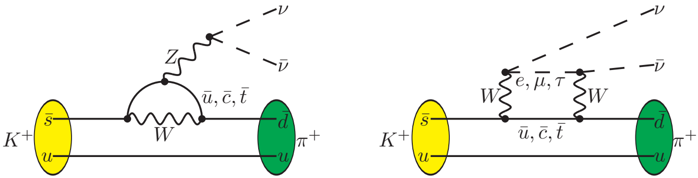
Long-distance contributions to the rare kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$



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Lat15@Kobe, 07/17/2015

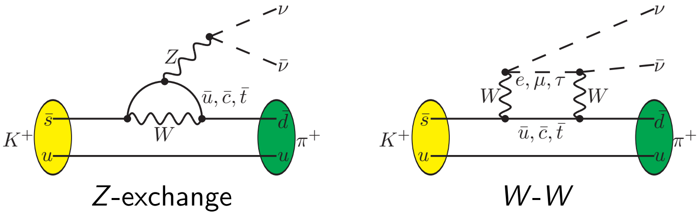
Collaborators

- on behalf of RBC-UKQCD collaboration
- people involved in this project

UKQCD	**RBC**
Andreas Jüttner (Southampton)	Norman Christ (Columbia)
Andrew Lawson (Southampton)	Xu Feng (Columbia)
Antonin Portelli (Southampton)	Christoph Lehner (BNL)
Chris Sachrajda (Southampton)	Amarjit Soni (BNL)

$K^+ \to \pi^+\nu\bar{\nu}$: Experiment vs Standard model

As FCNC process, $K \to \pi\nu\bar{\nu}$ decay through second-order weak interaction



SM effects highly suppressed in the second order → ideal probes for NP

Past experimental measurement is 2 times larger than SM prediction

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$ arXiv:0808.2459

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = 9.11 \pm 0.72 \times 10^{-11}$$ arXiv:1503.02693

but still consistent with $> 60\%$ exp. error

New experiments

New generation of experiment: NA62 at CERN aims at

- observation of $O(100)$ events in 2-3 years
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$

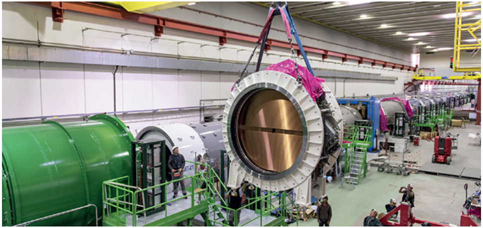
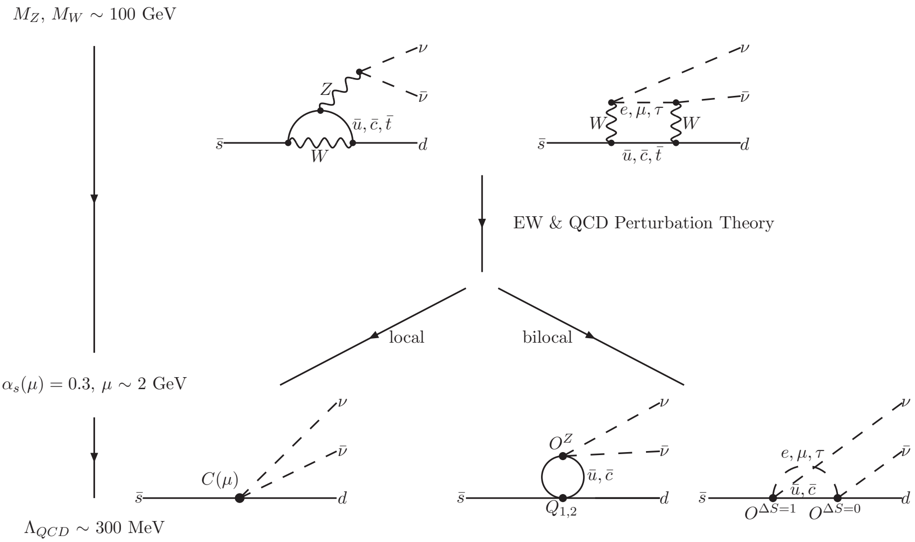


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

$K_L \to \pi^0 \nu\bar{\nu}$

- even more challenging since π^0 decays quickly to two photons
- only upper bound was set by KEK E391a in 2010
- new KOTO experiment at J-PARC designed to observe K_L decays

