Taste symmetry restoration in the sextet model with staggered fermions

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Lattice Higgs Collaboration ($\mathrm{L_{at}HC}$)

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Lattice 2015

Plan of the talk

Zoltan Fodor[†], Kieran Holland[*], Julius Kuti[††], Santanu Mondal[**], Daniel Nogradi[**], Chik Him Wong[†]

- Introduction
- Pion channels in staggered fermion.
- Discussion of the taste breaking effects in our data of pion spectra in the light of LO and NLO SχPT and comparison to those in QCD.
- Almost restoration of taste symmetry in the pion spectra at our smallest lattice spacing for a range of quark mass.
- Conclusion.

Introduction

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- Simulations with staggered fermion have the advantages of being very fast and a remnant of the chiral symmetry for the massless fermions is retained at finite lattice spacing.
- These advantages may be offset by taste breaking effects which if not negligible can introduce significant lattice artifacts in the final outcome.
- In particular any quantity sensitive to the chiral loops can be expected to show large taste breaking artifacts. In the present model (*) precise determination of the Goldstone decay constant F in the chiral limit is very important for scale setting and thus it is important to have good idea about the taste breaking effects in our simulations.

* Brief overview : Kuti (14/7 Tuesday, 16:30 PM),

Hadron spectroscopy in extended data set: Wong (14/7 Tuesday, 16:50 PM),

Study of the β function : Nogradi (14/7 Tuesday, 17:30 PM),

New method to study Dirac's spectrum and it's application in the sextet gauge model: Holland (Wednesday, Poster session).

Full pion spectra gives an excellent laboratory to study taste breaking effects because:

- We have a theory for it.
- Easily detectable: pion taste breaking effects are larger than those in the other states (e.g. vector mesons or baryons).
- Pion taste breaking effects feed into all quantities.

Full pion spectra with staggered fermion

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- In our simulation we do not take into account the disconnected diagrams and hence consider only non-diagonal flavour pions ($\pi^{+/-}$).
- The four tastes of staggered fermion give 16 tastes (including the taste singlet) of pions $\longrightarrow$ can be identified with the spin-taste structure $\gamma_5 \otimes \xi_F$ where $\xi_F = \{I, \xi_5, \xi_\mu, \xi_{\mu 5}, \xi_{\mu\nu} = \frac{1}{2}[\xi_\mu, \xi_\nu]\}$.
  $\Longrightarrow 1+1+4+4+6 = 16$
- At zero spatial momenta these states fall into 8 irreps of the lattice time slice group:
  $\{I, \xi_5, \xi_i, \xi_4, \xi_{i5}, \xi_{45}, \xi_{ij}, \xi_{i4}\}$

Properties of the pion spectra can be studied by using staggered chiral perturbation theory (SχPT) [1,2].
It is shown that [1], taste symmetry breaking happens in two steps:

1. At leading order in the joint expansion in $p^2 \sim a^2$ and m pion spectrum respects an SO(4) subgroup of full SU(4) taste symmetry of massive staggered fermion.

$$SU(4) \longrightarrow SO(4) \text{ at } O(a^2) \Longrightarrow \xi_F \in \{I, \xi_5, \xi_\mu, \xi_{\mu 5}, \xi_{\mu\nu}\}$$

2. At $O(a^2 p^2)$ SO(4) breaks down to discrete spin taste symmetry $\Longrightarrow \xi_F \in \{I, \xi_5, \xi_i, \xi_4, \xi_{i5}, \xi_{45}, \xi_{ij}, \xi_{i4}\}$.

1. W. J. Lee and S. R. Sharpe, Phys. Rev. D **60**, 114503 (1999) [hep-lat/9905023].

2. C. Aubin and C. Bernard, Phys. Rev. D **68**, 034014 (2003) [hep-lat/0304014].

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Operators

R. Altmeyer *et al.* [MT(c) Collaboration], Nucl. Phys. B **389**, 445 (1993).
channels: 1,2,7,8,13,14,17,18

representing $\{I, \xi_5, \xi_i, \xi_4, \xi_{i5}, \xi_{45}, \xi_{ij}, \xi_{i4}\}$

