Footprint of non-decoupling in chiral phase transition

based on T. S. and N. Yamada Phys. Rev. D 91, 034025 (2015)

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Goal : Understanding of the critical phenomena of two flavor QCD

When the $U_A(1)$ breaking is finite (but small) at T_C :

Our assumption

- How does it affect the nature of chiral phase transition?

→ 1st order or 2nd order?

- (If it is 2nd) What is the universality class?

3-d Linear sigma model (as an effective theory)

At the leading order of ε expansion

Results

2nd order phase transition?

→ Yes (depends on the parameters)

O(4) universality?

→ **One of the exponents differs**

$U_A(1)$ breaking	ν	η	ω
Infinite (O(4))	2/(4-ε)	0	ε
Finite	2/(4-ε)	0	2-5ε/3

!!

INTRODUCTION

Chiral Phase transition of 2flavor QCD

Chiral symmetry (Nf=2) $SU_L(2) \otimes SU_R(2) \otimes U_V(1) \otimes \cancel{U_A}(1)$

anomaly

Effective model

U(2) x U(2) Linear Sigma Model(LSM) + UA(1) breaking term

UA(1) symmetry

Infinitely large breaking

O(4) LSM

Effective restoration

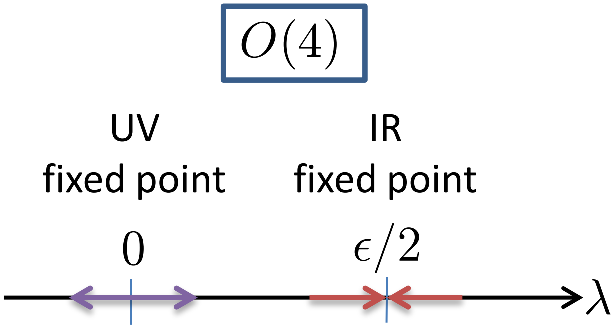
U(2)xU(2) LSM

R.D.Pisarski, F.Wilczek (1984)

Difference of RG flow

RG flow at 1-loop level in d=4-ε

Large UA(1) breaking



There is an IR fixed point

$$\lambda = \epsilon/2$$

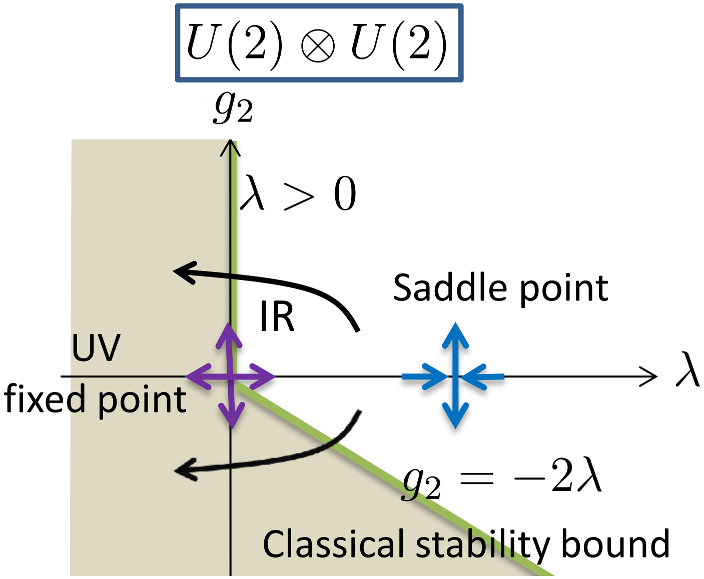
II

2nd order phase transition

※) Stability of this IRFP is well established in higher order

S. A. Antonenko and A. I. Sokolov (1998)

UA(1) restoration



No IR fixed point at 1-loop order

※) Existence of IR fixed point of U(2)xU(2) LSM in higher order is still under debate

Pelissetto and E. Vicari (2013)
Y. Nakayama and T. Ohtsuki (2014) *et .al.*

Chiral Phase transition of 2flavor QCD

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Infinitely large breaking

O(4) LSM

Effective restoration

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Chiral Phase transition of 2flavor QCD

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anomaly

Effective model

U(2) x U(2) Linear Sigma Model(LSM) + $U_A(1)$ breaking term

$U_A(1)$ symmetry

Infinitely large breaking

O(4) LSM

Finite breaking

? ? ? ?

Effective restoration

U(2)xU(2) LSM

Our work

METHOD

U(2)xU(2)LSM +~~UA(1)~~

U(2)xU(2) LSM+UA(1) breaking term (**UA(1) broken model**)

$$\mathcal{L}_{\rm total} = \mathcal{L}_{U(2)\times U(2)} + \mathcal{L}_{\rm breaking}$$

$$\Phi \to e^{2i\theta_A} L^\dagger \Phi R \ (L \in SU_L(2),\ R \in SU_R(2))$$

2x2 complex scalar

$$\mathcal{L}_{U(2)\times U(2)} = \frac{1}{2}{\rm tr}\left[\partial_\mu \Phi \partial^\mu \Phi^\dagger\right] + \frac{1}{2} m^2 {\rm tr}\left[\Phi\Phi^\dagger\right] + \frac{\pi^2}{3} g_1 \left({\rm tr}[\Phi\Phi^\dagger]\right)^2 + \frac{\pi^2}{3} g_2\, {\rm tr}\left[(\Phi\Phi^\dagger)^2\right]$$

$$\mathcal{L}_{\rm breaking} = -\frac{c_A}{4}(\det\Phi + \det\Phi^\dagger) + \frac{\pi^2}{3} x\, {\rm Tr}[\Phi\Phi^\dagger](\det\Phi + \det\Phi^\dagger) + \frac{\pi^2}{3} y\,(\det\Phi + \det\Phi^\dagger)^2$$
$$+ w\left({\rm tr}\left[\partial_\mu \Phi^\dagger\, t_2\, \partial^\mu \Phi^*\, t_2\right] + {\rm h.c.}\right)$$