Low energy effective field theory



Effective Hamiltonian for charm quark contribution

- SD part: $\mathcal{H}_{eff}$ described by a dim-6 operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$

$$\mathcal{H}_{eff}^{(6)} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{\ell=e,\mu,\tau} \lambda_c X_c^\ell \, (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

X_c^ℓ is the perturbative Inami-Lim function for charm quark loop

- LD part: bilocal effects from two four-fermion operator O_1 and O_2

$$\mathcal{H}_{eff}^{BL} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{\ell=e,\mu,\tau} \lambda_c \left(\frac{\pi^2}{M_W^2} \int d^4x \, O_1(x) \, O_2(0) \right)_{u-c}$$

- Define bilocal contribution X_{BL} as

$$X_{BL} = \frac{\langle \pi\nu\bar{\nu} | \left(\frac{\pi^2}{M_W^2} \int d^4x \, O_1(x) \, O_2(0) \right)_{u-c} | K \rangle}{\langle \pi\nu\bar{\nu} | (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} | K \rangle}$$

so that X_{BL} can be compared to X_c^ℓ directly

Lattice setup

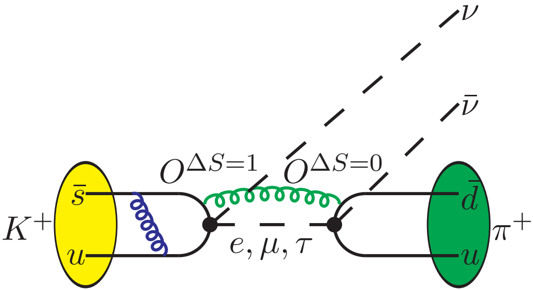
- $16^3 \times 32$, DWF+Iwasaki, $m_\pi \approx 420$ MeV, $m_K \approx 540$ MeV, $a^{-1} = 1.73$ GeV, $m_c = 860$ MeV, 800 configurations
- Construct 4-point correlator $\langle \phi_\pi(t_\pi) O_1(t_1) O_2(t_2) \phi_K^\dagger(t_K) \rangle$
  - wall source for ϕ_π and ϕ_K $\Rightarrow$ better overlap with ground state
  - O_1 and O_2: point source for one operator, the other is sink
- Perform time translation average $\rightarrow$ statistical error reduced by $\sqrt{T}$
  - low-mode deflation to reduce the time required by light quark CG
- One can extract the scalar amplitude $F_{BL}^\ell(p_K, p_\nu, p_{\bar\nu})$

$$\int dt \, \langle \pi^+ \nu \bar\nu | T\{O_1(t) O_2(0)\} | K^+ \rangle = F_{BL}^\ell(p_K, p_\nu, p_{\bar\nu}) \, \bar u(p_\nu) \not p_K (1 - \gamma_5) v(p_{\bar\nu})$$

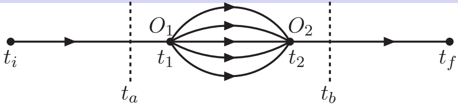
Preliminary results for W-W diagrams

(Z-exchange similar as γ-exchange, see A. Lawson's talk)

Type 1 diagram



Double integration



- Perform the double integration to gain a better precision

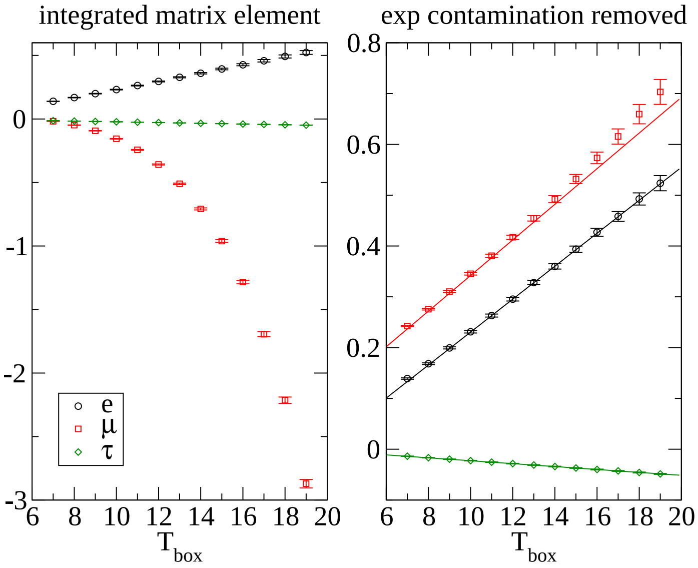
$$\sum_{t_1=t_a}^{t_b}\sum_{t_2=t_a}^{t_b}\langle f|T[O^{\Delta S=1}(t_2)O^{\Delta S=0}(t_1)]|K\rangle\, e^{m_K t_1}e^{-m_f t_1}$$

