The Nature of the Roberge-Weiss Transition in $N_f = 2$ QCD with Wilson Fermions on $N_t = 6$ Lattices

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in collaboration with
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Kobe, Japan

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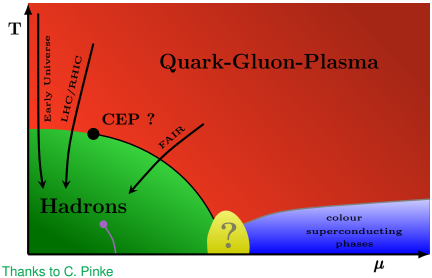
Outline

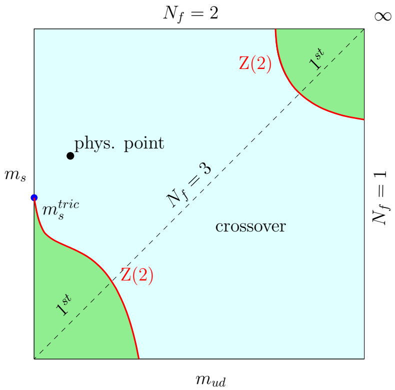
QCD Phase Diagram

QCD Phase Diagram



Thanks to C. Pinke

Columbia plot, $\mu = 0$



- Sign problem spoils Hybrid Monte Carlo simulations for $\mu > 0$.
- Simulate at $\mu = 0$ and apply e.g. extrapolation techniques.
- Choose purely imaginary chemical potential $\mu = i\mu_i$.

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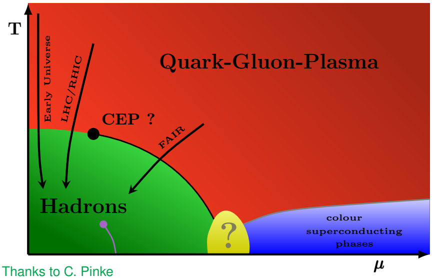

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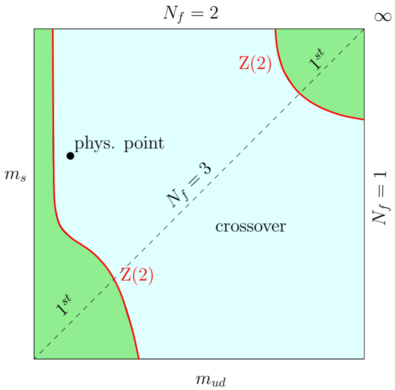

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Imaginary Chemical Potential and Roberge Weiss Symmetry

- No sign problem $\rightarrow$ Hybrid Monte Carlo applicable.
- QCD partition function symmetries (Roberge-Weiss symmetry)

$$Z(\mu) = Z(-\mu) \, , \qquad Z\left(\frac{\mu}{T}\right) = Z\left(\frac{\mu}{T} + i\frac{2\pi n}{3}\right) .$$

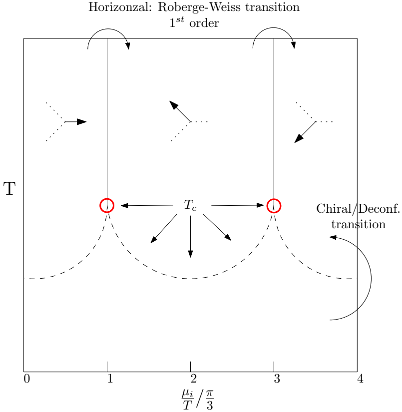
Roberge & Weiss, Nucl. Phys. B275 (1986)

- Adding $\mu_i/T = 2\pi n/3 \, , \; n \in \mathbb{Z}$ is equivalent to $Z(3)$ transformation.
- Describes completely equivalent physics.
- Centre symmetry is a good symmetry again.
- Centre sectors seperated by

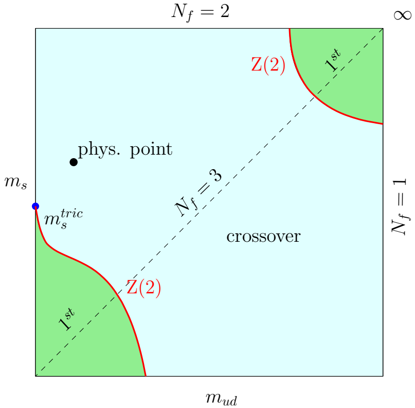
$$\mu_i^c = \frac{\pi T}{3}(2n+1) \, , \; n \in \mathbb{Z}.$$