In terms of components

$$\Phi = \sqrt{2}(\phi_0 - i\chi_0)t_0 + \sqrt{2}(\chi_i + i\phi_i)t_i \quad \left(t_0 = \frac{1}{2},\ t_i = \frac{\sigma_i}{2}\right)$$

$$(i = 1,2,3\ a = 0,1,2,3)$$

$$\mathcal{L}_{\rm total} = (1-w)\frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{1}{2}(m^2(T) - c_A/2){\phi_a}^2 + \frac{\pi^2}{3}\lambda({\phi_a}^2)^2$$
$$+ (1-w)\frac{1}{2}(\partial_\mu \chi_a)^2 + \frac{1}{2}(m^2(T) + c_A/2){\chi_a}^2$$
$$+ \frac{\pi^2}{3}\left[(\lambda - 2x)({\chi_a}^2)^2 + 2(\lambda + g_2 - z){\phi_a}^2{\chi_b}^2 - 2g_2(\phi_a\chi_a)^2\right]$$

U(2)xU(2)LSM +~~UA(1)~~

U(2)xU(2) LSM+UA(1) breaking term (**UA(1) broken model**)

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O(4) LSM!!

$$+(1-w)\frac{1}{2}(\partial_\mu \chi_a)^2 + \frac{1}{2}(m^2(T) + c_A/2){\chi_a}^2$$

$$+\frac{\pi^2}{3}\left[(\lambda - 2x)({\chi_a}^2)^2 + 2(\lambda + g_2 - z){\phi_a}^2{\chi_b}^2 - 2g_2(\phi_a\chi_a)^2\right]$$

Strategy

- Working hypothesis

  : $U_A(1)$ broken model undergoes 2nd order phase transition

  at $m_\phi^2(T_c) = m^2(T_c) - c_A/2 = 0$

  ⇒ 4 massless scalar ϕ_a and 4 massive scalar χ_a $m_\chi^2(T_c) = m^2(T_c) + c_A/2 = c_A$

- ε expansion with mass dependent scheme

⇓

- Check existence of an IR fixed point ⇒

  Yes : 2nd order

  No : 1st order?

- （When it is 2nd order）Calculate the critical exponents

RESULTS

β functions

β functions of U$_A$(1) broken model (1-loop) $\hat{g}_i = \mu^{-\epsilon} g_i \ (d = 4 - \epsilon)$

$$\beta_\lambda \equiv \mu \frac{d\hat{\lambda}}{d\mu} = -\epsilon\hat{\lambda} + 2\hat{\lambda}^2 + \frac{1}{6} f(\hat{\mu})(4\hat{\lambda}^2 + 6\hat{\lambda}\hat{g}_2 + 3\hat{g}_2^2 - 8\hat{\lambda}\hat{z} - 6\hat{g}_2\hat{z} + 4\hat{z}^2)$$

$$\beta_{g_2} \equiv \mu \frac{d\hat{g}_2}{d\mu} = -\epsilon\hat{g}_2 + \frac{1}{3}\hat{\lambda}\hat{g}_2 + \frac{1}{3} f(\hat{\mu})\hat{g}_2(\hat{\lambda} - 2\hat{x}) + \frac{1}{3} h(\hat{\mu})\hat{g}_2(4\hat{\lambda} + \hat{g}_2 - 4\hat{z})$$

$$\beta_x \equiv \mu \frac{d\hat{x}}{d\mu} = -\epsilon\hat{x} + \frac{1}{12}(1 - f(\hat{\mu}))(8\hat{\lambda}^2 - 6\hat{\lambda}\hat{g}_2 - 3\hat{g}_2^2 + 8\hat{\lambda}\hat{z} + 6\hat{g}_2\hat{z} - 4\hat{z}^2) + 4 f(\hat{\mu})(\hat{\lambda}\hat{x} - \hat{x}^2)$$

$$\beta_z \equiv \mu \frac{d\hat{z}}{d\mu} = -\epsilon\hat{z} + \frac{1}{2}(2\hat{\lambda}^2 - \hat{\lambda}\hat{g}_2 + 2\hat{\lambda}\hat{z}) - \frac{1}{6} h(\hat{\mu})(4\hat{\lambda}^2 + 3\hat{g}_2^2 - 8\hat{\lambda}\hat{z} + 4\hat{z}^2)$$
$$+ \frac{1}{6} f(\hat{\mu})(-2\hat{\lambda}^2 + 3\hat{\lambda}\hat{g}_2 + 3\hat{g}_2^2 - 2\hat{\lambda}\hat{z} - 6\hat{g}_2\hat{z} + 12\hat{\lambda}\hat{x} + 6\hat{g}_2\hat{x} - 12\hat{x}\hat{z} + 4\hat{z}^2)$$

$$\hat{\mu} = \frac{\mu}{\sqrt{c_A}}, \quad f(\hat{\mu}) = 1 - \frac{4}{\hat{\mu}\sqrt{4 + \hat{\mu}^2}} \arctan \frac{\hat{\mu}}{\sqrt{4 + \hat{\mu}^2}}, \quad h(\hat{\mu}) = 1 - \frac{1}{\hat{\mu}^2} \log[1 + \hat{\mu}^2]$$

$$\lim_{\hat{\mu}\to\infty} f(\hat{\mu}) = \lim_{\hat{\mu}\to\infty} h(\hat{\mu}) = 1, \ \lim_{\hat{\mu}\to 0} f(\hat{\mu}) = \lim_{\hat{\mu}\to 0} h(\hat{\mu}) = \mathcal{O}(\hat{\mu}^2)$$

β functions

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$c_A = m_\chi^2 \to \infty$ with fixed μ ⇒ $\beta_\lambda \to \beta_{O(4)}$

