Determination of ε_K using lattice QCD inputs

Jon Bailey, Yong-Chull Jang, and Weonjong Lee
(SWME Collaboration)

Lattice Gauge Theory Research Center
Department of Physics and Astronomy
Seoul National University

Kobe ICC, Japan, 2015/07/14∼18

ε_K and $\hat{B}_K$, V_{cb} I

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Relation between ε_K and $\hat{B}_K$ in standard model.

$$\varepsilon_K = \exp(i\phi_\varepsilon)\ \sqrt{2}\sin(\phi_\varepsilon)\ [C_\varepsilon\ X_{\mathsf{SD}}\ \hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\mathsf{LD}}]$$

$$+ \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0\ \Gamma_2/\Gamma_1)$$

$$X_{\mathsf{SD}} = \mathrm{Im}\lambda_t \Big[\mathrm{Re}\lambda_c\ \eta_{cc}\ S_0(x_c) - \mathrm{Re}\lambda_t\ \eta_{tt}\ S_0(x_t)$$

$$- (\mathrm{Re}\lambda_c - \mathrm{Re}\lambda_t)\ \eta_{ct}\ S_0(x_c, x_t)\Big]$$

ε_K and $\hat{B}_K$, V_{cb} II

$$\lambda_i = V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\,\pi^2 \Delta M_K}$$

$$\frac{\xi_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \approx -5\%$$

$\xi_{\text{LD}} =$ Long Distance Effect $\approx 2\%$ $\longrightarrow$ a systematic error.

- Inami-Lim functions:

$$S_0(x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} - \frac{3x_i^2 \ln x_i}{(1-x_i)^3} \right],$$

$$S_0(x_i, x_j) = \left\{ \frac{x_i x_j}{x_i - x_j} \left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right] \ln x_i - (i \leftrightarrow j) \right\} - \frac{3 x_i x_j}{4(1-x_i)(1-x_j)}$$

ε_K and $\hat{B}_K$, V_{cb} III

$$
\begin{aligned}
S_0(x_t) &\longrightarrow +70\% \\
S_0(x_c, x_t) &\longrightarrow +44\% \\
S_0(x_c) &\longrightarrow -14\%
\end{aligned}
$$

- Dominant contribution ($\approx 70\%$) comes with $|V_{cb}|^4$.

$$
\begin{aligned}
\mathrm{Im}\lambda_t \cdot \mathrm{Re}\lambda_t &= \bar{\eta}\lambda^2 |V_{cb}|^4 (1-\bar{\rho}) \\
\mathrm{Re}\lambda_c &= -\lambda(1-\frac{\lambda^2}{2}) + \mathcal{O}(\lambda^5) \\
\mathrm{Re}\lambda_t &= -(1-\frac{\lambda^2}{2})A^2\lambda^5(1-\bar{\rho}) + \mathcal{O}(\lambda^7) \\
\mathrm{Im}\lambda_t &= \eta A^2\lambda^5 + \mathcal{O}(\lambda^7)
\end{aligned}
$$

ε_K and $\hat{B}_K$, V_{cb} IV

- Definition of B_K in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu (1-\gamma_5) d][\bar{s}\gamma_\mu (1-\gamma_5) d] | K_0 \rangle}{\frac{8}{3} \langle \bar{K}_0 | \bar{s}\gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu \gamma_5 d | K_0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$

$$\phi_\varepsilon = 43.52(5)^\circ$$

Wolfenstein Parameters

Input Parameters for Angle-Only-Fit (AOF)

- ε_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of `UTfit` and `CKMfitter`.
- Instead, we can use angle-only-fit result for the UT apex $(\bar{\rho}, \bar{\eta})$.
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7) \,,$$

  which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ instead of A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

λ	0.22537(61)	[1] CKMfitter
	0.2255(6)	[1] UTfit
	0.2253(8)	[1] $|V_{us}|$ (AOF)
$\bar{\rho}$	0.117(21)	[1] CKMfitter
	0.124(24)	[1] UTfit
	0.139(29)	[2] UTfit (AOF)
$\bar{\eta}$	0.353(13)	[1] CKMfitter
	0.354(15)	[1] UTfit
	0.337(16)	[2] UTfit (AOF)

Input Parameters of B_K, V_{cb} and others

B_K

$\hat{B}_K$	$0.7661(99)$	[3] FLAG
	$0.7379(47)(365)$	[4] SWME

V_{cb}

V_{cb} $\times 10^{-3}$	$42.21(78)$	[5] Incl.
	$39.04(49)(53)(19)$	[6] Excl.