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TABLE 1

Irreducible representations and their operators for mesonic states which are local in time. $i \neq j \neq k \neq i$. The symmetric operator Δ is given in eq. (3.1), while η, ζ and ϵ have their usual definitions: $\eta_\mu(x) = (-1)^{x_1 + \cdots + x_{\mu-1}}$, see eq. (2.5), $\zeta_\mu(x) = (-1)^{x_{\mu+1} + \cdots + x_4}$, $\epsilon(x) = (-1)^{x_1 + \cdots + x_4}$. The sum over x is omitted. The meaning of the columns is described in the text in subsect. 3.1

No.	Operator	$r^{\sigma_s \sigma_{123}}$	τ_0	ξ_4	$\Gamma^D \otimes \Gamma^F$	State	Part.
1	$\bar{\chi}\chi$	1^{++}	+	+	$1 \otimes 1$	0_S^{++}	f_0
				−	$\gamma_4\gamma_5 \otimes \gamma_4\gamma_5$	0_A^{-+}	π
2	$\eta_4\zeta_4\bar{\chi}\chi$	1^{+-}	+	+	$\gamma_4 \otimes \gamma_4$	0_A^{+-}	–
				−	$\gamma_5 \otimes \gamma_5$	0_A^{-+}	π
3	$\eta_i\epsilon\zeta_i\bar{\chi}\chi$	$3''''^{+-}$	+	+	$\gamma_i\gamma_5 \otimes \gamma_i\gamma_5$	1_A^{++}	a_1
				−	$\gamma_i\gamma_4 \otimes \gamma_i\gamma_4$	1_A^{--}	ρ
4	$\eta_4\zeta_4\eta_i\epsilon\zeta_i\bar{\chi}\chi$	$3''''^{++}$	+	+	$\gamma_j\gamma_k \otimes \gamma_j\gamma_k$	1_A^{+-}	b_1
				−	$\gamma_i \otimes \gamma_i$	1_A^{--}	ρ
5	$\bar{\chi}\eta_i\, \Delta_i\chi$	3^{-+}	−	+	$\gamma_i \otimes 1$	1_S^{--}	ω
				−	$\gamma_j\gamma_k \otimes \gamma_4\gamma_5$	1_A^{+-}	b_1
6	$\eta_4\zeta_4\bar{\chi}\eta_i\, \Delta_i\chi$	3^{--}	+	+	$\gamma_i\gamma_4 \otimes \gamma_4$	1_A^{--}	ρ
				−	$\gamma_i\gamma_5 \otimes \gamma_5$	1_A^{++}	a_1
7	$\bar{\chi}\epsilon\zeta_i\, \Delta_i\chi$	$3''^{--}$	+	+	$\gamma_5 \otimes \gamma_i\gamma_5$	0_A^{-+}	π
				−	$\gamma_4 \otimes \gamma_i\gamma_4$	0_A^{+-}	–
8	$\eta_4\zeta_4\bar{\chi}\epsilon\zeta_i\, \Delta_i\chi$	$3''^{-+}$	−	+	$\gamma_4\gamma_5 \otimes \gamma_j\gamma_k$	0_A^{-+}	π
				−	$1 \otimes \gamma_i$	0_A^{++}	a_0
9	$\eta_i\epsilon\zeta_i\bar{\chi}\eta_j\, \Delta_j\chi$	6^{--}	−	+	$\gamma_k\gamma_4 \otimes \gamma_i\gamma_5$	1_A^{--}	ρ
				−	$\gamma_k\gamma_5 \otimes \gamma_i\gamma_4$	1_A^{++}	a_1