β functions

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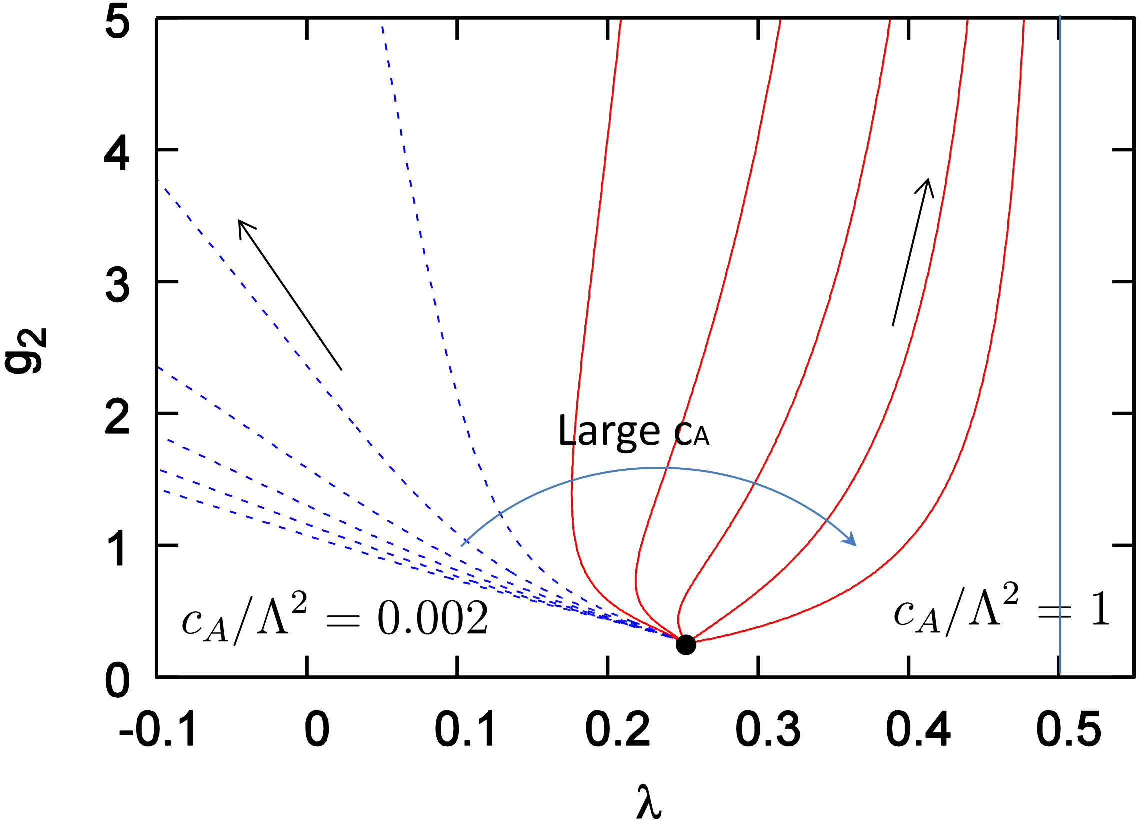
$c_A = m_\chi^2 \to \infty$ with fixed μ ➡ $\beta_\lambda \to \beta_{O(4)}$

$\mu \to 0$ limit with fixed c_A ➡ ?

RG flow

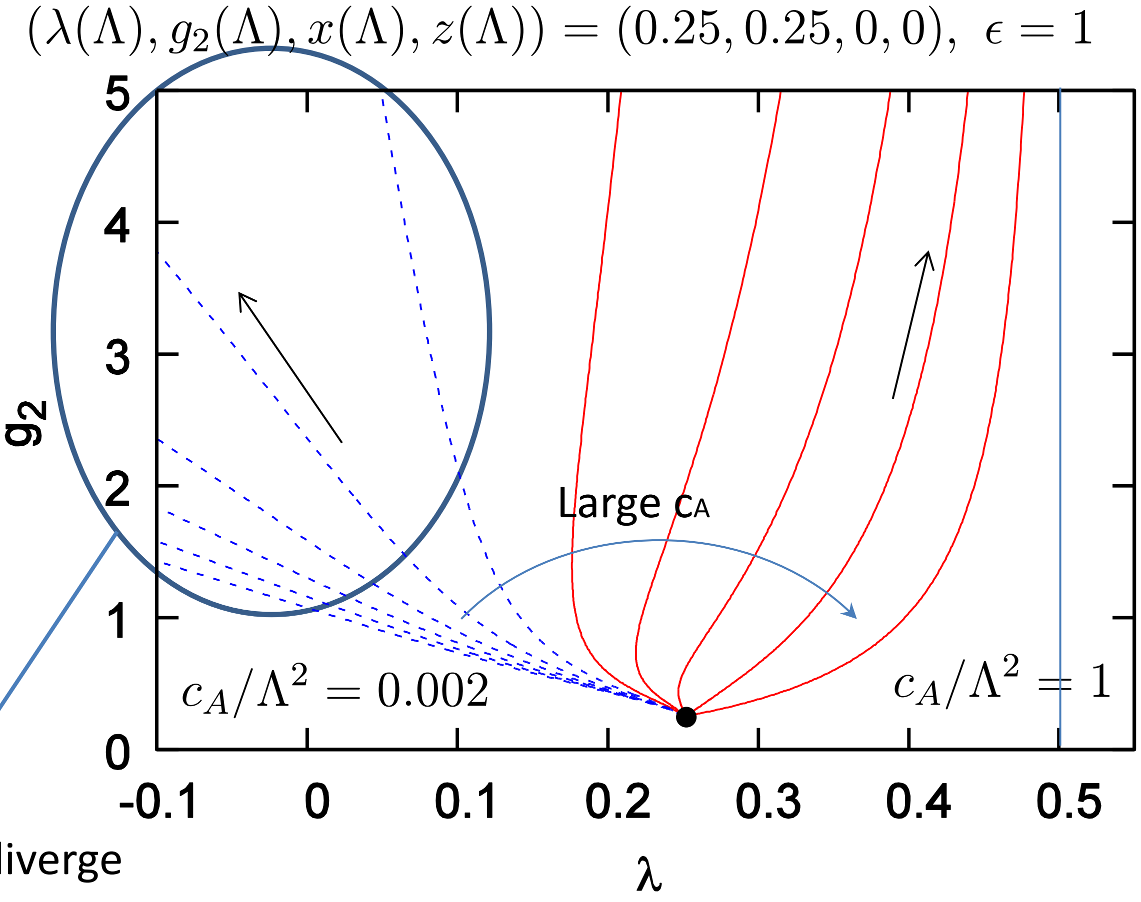
$$(\lambda(\Lambda), g_2(\Lambda), x(\Lambda), z(\Lambda)) = (0.25, 0.25, 0, 0), \;\; \epsilon = 1$$