Others

G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[1]
M_W	$80.385(15)$ GeV	[1]
$m_c(m_c)$	$1.275(25)$ GeV	[1]
$m_t(m_t)$	$163.3(2.7)$ GeV	[7]
η_{cc}	$1.72(27)$	[8]
η_{tt}	$0.5765(65)$	[9]
η_{ct}	$0.496(47)$	[10]
θ	$43.52(5)^\circ$	[1]
m_{K^0}	$497.614(24)$ MeV	[1]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[1]
F_K	$156.2(7)$ MeV	[1]

ξ_0
Input Parameters

$$\xi_0 = \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$$

ξ_0	$-1.63(19)(20) \times 10^{-4}$	[11]

- RBC-UKQCD collaboration performs lattice calculation of $\mathrm{Im} A_2$. From this result, ξ_0 can be obtained by the relation

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) = \frac{1}{\sqrt{2}|\varepsilon_K|}\omega\left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \xi_0\right).$$

  Other inputs ω, ε_K and $\varepsilon'_K/\varepsilon_K$ are taken from the experimental values.
- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$

ξ_{LD}

Input Parameters

- Definition of ξ_{LD}

$$\xi_{\mathsf{LD}} = \frac{m'_{\mathsf{LD}}}{\sqrt{2}\Delta M_K}$$

$$m'_{\mathsf{LD}} = -\mathrm{Im}\left[\mathcal{P}\sum_C \frac{\langle\overline{K}^0|H_{\mathsf{w}}|C\rangle\langle C|H_{\mathsf{w}}|K^0\rangle}{m_{K^0} - E_C}\right]$$

- We incorporate the contribution of ξ_{LD} as follows.

$$\xi_{\mathsf{LD}} = (0 \pm 1.6)\% \qquad \longleftarrow \quad \text{Ref. [12]}$$

ε_K: FLAG $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}

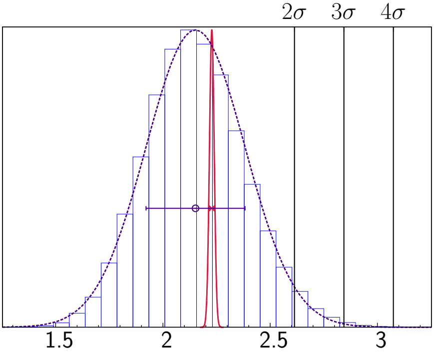

Figure: Inclusive V_{cb}

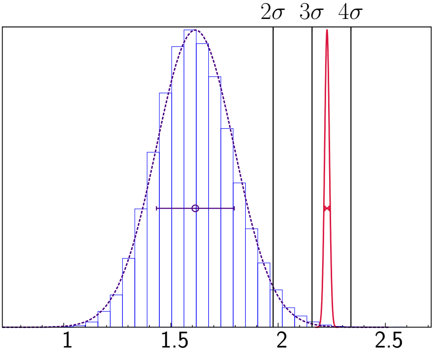

Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.4σ tension.

$$\varepsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\varepsilon_K^{SM} = 1.61(18) \times 10^{-3}$$

ε_K: SWME $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}

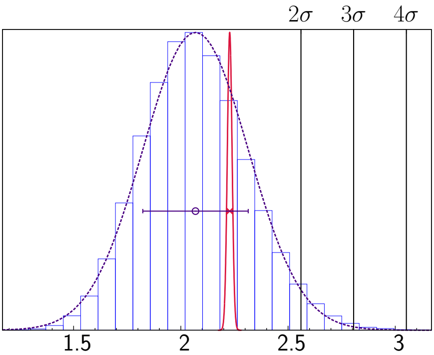

Figure: Inclusive V_{cb}

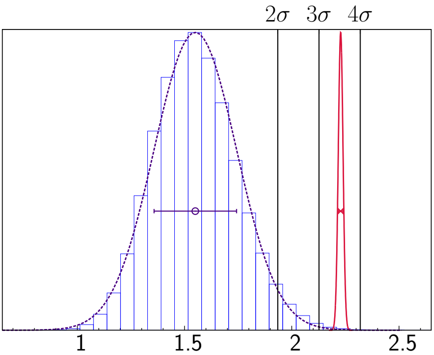

Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.5σ tension.