10	$\eta_4\zeta_4\eta_i\epsilon\zeta_i\bar{\chi}\eta_j\, \Delta_j\chi$	6^{-+}	+	+	$\gamma_k \otimes \gamma_j\gamma_k$	1_A^{--}	ρ
				−	$\gamma_i\gamma_j \otimes \gamma_i$	1_A^{+-}	b_1
11	$\epsilon_{ijk}\bar{\chi}\eta_i\, \Delta_i(\eta_j\, \Delta_j\chi)$	3^{++}	−	+	$\gamma_i\gamma_j \otimes 1$	1_S^{+-}	h_1
				−	$\gamma_k \otimes \gamma_4\gamma_5$	1_A^{--}	ρ
12	$\epsilon_{ijk}\eta_4\zeta_4\bar{\chi}\eta_i\, \Delta_i(\eta_j\, \Delta_j\chi)$	3^{+-}	−	+	$\gamma_k\gamma_5 \otimes \gamma_4$	1_A^{++}	a_1
				−	$\gamma_k\gamma_4 \otimes \gamma_5$	1_A^{--}	ρ
13	$\epsilon_{ijk}\bar{\chi}\zeta_i\, \Delta_i(\zeta_j\, \Delta_j\chi)$	$3''^{++}$	−	+	$1 \otimes \gamma_i\gamma_j$	0_A^{++}	a_0
				−	$\gamma_4\gamma_5 \otimes \gamma_k$	0_A^{-+}	π
14	$\epsilon_{ijk}\eta_4\zeta_4\bar{\chi}\zeta_i\, \Delta_i(\zeta_j\, \Delta_j\chi)$	$3''^{+-}$	−	+	$\gamma_4 \otimes \gamma_k\gamma_5$	0_A^{+-}	–
				−	$\gamma_5 \otimes \gamma_k\gamma_4$	0_A^{-+}	π
15	$\eta_k\zeta_k\bar{\chi}\eta_i\, \Delta_i(\zeta_j\, \Delta_j\chi)$	6^{++}	+	+	$\gamma_i\gamma_k \otimes \gamma_j\gamma_k$	1_A^{+-}	b_1
				−	$\gamma_j \otimes \gamma_i$	1_A^{--}	ρ
16	$\eta_4\zeta_4\eta_k\zeta_k\bar{\chi}\eta_i\, \Delta_i(\zeta_j\, \Delta_j\chi)$	6^{+-}	+	+	$\gamma_j\gamma_5 \otimes \gamma_i\gamma_5$	1_A^{++}	a_1
				−	$\gamma_j\gamma_4 \otimes \gamma_i\gamma_4$	1_A^{--}	ρ
17	$\bar{\chi}\eta_1\, \Delta_1(\eta_2\, \Delta_2(\eta_3\, \Delta_3\chi))$	1^{-+}	+	+	$\gamma_4\gamma_5 \otimes 1$	0_S^{-+}	η'
				−	$1 \otimes \gamma_4\gamma_5$	0_A^{++}	a_0
18	$\eta_4\zeta_4\bar{\chi}\eta_1\, \Delta_1(\eta_2\, \Delta_2(\eta_3\, \Delta_3\chi))$	1^{--}	−	+	$\gamma_5 \otimes \gamma_4$	0_A^{-+}	π
				−	$\gamma_4 \otimes \gamma_5$	0_A^{+-}	–
19	$\eta_i\epsilon\zeta_i\bar{\chi}\eta_1\, \Delta_1(\eta_2\, \Delta_2(\eta_3\, \Delta_3\chi))$	$3''''^{--}$	−	+	$\gamma_i\gamma_4 \otimes \gamma_i\gamma_5$	1_A^{--}	ρ
				−	$\gamma_i\gamma_5 \otimes \gamma_i\gamma_4$	1_A^{++}	a_1
20	$\eta_4\zeta_4\eta_i\epsilon\zeta_i\bar{\chi}\eta_1\, \Delta_1(\eta_2\, \Delta_2(\eta_3\, \Delta_3\chi))$	$3''''^{-+}$	+	+	$\gamma_i \otimes \gamma_j\gamma_k$	1_A^{--}	ρ
				−	$\gamma_j\gamma_k \otimes \gamma_i$	1_A^{+-}	b_1