Λ : Initial value of the renormalization scale μ

No IR fixed point

RG flow



All couplings diverge

Λ : Initial value of the renormalization scale μ

RG flow

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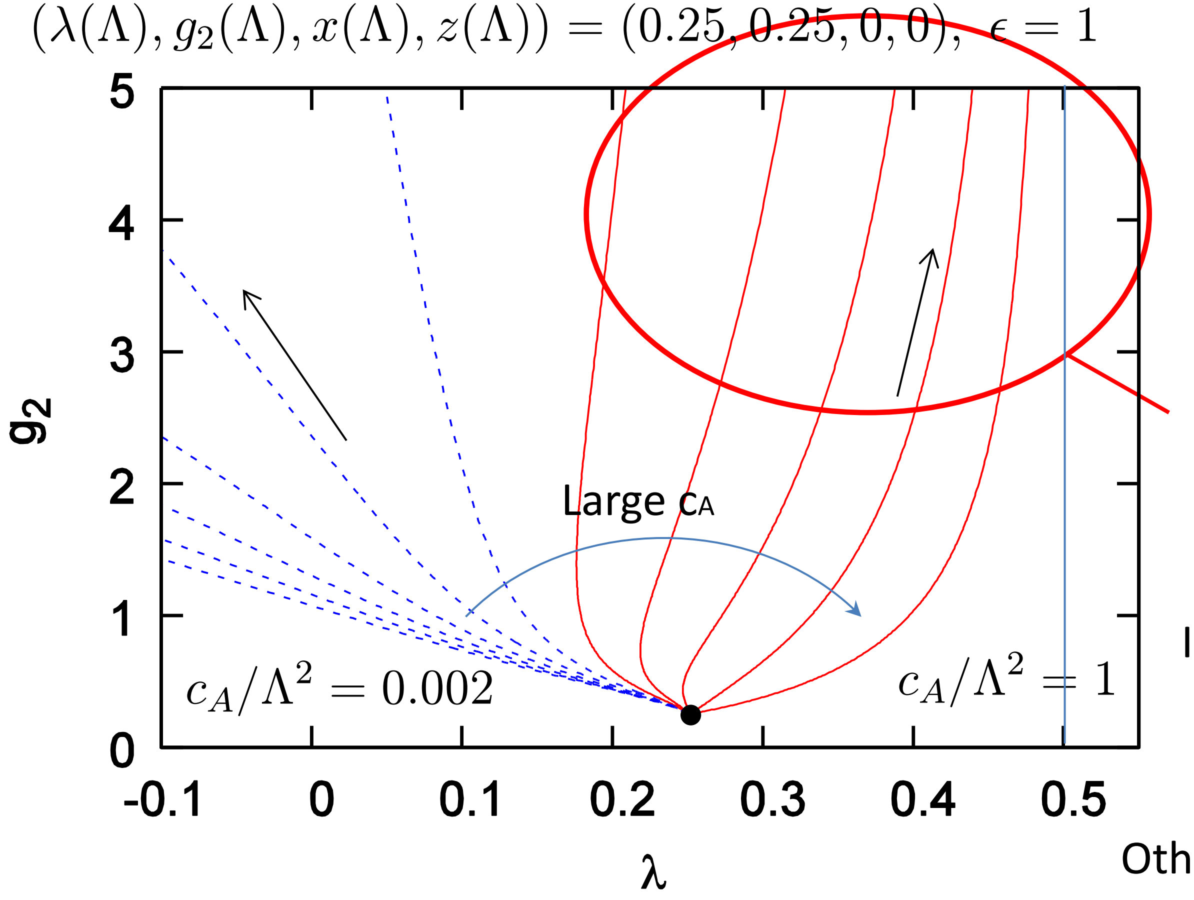


Converges to

$\hat{\lambda}_* = \epsilon/2$

IR fixed point of
O(4) LSM

Other couplings diverge

ex) $\lim_{\mu \to 0} \hat{g}_2 \sim \mu^{-\frac{5}{6}\epsilon}$

Λ : Initial value of the renormalization scale μ

Decoupling?

N point function

Any correlation function has same IR limit with O(4) LSM

⇒ **Same IR physics = 2nd order phase transition**

IR limit

$$G_\phi^{(N)} = \langle \phi_a(c_1 P)\, \phi_a(c_2 P)\, \phi_b(c_3 P) \cdots \phi_c(c_N P) \rangle \xrightarrow{P\to 0} G_{O(4)}^{(N)} \ ?$$

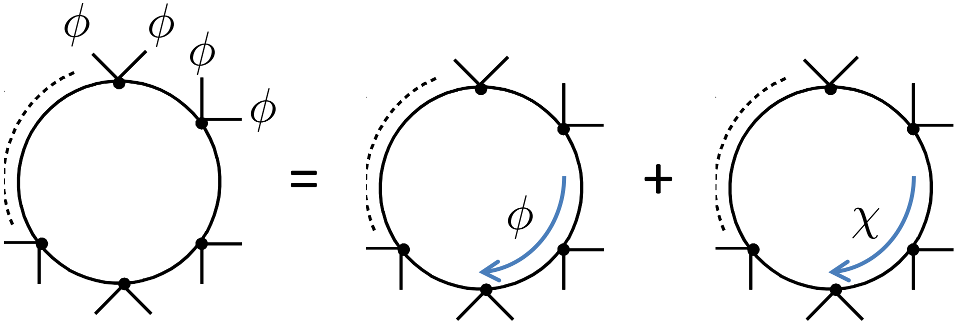
⇓

$$G_\phi^{(2)} \xrightarrow{P\to 0} G_{O(4)}^{(2)}, \quad G_\phi^{(4)} \xrightarrow{P\to 0} G_{O(4)}^{(4)}$$

N≧6?