$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(T_{\text{box}}-\frac{1-e^{(E_f-E_{n_s})T_{\text{box}}}}{E_{n_s}-E_f}\right)$$

$$-\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(T_{\text{box}}+\frac{1-e^{(E_K-E_n)T_{\text{box}}}}{E_K-E_n}\right)$$

  here $T_{\text{box}} = t_b - t_a + 1$ is defined as size of the integral window
- Remove the exponential growing contamination, and fit with $a + bT_{box}$, the slope b is what we want
- On how to remove exp contamination, see also A. Lawson's talk

Integrated matrix element



Right figure: the slope of the curve gives $F^{\ell}_{BL}(p_K, p_\nu, p_{\bar{\nu}})$

F_{BL}^{ℓ} for type 1 diagram

F_{BL}^{ℓ}	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

- Vacuum saturation approximation assumes only single-lepton contribution in the intermediate state

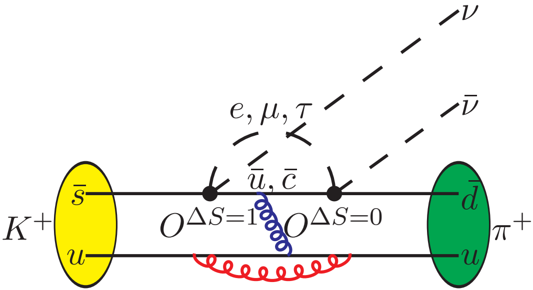
$$f_K p_{K,\mu} \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)\frac{\not{q}}{q^2 - m_\ell^2}\gamma_\nu(1-\gamma_5)v(p_{\bar{\nu}}) f_\pi p_{\pi,\nu}$$

$$= f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2}\bar{u}(p_\nu)\not{p}_K(1-\gamma_5)v(p_{\bar{\nu}})$$

with $q = p_K - p_\nu = p_\pi + p_{\bar{\nu}}$

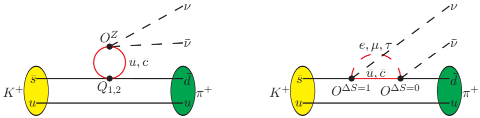
- In the above table, model results are given by $Z_A^{-2} f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2}$

Type 2 diagram



Short-distance divergence

- By dimensional counting the loop integrals are quadratically divergent



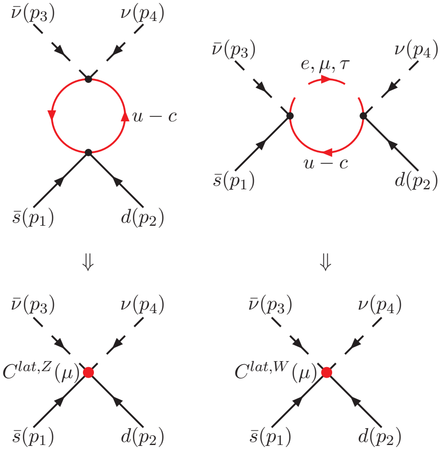
- GIM mechanism reduces the divergence to logarithmic
- In the physical world, the SD divergence is cut off by physical M_W
- In the lattice calculation it is cut off by an energy scale $\Lambda_{lat} \sim \frac{1}{a}$
- Correction can be made through $A - A_{SD}^{lat} + A_{SD}^{cont} =$

$$\int d^4x \, \langle f|T\{O_1(x)O_2(0)\}|K\rangle - \langle f|C^{lat}(\mu)O_{SD}|K\rangle + \langle f|C^{cont}(\mu)O_{SD}|K\rangle$$

