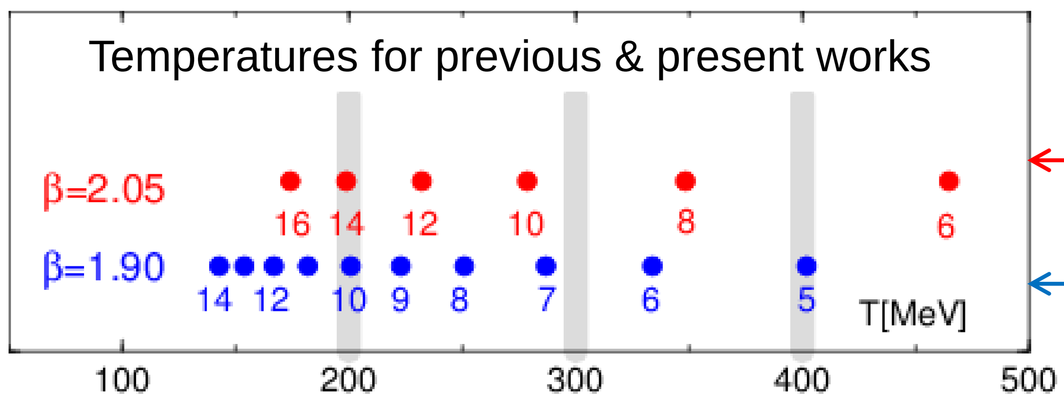


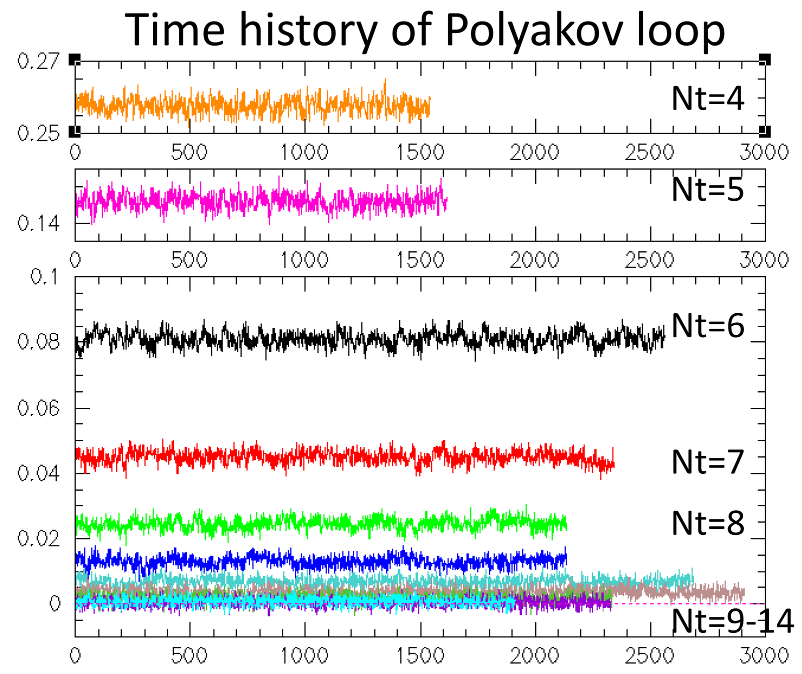
FIG. 13 (color online). Determination of the physical point with m_π/m_Ω and m_K/m_Ω inputs in $(1/\kappa_{ud}, 1/\kappa_s)$ plane. Solid and open black circles denote the original and target points, respectively. Green symbols represent $(\kappa_{ud}, \kappa_s) =$ (0.137 810 00, 0.136 400 00) and (0.137 700 00, 0.136 400 00) which are lightest two simulation points in Ref. [9].

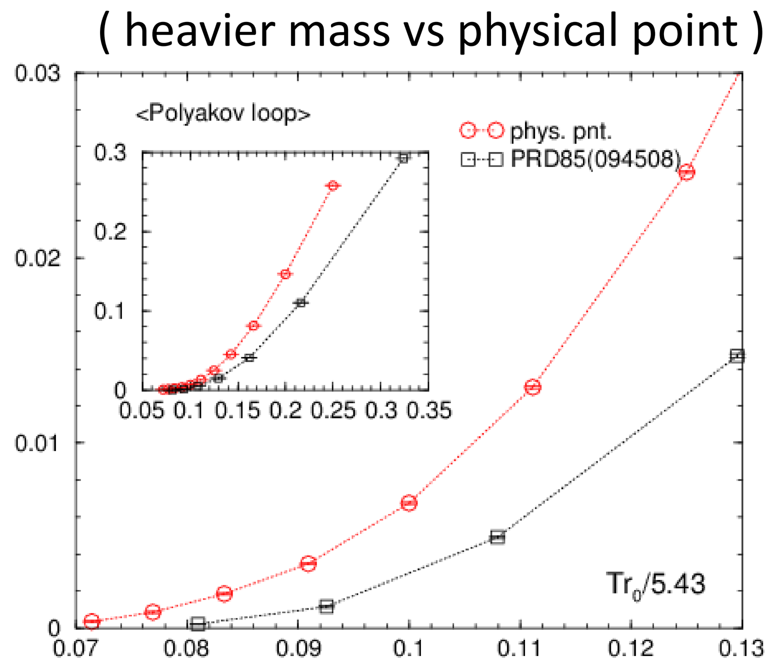
Phys. Rev. D81, 074503 (2010), PACS-CS
Iwasaki gauge + improved Wilson
β=1.90, c_{sw}=1.715, 32^3 x 64 lattice
Reweighting to physical point
original hopping param.
$(\kappa_{ud}, \kappa_s) = (0.137785, 0.136600)$
target hopping param.
$(\kappa_{ud}, \kappa_s) = (0.13779625, 0.13663375)$
MD time = 2000 (#conf = 80)
Configs. are open to the public on ILDG