Imaginary Chemical Potential and Roberge Weiss Symmetry

Roberge-Weiss phase diagram



Columbia plot at $\mu = 0$



- Polyakov loop on the lattice: $L(n) = \frac{1}{V}\prod_{\tau_0}^{N_\tau} U_0(\tau, n)$
- Phase of the Polyakov is sensitive to centre sector transitions

$$\mathrm{Tr}L^g = e^{-i\frac{2\pi n}{3}}\mathrm{Tr}L$$

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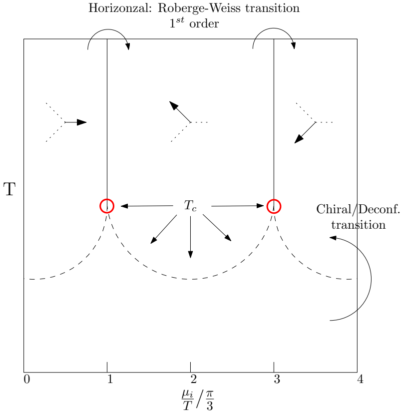
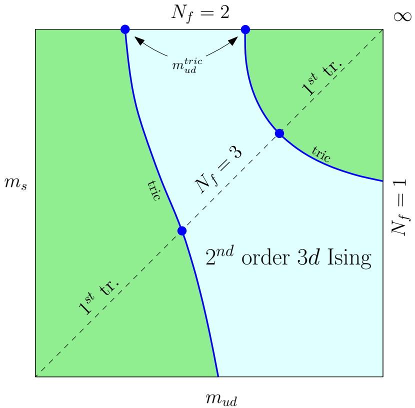
Roberge-Weiss phase diagram



Columbia plot at $\mu_c = i\pi T/3$



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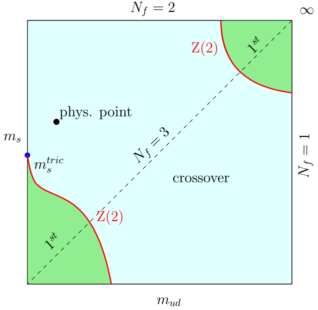
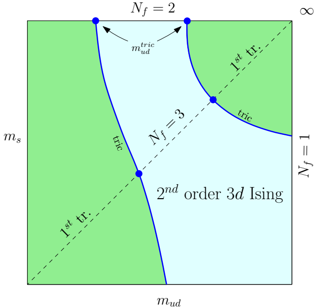
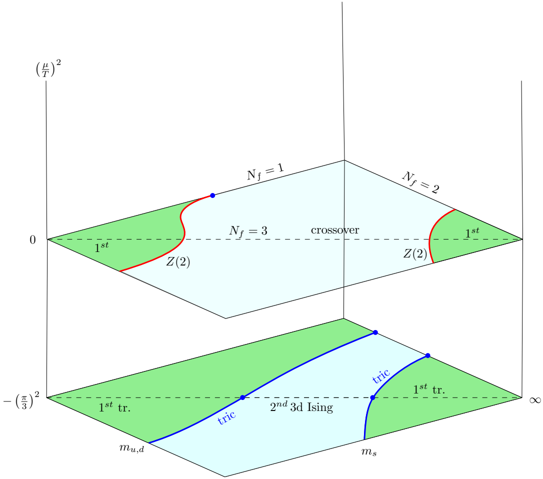
The QCD Phase Structure for Imaginary μ







P. de Forcrand and O. Philipsen, JHEP (2007)
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H. Saito et al., arXiv:1309.2445 (hep-lat)

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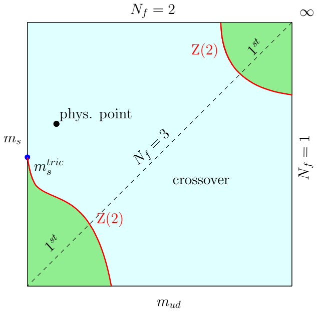