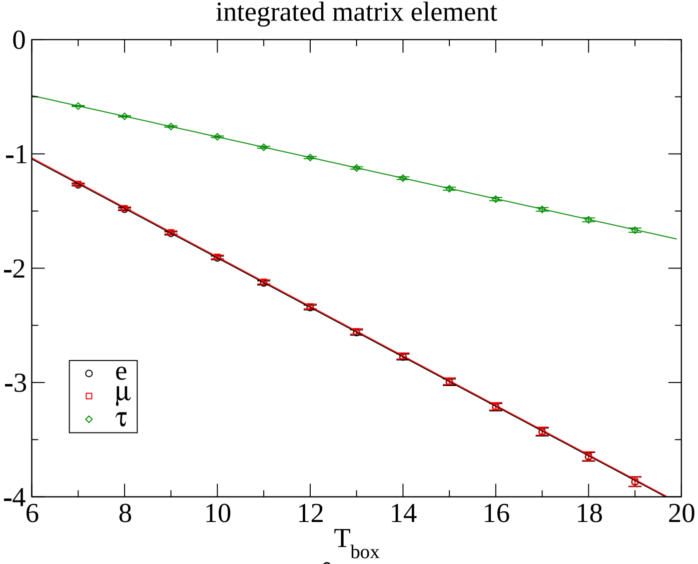
- $C^{lat}(\mu)$ is determined non-perturbatively using RI/SMOM approach
- $C^{cont}(\mu)$ can be calculated perturbatively, currently in LO

Rome-Southampton method (RI/SMOM)

- Evaluate off-shell Green's function with $p_i^2 \gg \Lambda_{QCD}^2$
- Energy scale of internal momentum, μ^2, is forced to be larger than p_i^2
- At high energy scale μ, mainly SD contribution to off-shell Green's function
- Correctly represented by a SD operator multiplying with Wilson coefficient $C^{lat}(\mu)$



Type 2 diagram



Intermediate state is given by $\ell + \pi^0$, since pion is heavy, we don't observe significant exponential growing effects

Preliminary results

- Type 1 diagram

F_{BL}^{ℓ}	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

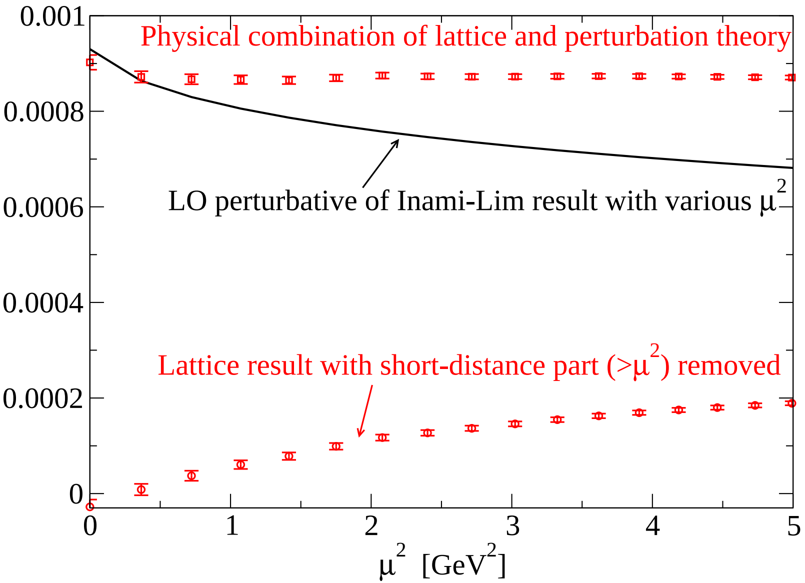
- Type 2 diagram

F_{BL}^{ℓ}	lattice
e	$-2.164(31) \times 10^{-1}$
μ	$-2.164(31) \times 10^{-1}$
τ	$-9.03(14) \times 10^{-2}$