$$\varepsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\varepsilon_K^{SM} = 1.55(19) \times 10^{-3}$$

Current Status of ε_K

- FLAG: (in units of 1.0×10^{-3}, AOF)

$$\varepsilon_K = 1.61 \pm 0.18 \qquad \text{for Exclusive } V_{cb} \text{ (Lattice QCD)}$$
$$\varepsilon_K = 2.15 \pm 0.23 \qquad \text{for Inclusive } V_{cb} \text{ (QCD Sum Rule)}$$

- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

- Hence, we observe $3.4\ \sigma$ difference between the SM theory (Lattice QCD) and experiments.
- What does this mean? $\longrightarrow$ Breakdown of SM ?

Error Budget of Exclusive ε_K

cause	error (%)	memo
V_{cb}	39.3	Exclusive (FNAL/MILC)
$\bar{\eta}$	20.4	AOF
η_{ct}	16.9	$c - t$ Box
η_{cc}	7.1	$c - c$ Box
$\bar{\rho}$	5.4	AOF
m_t	2.4	
ξ_0	2.2	$\text{Im}(A_0)/\text{Re}(A_0)$
ξ_{LD}	2.0	Long-distance
$\hat{B}_K$	1.5	FLAG
m_c	1.0	Charm quark mass
$\vdots$	$\vdots$	$\vdots$

Conclusion and Future Outlook

1. Lattice determination of ε_K from the standard model with the exclusive V_{cb} channel shows $3.4\ \sigma$ tension compared with the experiment.

2. However, in the inclusive V_{cb} channel determined from the QCD sum rules, we observe no tension.

3. The dominant systematic error in ε_K comes from V_{cb} in the exclusive channel.

4. Hence, it becomes crucial to reduce the theoretical error of V_{cb} down to $\approx 0.5\%$ level: $\longleftrightarrow$ the OK action.

5. Thank God very much for your help!!!

References for the Input Parameters I

[1] K.A. Olive et al.
Review of Particle Physics.
Chin.Phys., C38:090001, 2014.

[2] `http://www.utfit.org/UTfit/ResultsSummer2014PostMoriondSM`.

[3] Sinya Aoki, Yasumichi Aoki, Claude Bernard, Tom Blum, Gilberto Colangelo, et al.
Review of lattice results concerning low energy particle physics.
2013.

[4] Taegil Bae et al.
Improved determination of B_K with staggered quarks.
Phys.Rev., D89:074504, 2014.

[5] Andrea Alberti, Paolo Gambino, Kristopher J. Healey, and Soumitra Nandi.
Precision Determination of the Cabibbo-Kobayashi-Maskawa Element V_{cb}.
Phys.Rev.Lett., 114(6):061802, 2015.

[6] Jon A. Bailey, A. Bazavov, C. Bernard, et al.
Phys.Rev., D89:114504, 2014.

References for the Input Parameters II

[7] S. Alekhin, A. Djouadi, and S. Moch.
The top quark and Higgs boson masses and the stability of the electroweak vacuum.
Phys.Lett., B716:214–219, 2012.

[8] Jon A. Bailey, Yong-Chull Jang, Weonjong Lee, and Sungwoo Park.
Standard Model evaluation of ε_K using lattice QCD inputs for $\hat{B}_K$ and V_{cb}.
hep-lat/1503.05388, 2015.

[9] Andrzej J. Buras and Diego Guadagnoli.
Phys.Rev., D78:033005, 2008.

[10] Joachim Brod and Martin Gorbahn.
ϵ_K at Next-to-Next-to-Leading Order: The Charm-Top-Quark Contribution.
Phys.Rev., D82:094026, 2010.

[11] T. Blum, P.A. Boyle, N.H. Christ, N. Garron, E. Goode, et al.
Phys.Rev.Lett., 108:141601, 2012.

[12] N.H. Christ, T. Izubuchi, C.T. Sachrajda, A. Soni, and J. Yu.
Long distance contribution to the KL-KS mass difference.
Phys.Rev., D88(1):014508, 2013.