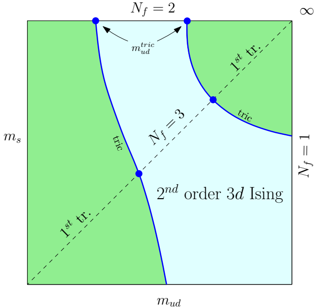

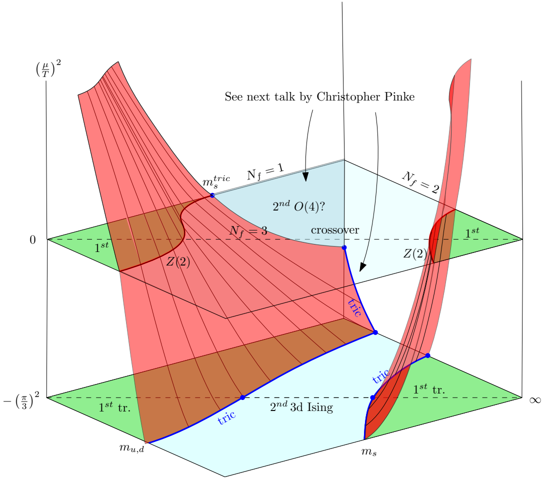

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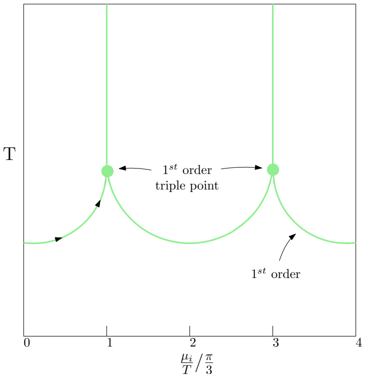
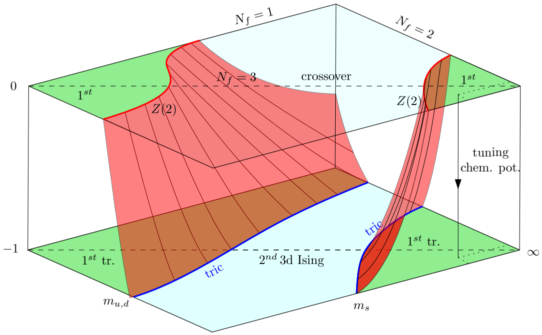
The QCD Phase Structure for Imaginary μ

1^{st} order $\longrightarrow$ 1^{st} order triple



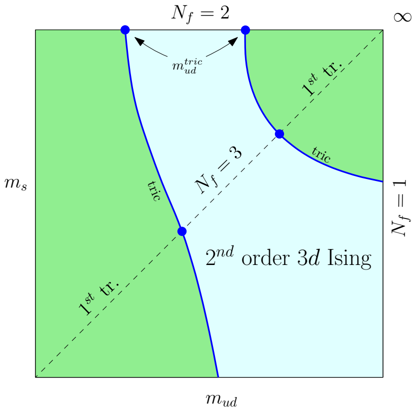


D'Elia, M. and Sanfilippo, F. Phys. Rev. D80 (2009)
C. Pinke, O. Philipsen, Phys.Rev. D89 (2014)

Previous and Ongoing Studies

Studies done on $N_\tau = 4$:

- Staggered, $N_f = 2,\ 2+1,\ 3$
- Wilson, $N_f = 2$



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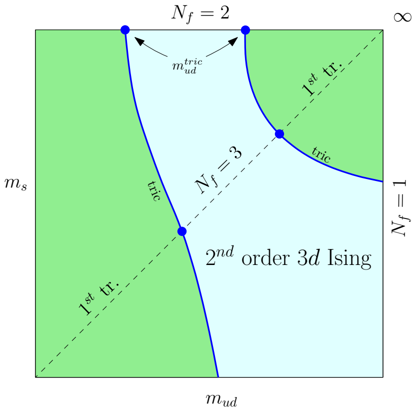
Ongoing for $N_\tau = 6$:

- Wilson, $N_f = 2$
- $\kappa \in (0.1000, 0.1650)$
- $\mathcal{O}(15)\beta$ for 3-4 $N_\sigma \in (16, 36)$
- $\mathcal{O}(80k - 200k)$ trajectories
- OpenCL based CL²QCD code

  Bach, Philipsen, Pinke (arxiv:1411.5219v2)(2014)
  https://github.com/CL2QCD/cl2qcd.git



