Lattice QCD studies of baryon interactions from HAL QCD method and Lüscher's finite volume method

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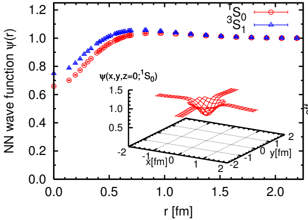
2 Lattice QCD Approaches for Hadron Interactions

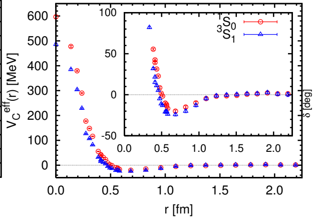
- Lüscher's finite volume method — Lüscher '86, '91
  energy shift of two-particle system in "finite box" ⟹ phase shift

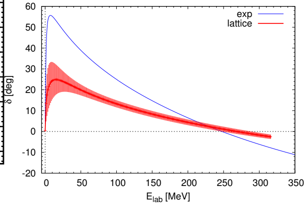
$$\tan\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2-(kL/2\pi)^2}$$

  with $\Delta E = 2\sqrt{k^2+m^2}-2m$

- HAL QCD method — Ishii-Aoki-Hatsuda '07
  NBS wave function ⟹ potential ⟹ phase shift



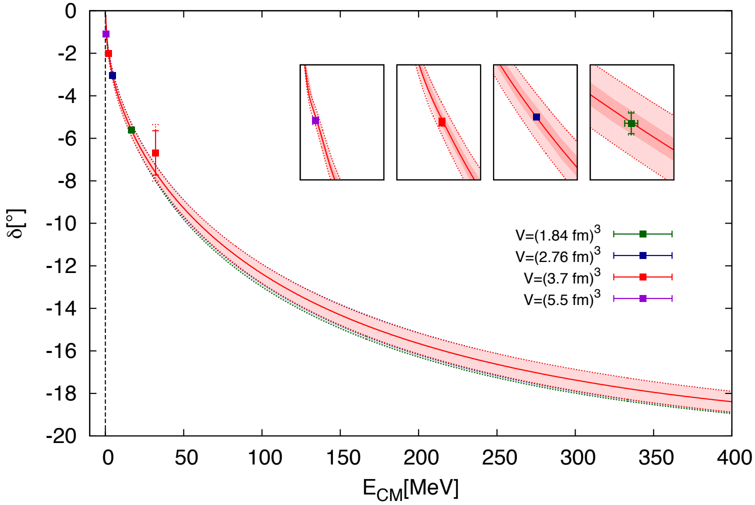




Consistency of Two Methods

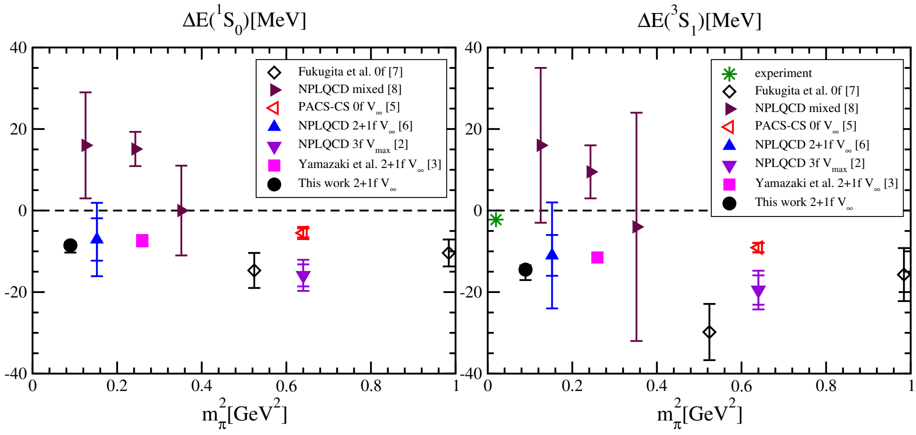
■ $I = 2\ \pi\pi$ scattering — Kurth-Ishii-Doi-Aoki-Hatsuda '13

$\Rightarrow$ good agreement



NN Interactions from Lüscher's Method

Both $^1S_0$ and $^3S_1$ channels are bound or unbound states ?