Divergence vs. suppression

N point 1PI diagram of ϕ with typical scale P (N≧6)



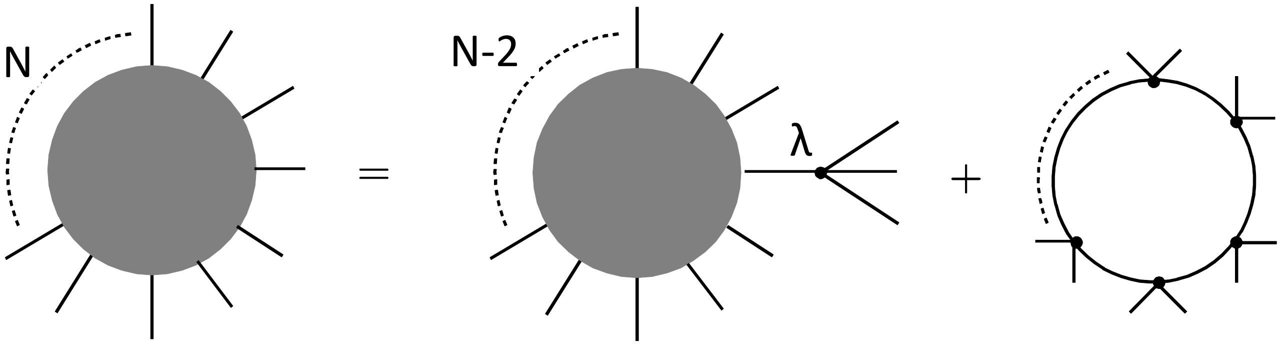
$$\sim \underbrace{\hat{\lambda}^{N/2}\left(\frac{1}{P^2}\right)^{\frac{N-d}{2}}}_{\text{ϕ's contribution}} \underbrace{+\hat{g}_2^{N/2}\left(\frac{1}{P^2+c_A}\right)^{\frac{N-d}{2}}}_{\text{χ's contribution}}$$

$$\xrightarrow{P\to 0} P^{-N+d}\left\{\hat{\lambda}^{N/2} + \underbrace{\overset{\substack{\text{Diverge}\\ \downarrow}}{\hat{g}_2^{N/2}}\overset{\substack{\text{Suppress}\\ \downarrow}}{\left(\frac{P^2}{c_A}\right)^{\frac{N-d}{2}}}}_{\xrightarrow{d=4-\epsilon} P^{N-4+\epsilon-\frac{5}{12}N\epsilon}}\right\}$$

Contribution from massive components vanishes with $N > \dfrac{4-\epsilon}{1-\frac{5}{12}\epsilon} \xrightarrow{\epsilon\to 1} \dfrac{36}{7}$

= N(≧6) point 1PI diagram **converges to O(4)**

Decoupling



Any correlation function has same IR limit with O(4)

$$G_{\phi}^{(N)} = \langle \phi_a(c_1P)\,\phi_a(c_2P)\,\phi_b(c_3P)\cdots\phi_c(c_NP)\rangle \xrightarrow{P\to 0} G_{O(4)}^{(N)}$$

IR physics agree with O(4) LSM

||

$U_A(1)$ broken model undergoes 2nd order phase transition

O(4) universality?

Critical exponents

ν : Power index of a correlation length $\xi \sim \left(\frac{|T-T_c|}{T_c}\right)^{\nu}$

η : Anomalous dimension of correlation function $\langle\phi(x)\phi(0)\rangle \sim |x|^{-d+2-\eta}$

ω : Scaling dimension of the leading irrelevant operator $|\hat{\lambda}-\hat{\lambda}_*| \sim \mu^{\omega}$

Results at the leading order of ε expansion

$U_A(1)$ breaking	ν	η	ω
Infinite (O(4))	2/(4-ε)	0	ε
Finite	2/(4-ε)	0	2-5ε/3

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Non-decoupling !?

Nature of the chiral phase transition

Possible pattern of chiral phase transition of two-flavor QCD
depends on c_A

1. 1st order ($c_A \fallingdotseq 0$)

2. 2nd order with U(2)xU(2) universality class ($c_A=0$)

3. 2nd order with O(4) universality class ($c_A \to \infty$)

Nature of the chiral phase transition

Possible pattern of chiral phase transition of two-flavor QCD depends on c_A

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2. 2nd order with U(2)xU(2) universality class ($c_A=0$)

3. 2nd order with O(4) universality class ($c_A \to \infty$)

4. 2nd order with the $U_A(1)$ broken scaling ($c_A \neq 0, \lambda \to \varepsilon/2$)