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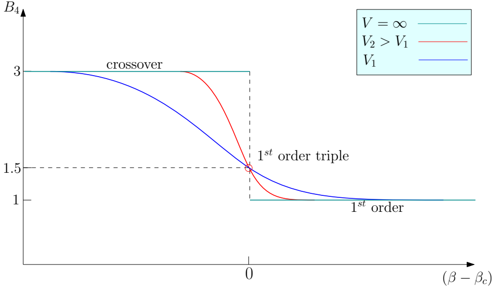
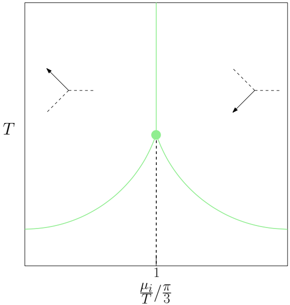
Extracting the Order of a Phase Transition

$$B_4(\beta) = \frac{\left\langle (L_{\mathrm{Im}} - \langle L_{\mathrm{Im}} \rangle)^4 \right\rangle}{\left\langle (L_{\mathrm{Im}} - \langle L_{\mathrm{Im}} \rangle)^2 \right\rangle^2} \qquad \lim_{V\to\infty} B_4(\beta_c) = \begin{cases} 1 & , 1^{st} \text{ order} \\ 1.5 & , 1^{st} \text{ order triple} \\ 1.604 & , 2^{nd} \text{ order Z(2)} \\ 3 & , \text{crossover} \end{cases}$$





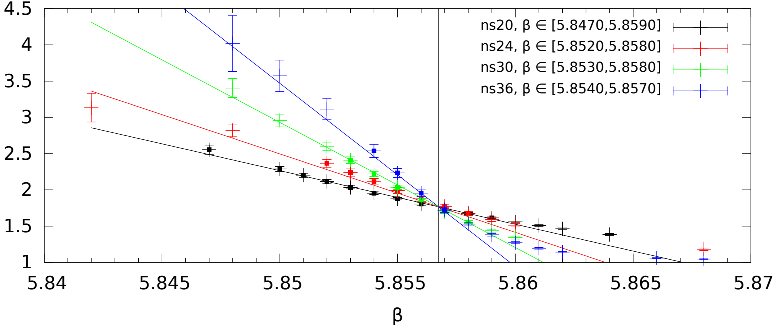
$$B_4(\beta, N_\sigma) = B_4(\beta_c, \infty) + a\,(\beta - \beta_c) N_\sigma^{1/\nu} + \ldots$$

Finite Size Scaling

- Smoothing/interpolating Data points with Ferrenberg-Swensen reweighting.
- Fitting of

$$B_4(\beta_c, N_\sigma) = B_4(\beta_c, \infty) + a(\beta - \beta_c) N_\sigma^{1/\nu}$$

to reweighted points.

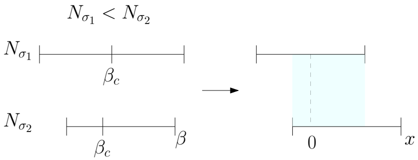


$\kappa = 0.11, \quad N_\tau = 6$

Finite Size Scaling

$$B_4(\beta_c, N_\sigma) = B_4(\beta_c, \infty) + a\,(\beta - \beta_c) N_\sigma^{1/\nu} + \dots$$
$$= B_4(\beta_c, \infty) + a \qquad x \qquad + \dots$$

- Fit criteria:
  - ✓ $Q \approx 50\%$
  - ✓ $\chi^2 \approx 1$
  - ✓ Overlap in $x \geq 80\%$
  - ✓ Symmetry of x around 0



Finite Size Scaling

$$
\begin{aligned}
B_4(\beta_c, N_\sigma) &= B_4(\beta_c, \infty) + a\,(\beta - \beta_c) N_\sigma^{1/\nu} + \ldots \\
&= B_4(\beta_c, \infty) + a \qquad x \qquad + \ldots
\end{aligned}
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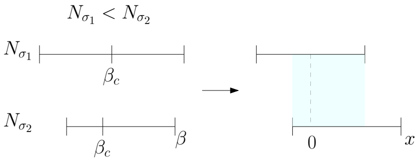
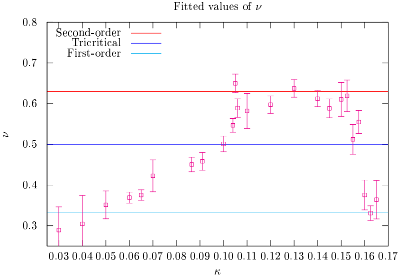