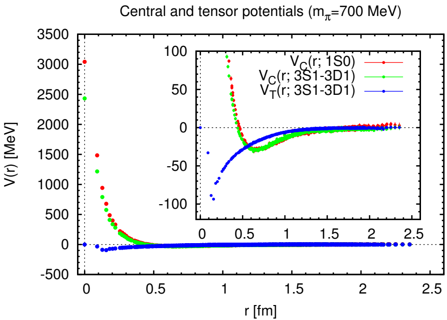


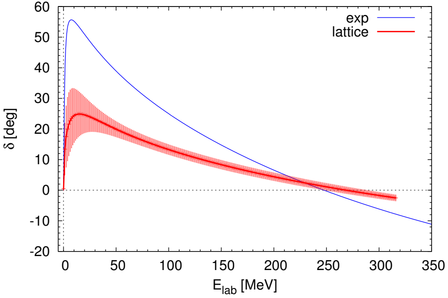
Figs. Yamazaki-Ishikawa-Kuramashi-Ukawa '15 [arXiv:1502.04182]

NN Interactions from HAL QCD Method

□ HAL QCD Coll. — NN potential and NN($^1S_0$) phase shift

⇒ scattering states





○ qualitatively consistent with the experimental phase shift

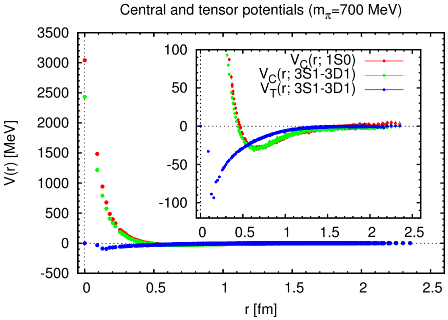
Aim of This Work

For deeper understanding of hadron interactions from lattice QCD, i.e.,

HAL QCD method and Lüscher's finite volume method

$\Rightarrow$ we investigate baryon interactions
from both methods using the same lattice setup

■ analyze baryon potential



■ measure energy shift

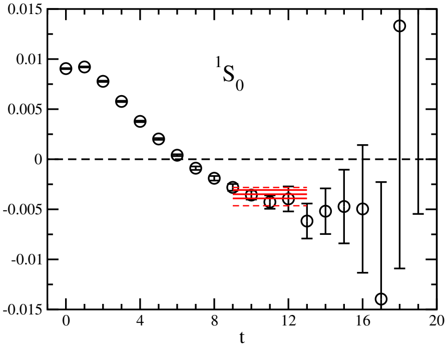


Fig. Yamazaki et al. '15

Time-dependent HAL QCD Method

- normalized 4pt correlator

$$R(\vec{r}, t - t_0) \equiv e^{2mt} \left\langle 0 | T\{B(\vec{x} + \vec{r}, t) B(\vec{x}, t)\} \bar{\mathcal{J}}(t_0) | 0 \right\rangle$$
$$= \sum_n A_n \phi^{W_n}(\vec{r}) e^{-\Delta W_n (t - t_0)}$$

- R-correlator satisfies "time-dependent" Schrödinger-like equation

$$\left[\frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\vec{r}, t) = \int d\vec{r'} U(\vec{r}, \vec{r'}) R(\vec{r'}, t)$$

- using velocity expansion, "potential" is given by

$$V^{\mathrm{LO}}(\vec{r}) = \frac{1}{4m} \frac{(\partial/\partial t)^2 R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{(\partial/\partial t) R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{H_0 R(\vec{r}, t)}{R(\vec{r}, t)}$$

- This method does not require the ground state saturation.