Taste breaking in pion spectra at the tree level of SχPT

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At tree level the general expression of masses of non-diagonal flavour pions (π^+/π^-) is given by,

$$M^2_{\pi_b^{+/-}} = 2B(m_u + m_d) + a^2\Delta(\xi_b) \text{ taste label b } \in \{5, \mu 5, \mu\nu, \mu, I\}.$$

The mass splits are:

$$\begin{aligned}
\Delta(\xi_5) &\equiv \Delta_P = 0 \longrightarrow \; \textit{axial } U(1) \textit{ symmetry} \\
\Delta(\xi_{\mu 5}) &\equiv \Delta_A = \frac{16}{f^2}(C_1 + 3C_3 + C_4 + 3C_6) \\
\Delta(\xi_{\mu\nu}) &\equiv \Delta_T = \frac{16}{f^2}(2C_3 + 2C_4 + 4C_6) \\
\Delta(\xi_{\mu}) &\equiv \Delta_V = \frac{16}{f^2}(C_1 + C_3 + 3C_4 + 3C_6) \\
\Delta(\xi_5) &\equiv \Delta_I = \frac{16}{f^2}(4C_3 + 4C_4)
\end{aligned}$$

Full pion spectra

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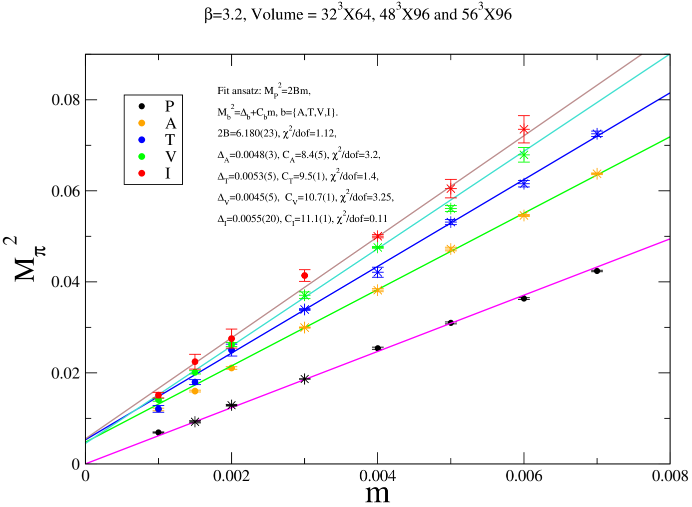

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- The Δ s are of comparable size for non-Goldstone pions.
- Equating all four Δ s of non-Goldstone pions with a constant (Δ) we get

$$C_1 = C_3 = C_4 = C_6 = \frac{\Delta f^2}{128}$$

$\Longrightarrow$ Preliminary indication of all four coefficients having comparable values.

Very different from QCD which gives approximately equal splittings of mass squares of the non-Goldstone pions in the order of P, A, T, V, I.

$\Longrightarrow C_4$ is the dominant coefficient.

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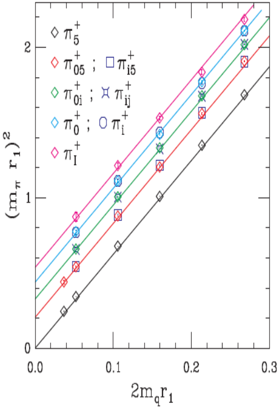

T. Bae, D. H. Adams, C. Jung, H. J. Kim, 2, J. Kim, K. Kim, W. Lee and S. R. Sharpe, Phys. Rev. D **77**, 094508 (2008).

Taste breaking at NLO SχPT

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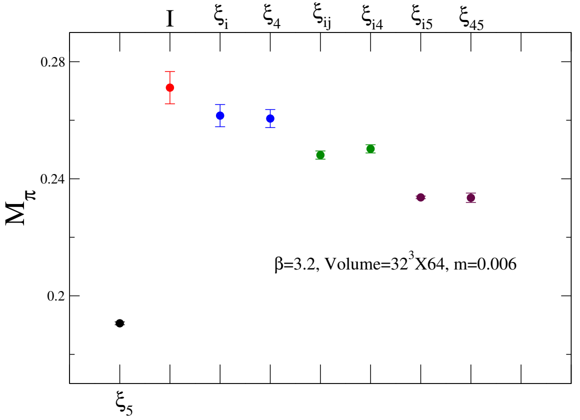
The SχPT prediction for taste breaking of Goldstone and non-Goldstone pion masses at NLO can be expressed as,

$$M^2_{NLO} = M^2_{LO}(1+\delta_{b'}) \text{ taste label b}' \in \{5, i5, 45, ij, i4, i, 4, I\},$$
$$\text{and } \delta_{b'} \sim \mathscr{O}(\mathrm{a}^2).$$

S. R. Sharpe and R. S. Van de Water, Phys. Rev. D **71**, 114505 (2005) [hep-lat/0409018].

Zoltan Fodor[†], Kieran Holland[*], Julius Kuti[††], Santanu Mondal[**], Daniel Nogradi[**], Chik Him Wong[†]