Summary

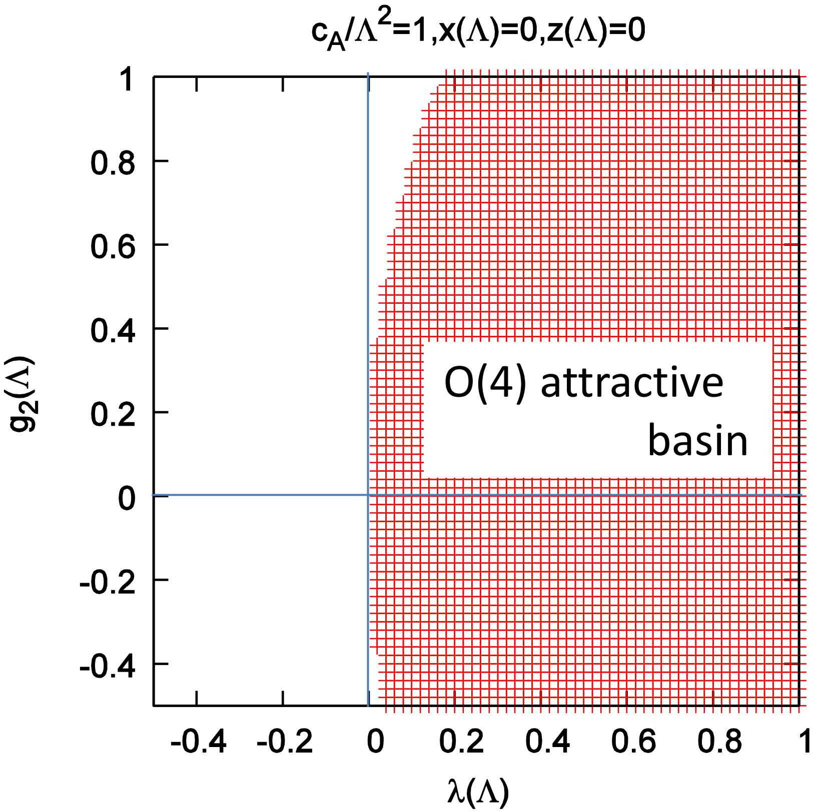
- We study the nature of chiral phase transition of massless two-flavor QCD using the $U_A(1)$ broken model
- Depending on the parameters, the model shows the 2nd order phase transition
- We find that at least one of the critical exponents has different value from the O(4) LSM. It suggests novel possibility of chiral phase transition of two-flavor QCD.

Thank you for your attention!