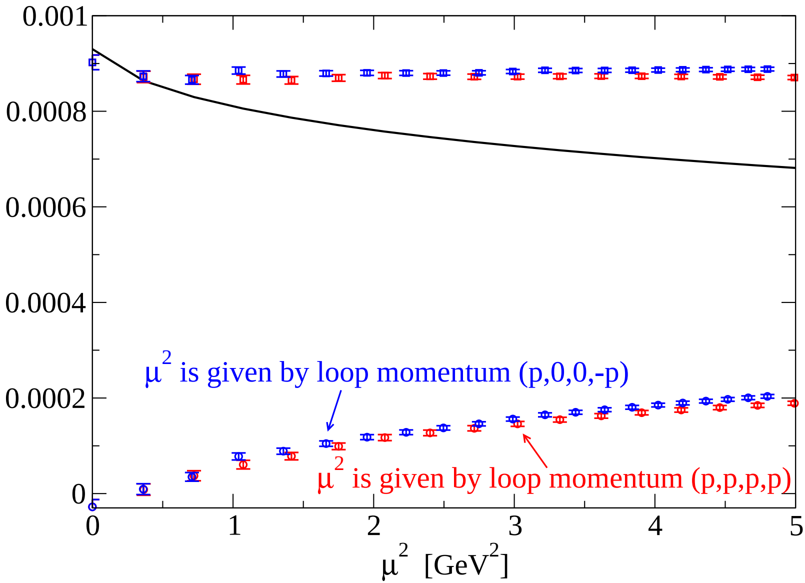
- It seem that type 2 contribution is much larger than type 1, but

type 2 diagram contains large lattice cutoff effects due to SD divergence

Short-distance matching and correction



SD matching: different loop momentum



Pauli-Villars method

- Rome-Southampton (RI/SMOM)

$$\int d^4x \langle f|T\{O_1(x)O_2(0)\}|K\rangle - \langle f|C^{lat}(\mu^2)O_{SD}|K\rangle + \langle f|C^{cont}(\mu^2)O_{SD}|K\rangle$$

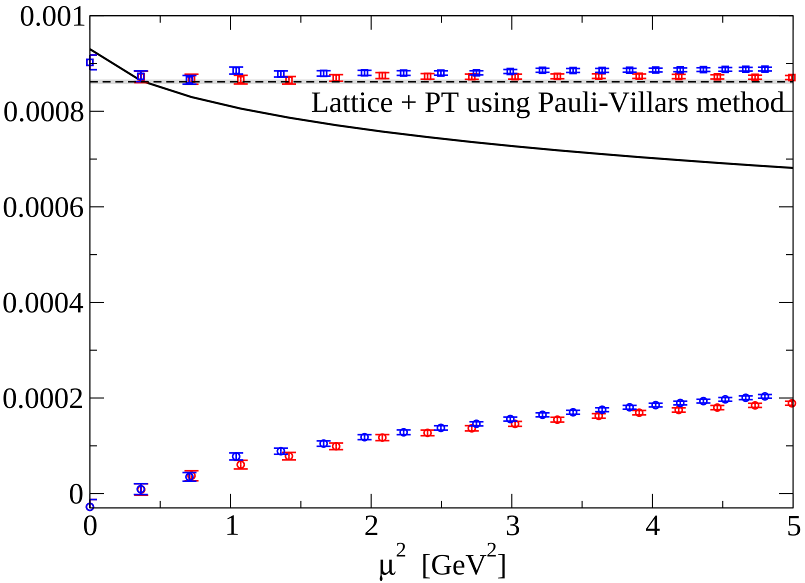
  - μ^2 is the scale of loop momenta
  - $C^{lat}(\mu^2)$ contains the lattice cutoff effects, but replaced by $C^{cont}(\mu^2)$

- Pauli-Villars

$$X^{\ell}_{BL} - X^{M}_{BL} + X^{M}_{c}$$

  - we use a heavy lepton M as a regulator
  - X^M_{BL} contains the lattice cutoff effects, but replaced by correct SD X^M_c
  - we calculated X^{ℓ}_{BL} with $\ell = e, \mu, \tau$, thus we use $M = \tau$ as a regulator

SD matching: Rome-Southampton vs Pauli-Villars



Outlook

- Calculation of the non-local matrix element is highly non-trivial, see also
  - A. Lawson's talk on $K^+ \to \pi^+\ell^-\ell^-$
  - N. Christ's talk on ϵ_K
  - C. Sachrajda's talk on EM correction to hadronic process
  - A. Jüttner's plenary talk
- Our exploratory study sheds light on the feasibility of lattice calculation of $K^+ \to \pi^+\nu\bar{\nu}$
- We are starting the calculation at $m_\pi = 170$ MeV. In the future, physical charm quark will also be included.
- NA62 will confront SM soon $\Rightarrow$ It's in a timely fashion for lattice QCD to make impact on $K^+ \to \pi^+\nu\bar{\nu}$