Data show three almost degenerate pairs, {i5,45}, {ij, i4}, {i,4}



Taste symmetry restoration in the sextet model with staggered fermions

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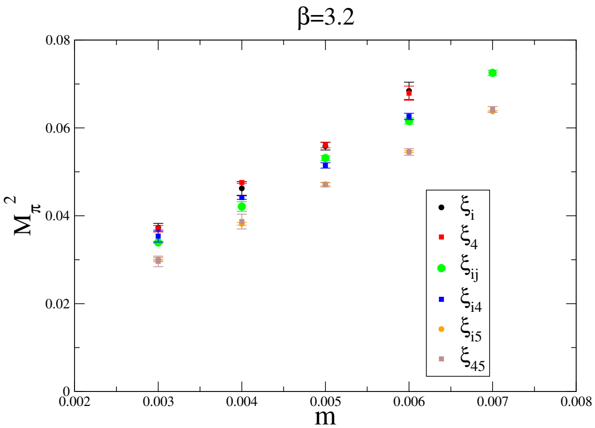
Data show three almost degenerate pairs, {i5,45}, {ij, i4}, {i,4}



Non-parallel slopes: fan out structure

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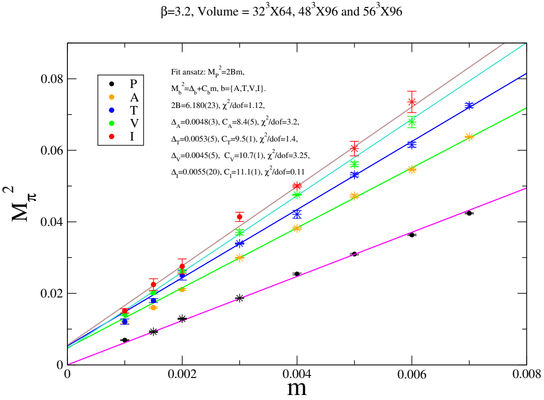
Very different from QCD where non-Goldstone pions have similar slopes slightly different from the Goldstone pion.



Decreasing the lattice spacing

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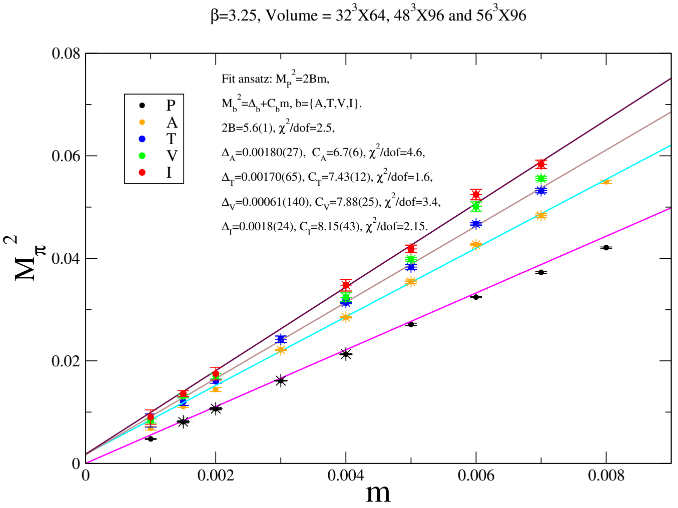
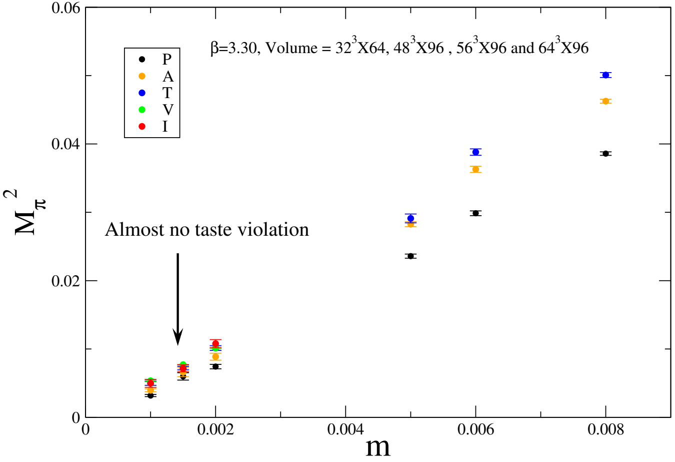


Decreasing further



Summary

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- Sextet model gives different kind of taste breaking pattern in the full pion spectra than QCD.
- Preliminary indication from our data is that the coefficients of some taste breaking terms in LO SχPT Lagrangian are of comparable values.
- At our smallest lattice spacing taste symmetry in pion spectra is almost restored for a range of quark mass within the precision of the data.