BACKUP

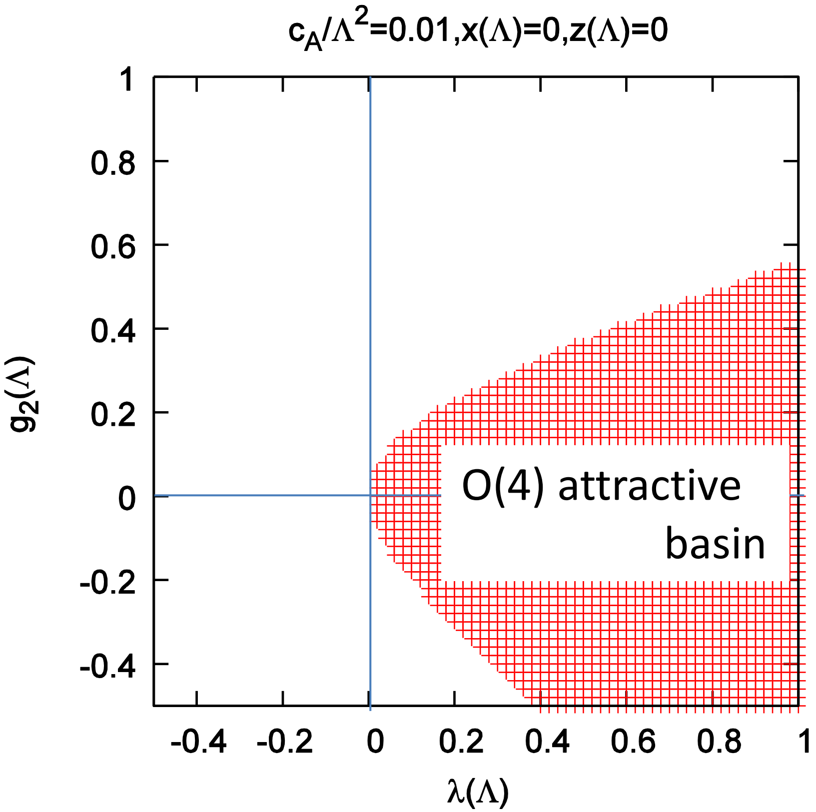
O(4) attractive basin

Region of initial values λ(Λ), g2(Λ) flowing into O(4) fixed point



$c_A/\Lambda^2=1, x(\Lambda)=0, z(\Lambda)=0$

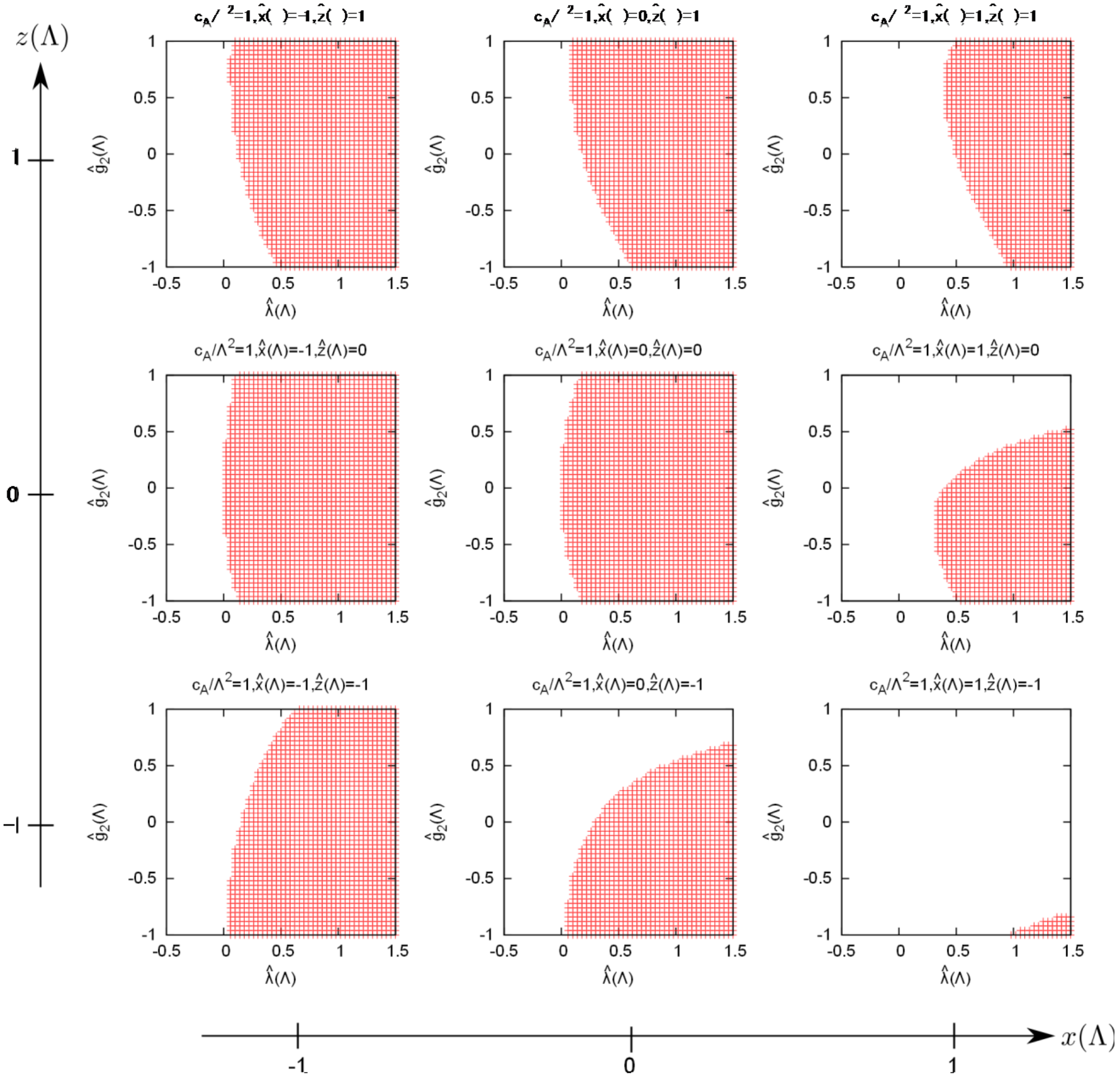
Small cA



$c_A/\Lambda^2=0.01, x(\Lambda)=0, z(\Lambda)=0$

O(4) attractive basin shrinks as c_A vanishes

O(4) attractive basin



Asymptotic behavior

- β functions of g2 and z ($\hat{\lambda}(\mu \to 0) \sim \epsilon/2$)

$$\begin{aligned}
\beta_{\hat{g}_2}(\mu \to 0) &\sim -\frac{5}{6}\epsilon\hat{g}_2 & \hat{g}_2(\mu \to 0) &\sim c\left(\frac{\mu}{\sqrt{c_A}}\right)^{-5\epsilon/6} \\
\beta_{\hat{x}}(\mu \to 0) &\sim -\epsilon\hat{x} + \frac{1}{12}(-3\hat{g}_2 + 6\hat{g}_2\hat{z} - 4\hat{z}^2) \quad\Longrightarrow & \hat{x}(\mu \to 0) &\sim \frac{3}{32}\hat{g}_2^2(\mu) \\
\beta_{\hat{z}}(\mu \to 0) &\sim -\frac{1}{2}\epsilon\hat{z} - \frac{1}{4}\epsilon\hat{g}_2 & \hat{z}(\mu \to 0) &\sim \frac{3}{4}\hat{g}_2(\mu)
\end{aligned}$$

- β function of $\hat{\lambda}(\mu \to 0)$

$$\begin{aligned}
\beta_\lambda &= -\epsilon\hat{\lambda} + 2\hat{\lambda}^2 + \frac{1}{6}f(\hat{\mu})(4\hat{\lambda}^2 + 6\hat{\lambda}\hat{g}_2 + 3\hat{g}_2^2 - 8\hat{\lambda}\hat{z} - 6\hat{g}_2\hat{z} + 4\hat{z}^2) \\
&\approx -\epsilon\lambda + 2\lambda^2 + \frac{c^2}{24}\mu^{2-\frac{5}{3}\epsilon} \quad\Longrightarrow\quad \lim_{\mu\to 0}\left[\hat{\lambda} - \hat{\lambda}_*\right] \sim -\frac{c^2}{8(5\epsilon - 3)}\left(\frac{\mu}{\sqrt{c_A}}\right)^{2-\frac{5}{3}\epsilon}
\end{aligned}$$

Four point correlation function

$G^{(4)}_{\phi}(\phi_a(p_1),\phi_a(p_2);\phi_b(p_3),\phi_b(p_4))|_{s=t=u=P^2}$ $\quad (s=(p_1+p_2)^2,\ t=(p_1-p_3)^2,\ u=(p_1-p_4)^2)$

$$
\begin{aligned}
&= \left(\prod_{i=1}^{4}\frac{-1}{p_i^2}\right)\mu^{\epsilon}\Bigg[-\frac{8}{3}\pi^2\hat{\lambda}-\frac{8}{3}\pi^2\hat{\lambda}\log[P^2/\mu^2] \\
&\qquad -\frac{2}{9}\pi^2(4\hat{\lambda}^2+6\hat{\lambda}\hat{g}_2+3\hat{g}_2^2-8\hat{\lambda}\hat{z}-6\hat{g}_2\hat{z}+4\hat{z}^2)\int_0^1 d\xi\log\left[\frac{c_A+\xi(1-\xi)P^2}{c_A+\xi(1-\xi)\mu^2}\right]\Bigg]
\end{aligned}
$$

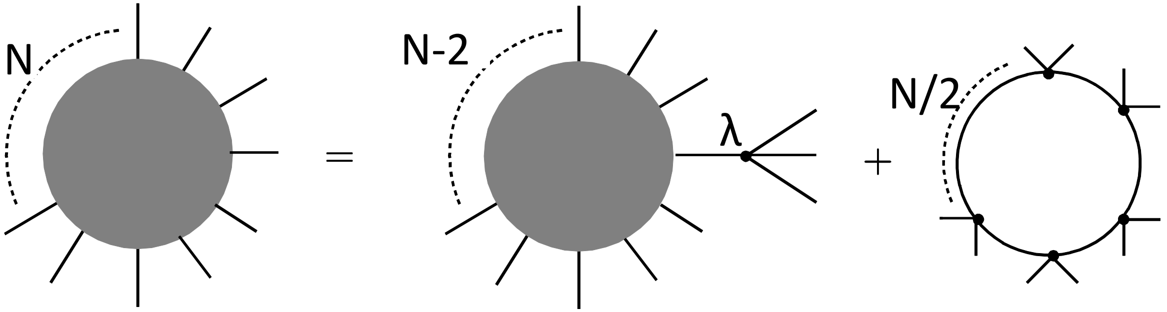
RG improvement

$$
G^{(4)}_{\phi}|_{s=t=u=P^2} = \left(\prod_{i=1}4\frac{-1}{p_i^2}\right)P^{\epsilon}\left[-\frac{8}{3}\pi^2\bar{\lambda}(P)\right], \qquad P\frac{d}{dP}\bar{\lambda}(P) = \begin{cases} \beta_{\lambda}^{U_A(1)\,\mathrm{broken}} & \mathrm{U_A(1)\ broken} \\ \beta_{O(4)} & \mathrm{O(4)} \end{cases}
$$