Lüscher's Finite Volume Method

- $\Delta E = 2\sqrt{k^2 + m_N^2} - 2m_N$

$$\tan\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2 - (kL/2\pi)^2}$$

- we measure "energy shift" $\Delta E = E_{NN} - 2m_N$ at finite volume

• NN(^{1}S$_0$) energy shift

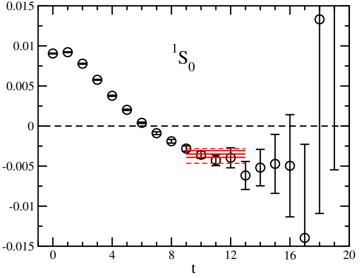


Fig. Yamazaki et al. '15

$\Rightarrow$ use effective mass plot

$$\Delta E(t) = \ln\frac{R(t)}{R(t+1)}$$

with

$$R(t) = \frac{G_{NN}(t)}{(G_N(t))^2}$$

• standard method in "single particle state"

Lattice Setup

- 2+1 flavor improved Wilson quark + Iwasaki gauge configuration†
  lattice spacing $a = 0.08995(40)$ fm, $a^{-1} = 2.194(10)$ GeV
- 2-type quark sources
  - (exponential) smeared source — the same as Yamazaki *et al.*
  - wall source — commonly used in HAL QCD method

☐ Analyze $\Xi\Xi$-channel potential and energy shift

volume	smeared src.	wall src.
$40^3 \times 48$	200 conf. × 192 meas.	200 conf. × 48 meas.
$48^3 \times 48$	800 conf. × 256 meas.	800 conf. × 48 meas.
$64^3 \times 64$	327 conf. × 48 meas.	327 conf. × 128 meas.

Table: Lattice configurations (we mainly use $48^3 \times 48$ volume)

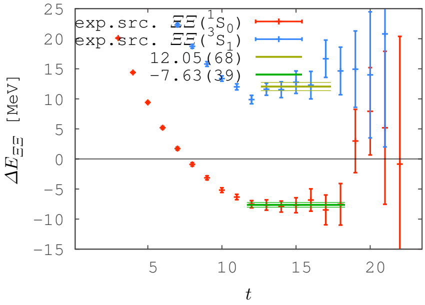
$m_\pi = 0.51$ GeV, $m_N = 1.32$ GeV, $m_K = 0.62$ GeV, $m_\Xi = 1.46$ GeV
† Yamazaki, Ishikawa, Kuramashi, Ukawa, arXiv:1207.4277, 1502.04182.

$\Xi\Xi$ Energy Shift in 48^3 Lattice Volume

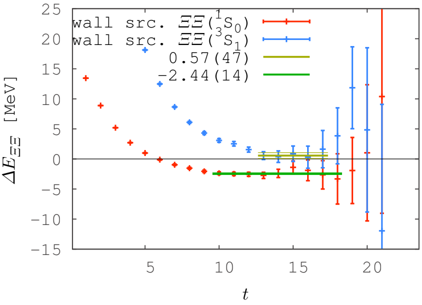
- $\Xi\Xi(^1S_0)$ (the same rep. as $NN(^1S_0)$) $\Longrightarrow$ negative energy shift
- $\Xi\Xi(^3S_1)$ $\Longrightarrow$ positive energy shift

$\Longrightarrow$ but $\Delta E(t)$ depends on quark sources ?