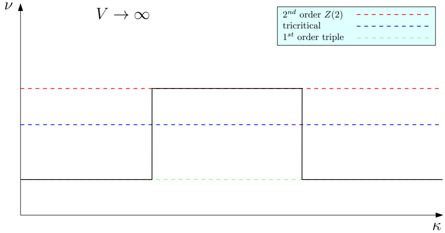
- ν less prone to finite size effects
  $\rightarrow$ better suited to extract order of phase transition
- Determine order of phase transition according to

$$
\nu = \begin{cases} 1/3 & 1^{st} \text{ order triple} \\ 1/2 & \text{tricritical} \\ 0.63 & 2^{nd} \text{ order } 3D \text{ Ising} \end{cases}
$$

Results

Results for studies with $N_f = 2$ flavors of Wilson fermions on $N_\tau = 4$ lattices





- κ as function of the bare quark mass: $\kappa = (2\,(am + 4))^{-1}$

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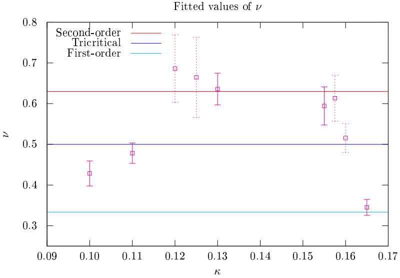

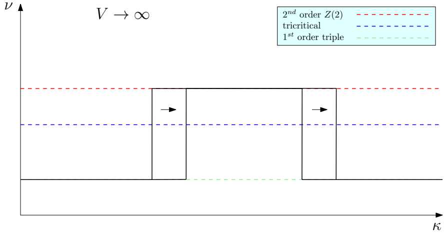

- κ as function of the bare quark mass: $\kappa = (2\,(am + 4))^{-1}$
- Shift in 1^{st} order region to smaller kappa: $T = 1/(aN_\tau)$
  $\Rightarrow$ Tricritical mass: $m_\pi^{tric} \approx 730\,\text{MeV} \to m_\pi^{tric} \approx 660\,\text{MeV}$

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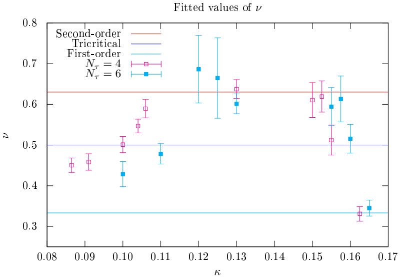

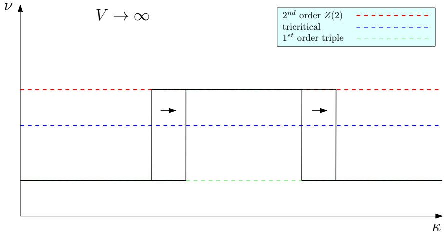

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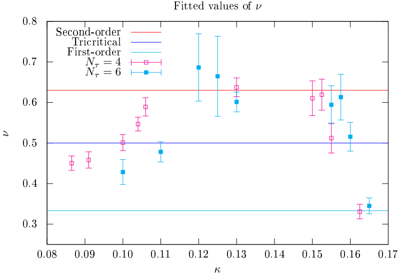

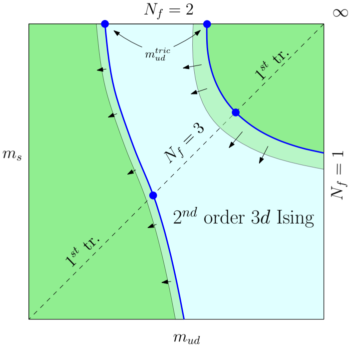

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Summary and Perspectives

Summary

- No sign problem for imaginary chemical potential.
- QCD phase diagram constrained by imaginary chemical potential region.
- $N_\tau = 6$ studies with Wilson fermions ongoing.
- Shift in $1^{st}/2^{nd}$ order region.

Perspectives

- Compare Wilson results to $N_\tau = 6$ staggered results (ongoing).
- Start studies for $N_\tau = 8$.
- Extend Wilson fermion studies to $N_f = 3$.