3-2. Finite-temperatures

We generated only T>0 configs by Bridge++ code on BlueGene/Q (KEK)
RHMC algorithm, threefold-mass-preconditioning
Lattice size : 32^3 x N_t, (N_t=14, 13, …, 5,4)
The same coupling parameters as T=0 PACS-CS
at the target hopping param. $(\kappa_{ud}, \kappa_s) = (0.13779625, 0.13663375)$



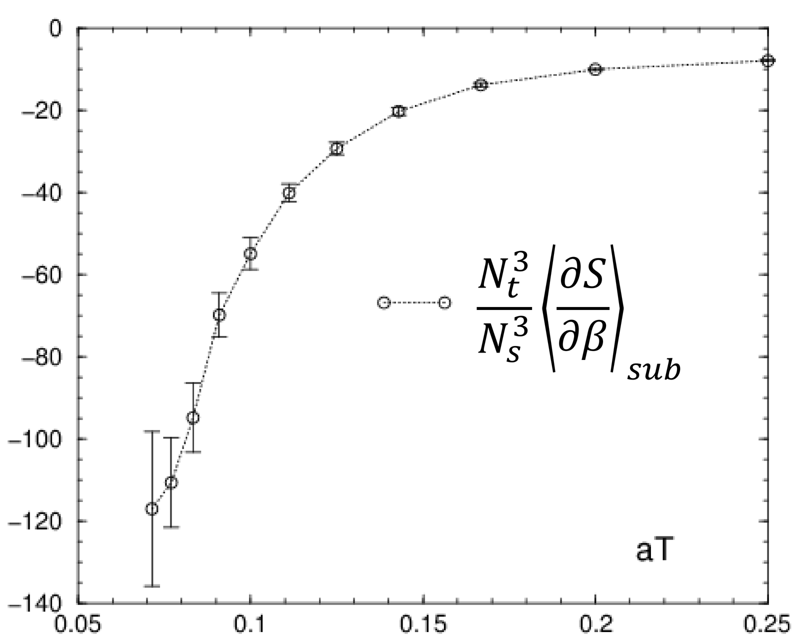


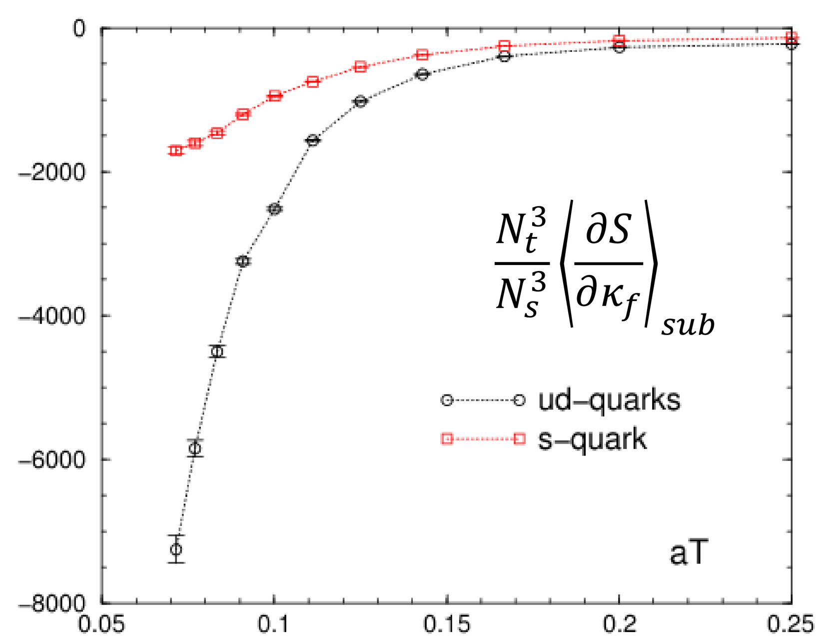


- Finite temperature configs 1500-3000 MD traj. at each T
- Polyakov loop starts to increase from the lower T at the physical point

3-2. Observables for the EOS

T-dependence of the observables in the trace anomaly (see Sec. 2-1)





3-3. Beta-function at the physical point

Beta-functions are necessary to calculate the EOS

We are planning to calculate the beta-function using the following methods

- Direct fit method using PACS-CS spectrum data
  Phys. Rev. D85, 094508 (2012), WHOT-QCD
- Single-point reweighting using PACS-CS reweighting data
- Multi-point reweighting
  R. Iwami et al. (in preparation)

4. Summary

1) Physical point simulations are carried out at T>0.
2) Pseudo-critical temperature decreases as quark masses decreases.
3) Observables for the EOS are measured at the physical point.
4) Beta-functions are needed to obtain the EOS.