The Nature of the Roberge-Weiss Transition in $N_f = 2$ QCD with Wilson Fermions on $N_t = 6$ Lattices

Christopher Czaban
in collaboration with
O. Philipsen, C. Pinke, F. Cuteri, A. Sciarra

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Outline

QCD Phase Diagram

Imaginary Chemical Potential and Roberge Weiss Symmetry

The QCD Phase Structure for Imaginary $\mu$

Previous and Ongoing Studies

Summary and Perspectives
QCD Phase Diagram

- Sign problem spoils Hybrid Monte Carlo simulations for $\mu > 0$.
- Simulate at $\mu = 0$ and apply e.g. extrapolation techniques.
- Choose purely imaginary chemical potential $\mu = i\mu_i$.偏偏
Sign problem spoils Hybrid Monte Carlo simulations for $\mu > 0$.
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Imaginary Chemical Potential and Roberge Weiss Symmetry

- No sign problem $\rightarrow$ Hybrid Monte Carlo applicable.
- QCD partition function symmetries (Roberge-Weiss symmetry)

\[ Z(\mu) = Z(-\mu), \quad Z\left(\frac{\mu}{T}\right) = Z\left(\frac{\mu}{T} + i\frac{2\pi n}{3}\right). \]

- Adding $\mu_i/T = 2\pi n/3$, $n \in \mathbb{Z}$ is equivalent to $Z(3)$ transformation.
- Describes completely equivalent physics.
- Centre symmetry is a good symmetry again.
- Centre sectors separated by

\[ \mu^c_i = \frac{\pi T}{3} (2n + 1), \quad n \in \mathbb{Z}. \]

Imaginary Chemical Potential and Roberge Weiss Symmetry

Roberge-Weiss phase diagram

Columbia plot at $\mu = 0$

- Polyakov loop on the lattice: $L(n) = \frac{1}{V} \prod_{\tau = 0}^{N_{\tau}} U_0(\tau, n)$
- Phase of the Polyakov is sensitive to centre sector transitions

$$\text{Tr}L^g = e^{-i\frac{2\pi n}{3}} \text{Tr}L$$
Imaginary Chemical Potential and Roberge Weiss Symmetry

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Columbia plot at $\mu_c = i\pi T/3$

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The QCD Phase Structure for Imaginary $\mu$

$N_f = 2$

$N_f = 1$

phys. point

$m_s$

$m_{ud}$

$crossover$

$Z(2)$

$1^{st}$

$m_{ud}^{tric}$

$m_s^{tric}$

$N_f = 3$

$1^{st}$

$N_f = 3$

$1^{st}$

$Z(2)$

$2^{nd}$ order $3d$ Ising

$N_f = 3$

$1^{st}$

$Z(2)$

$crossover$

$0$

$-(\frac{\pi}{3})^2$

$N_f = 1$

$N_f = 2$

$-(\frac{\pi}{3})^2$

$1^{st}$

$2^{nd}$ $3d$ Ising

$1^{st}$

$1^{st}$

$1^{st}$

$1^{st}$

$m_{ud}$

$m_s$

P. de Forcrand and O. Philipsen, JHEP (2007)
P. de Forcrand and O. Philipsen, JHEP (2008)
H. Saito et al., arXiv:1309.2445 (hep-lat)
The QCD Phase Structure for Imaginary $\mu$

$N_f = 2$

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$1^{st}$

$2^{nd}$ order $3d$ Ising

See next talk by Christopher Pinke

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The QCD Phase Structure for Imaginary $\mu$

$1^{st}$ order $\rightarrow$ $1^{st}$ order triple

Previous and Ongoing Studies

Studies done on $N_T = 4$:
- Staggered, $N_f = 2, 2 + 1, 3$
- Wilson, $N_f = 2$

Ongoing for $N_T = 6$:
- Wilson, $N_f = 2$
- $\kappa \in (0.1000, 0.1650)$
- $O(15)\beta$ for $3-4\ N_s \in (16, 36)$
- $O(80 k - 200 k)$ trajectories

OpenCL based CLQCD code
Bach, Philipsen, Pinke (arxiv:1411.5219v2)(2014)
https://github.com/CL2QCD/cl2qcd.git

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Extracting the Order of a Phase Transition

\[ B_4(\beta) = \frac{\langle (L_{\text{Im}} - \langle L_{\text{Im}} \rangle)^4 \rangle}{\langle (L_{\text{Im}} - \langle L_{\text{Im}} \rangle)^2 \rangle^2} \]

\[ \lim_{V \to \infty} B_4(\beta_c) = \begin{cases} 
1, & 1^{st} \text{ order} \\
1.5, & 1^{st} \text{ order triple} \\
1.604, & 2^{nd} \text{ order } Z(2) \\
3, & \text{crossover}
\end{cases} \]

\[ B_4(\beta, N_\sigma) = B_4(\beta_c, \infty) + a(\beta - \beta_c)N_\sigma^{1/\nu} + \ldots \]
Finite Size Scaling

- Smoothing/interpolating Data points with Ferrenberg-Swensen reweighting.
- Fitting of

\[ B_4(\beta_c, N_\sigma) = B_4(\beta_c, \infty) + a(\beta - \beta_c) N_\sigma^{1/\nu} \]

to reweighted points.

\[ \kappa = 0.11, \quad N_\tau = 6 \]
Finite Size Scaling

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\[ = B_4(\beta_c, \infty) + ax + \ldots \]

- **Fit criteria:**
  - ✓ \( Q \approx 50\% \)
  - ✓ \( \chi^2 \approx 1 \)
  - ✓ Overlap in \( x \geq 80\% \)
  - ✓ Symmetry of \( x \) around 0

![Diagram](image-url)
Finite Size Scaling

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- \( \nu \) less prone to finite size effects
  → better suited to extract order of phase transition

- Determine order of phase transition according to

\[
\nu = \begin{cases} 
1/3 & \text{1st order triple} \\
1/2 & \text{tricritical} \\
0.63 & \text{2nd order 3D Ising}
\end{cases}
\]
Results for studies with $N_f = 2$ flavors of Wilson fermions on $N_T = 4$ lattices

- $\kappa$ as function of the bare quark mass: $\kappa = (2(am + 4))^{-1}$
Results for studies with $N_f = 2$ flavors of Wilson fermions on $N_\tau = 6$ lattices

- $\kappa$ as function of the bare quark mass: $\kappa = (2 (am + 4))^{-1}$
- Shift in 1\textsuperscript{st} order region to smaller kappa: $T = 1/(aN_\tau)$
  \[\Rightarrow\] Tricritical mass: $m_{\pi}^{\text{tric}} \approx 730$ MeV $\rightarrow m_{\pi}^{\text{tric}} \approx 660$ MeV
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Summary

- No sign problem for imaginary chemical potential.
- QCD phase diagram constrained by imaginary chemical potential region.
- $N_\tau = 6$ studies with Wilson fermions ongoing.
- Shift in $1^{st}/2^{nd}$ order region.

Perspectives

- Compare Wilson results to $N_\tau = 6$ staggered results (ongoing).
- Start studies for $N_\tau = 8$.
- Extend Wilson fermion studies to $N_f = 3$. 