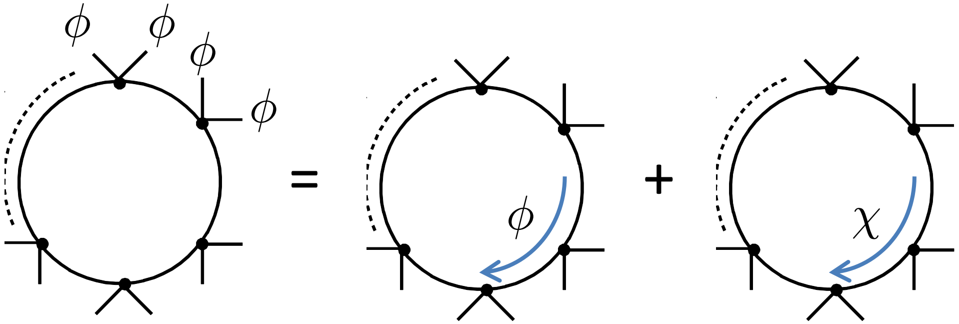
When $\lim_{P\to 0}\bar{\lambda}(P)|_{U_A(1)\,\mathrm{broken}} = \lim_{P\to 0}\bar{\lambda}(P)|_{O(4)}$,

the four point function of the U$_A$(1) broken model agrees with O(4)

Divergence vs. suppression



N point 1PI diagram of ϕ with typical scale P (N>4)



$$\xrightarrow{P\to 0} P^{-N+d}\left\{\hat{\lambda}^{N/2}+\underline{\overset{\overset{\text{Diverge}}{\downarrow}}{\hat{g}_2^{N/2}}\left(\overset{\overset{\text{Suppress}}{\downarrow}}{\frac{P^2}{c_A}}\right)^{\frac{N-d}{2}}}\right\}$$

$$\xrightarrow{d=4-\epsilon} P^{N-4+\epsilon-\frac{5}{12}N\epsilon}$$

Contribution from massive components vanishes with $N > \dfrac{4-\epsilon}{1-\frac{5}{12}\epsilon} \xrightarrow{\epsilon\to 1} \dfrac{36}{7}$

= N(≧6) point 1PI diagram **converges to O(4)**

Footprint of non-decoupling

4 point functions of UA(1) broken model and O(4) LSM

$$G_{\phi}^{(4)}|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2 \left(\prod_{i=1} 4\frac{-1}{p_i^2} \right) P^{\epsilon} \left\{ \frac{\epsilon}{2} - c \left(\frac{P}{\mu} \right)^{2-\frac{5}{3}\epsilon} \right\},$$

$$G_{O(4)}^{(4)}|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2 \left(\prod_{i=1} 4\frac{-1}{p_i^2} \right) P^{\epsilon} \left\{ \frac{\epsilon}{2} - c' \left(\frac{P}{\mu} \right)^{\epsilon} \right\},$$

Approaching rate

4 point functions of UA(1) broken model and O(4) LSM

$$G^{(4)}_{\phi}|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2 \left(\prod_{i=1} 4\frac{-1}{p_i^2}\right) P^{\epsilon} \left\{ \underline{\frac{\epsilon}{2}} - c\left(\frac{P}{\mu}\right)^{2-\frac{5}{3}\epsilon} \right\},$$

Agree

$$G^{(4)}_{O(4)}|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2 \left(\prod_{i=1} 4\frac{-1}{p_i^2}\right) P^{\epsilon} \left\{ \overline{\frac{\epsilon}{2}} - c'\left(\frac{P}{\mu}\right)^{\epsilon} \right\},$$

Approaching rate

4 point functions of UA(1) broken model and O(4) LSM

$$G_{\phi}^{(4)}|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2 \left(\prod_{i=1} 4\frac{-1}{p_i^2}\right) P^{\epsilon} \left\{ \underline{\frac{\epsilon}{2}} - c\left(\frac{P}{\mu}\right)^{\boxed{2-\frac{5}{3}\epsilon}} \right\},$$

Agree

Disagree

$$G_{O(4)}^{(4)}|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2 \left(\prod_{i=1} 4\frac{-1}{p_i^2}\right) P^{\epsilon} \left\{ \overline{\frac{\epsilon}{2}} - c'\left(\frac{P}{\mu}\right)^{\boxed{\epsilon}} \right\},$$

Same convergence point, but **different approaching rate**

Approaching rate

4 point functions of UA(1) broken model and O(4) LSM

$$G^{(4)}_{\phi}\big|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2\left(\prod_{i=1} 4\frac{-1}{p_i^2}\right)P^{\epsilon}\left\{\frac{\epsilon}{2}-c\left(\frac{P}{\mu}\right)^{\boxed{2-\frac{5}{3}\epsilon}}\right\},$$

$\Uparrow$

ω

$\Downarrow$

$$G^{(4)}_{O(4)}\big|_{s=t=u=P^2} \xrightarrow{P\to 0} -\frac{8}{3}\pi^2\left(\prod_{i=1} 4\frac{-1}{p_i^2}\right)P^{\epsilon}\left\{\frac{\epsilon}{2}-c'\left(\frac{P}{\mu}\right)^{\boxed{\epsilon}}\right\},$$