□ smeared source

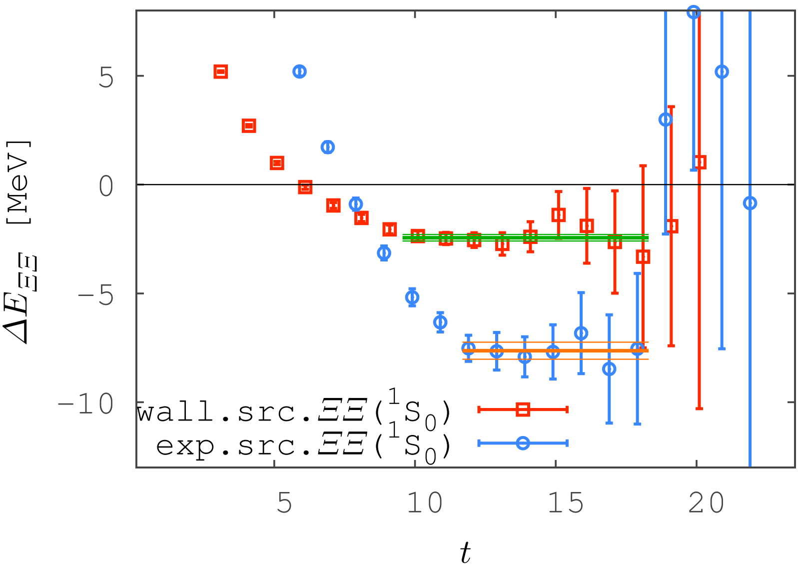


■ wall source



Effective Mass of ΞΞ states

$$\Delta E = M_{\Xi\Xi} - 2m_{\Xi}$$



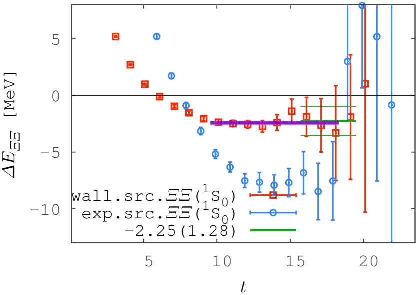
Effective Mass of ΞΞ states

- Be careful with effective mass plot

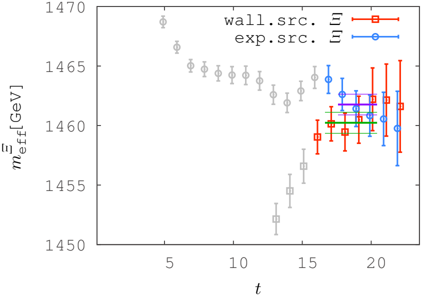
$\Rightarrow$ fictious "plateau" appears

- ∘ at least $t \gtrsim 16$
- ∘ smeared src.
  - — need more statistics?

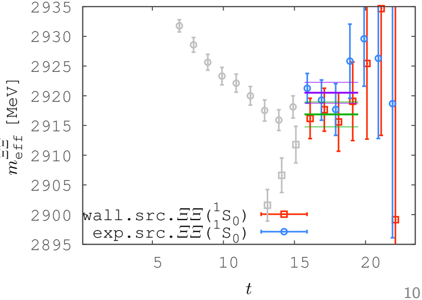
$$\Delta E = M_{\Xi\Xi} - 2m_{\Xi}$$



Ξ effective mass



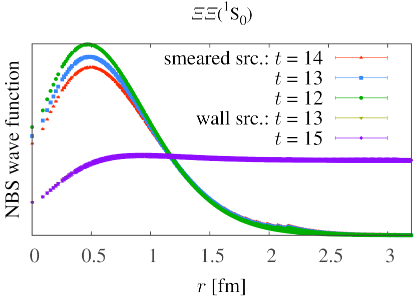
$\Xi\Xi(^1S_0)$ effective mass



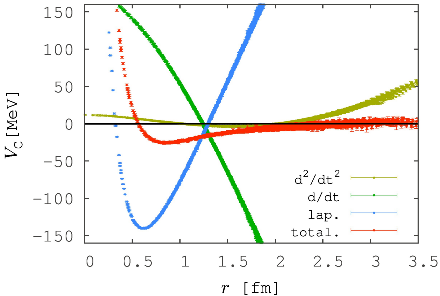
NBS Wave Function and $\Xi\Xi(^1S_0)$ Potential

$$V(\vec{r}) = \frac{(\partial/\partial t)^2 R(\vec{r},t)}{4mR(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

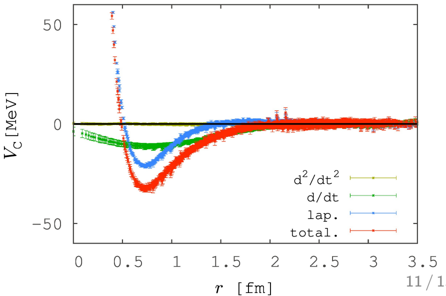
- ○ smeared. src. — strong t-dep.
  - $\Rightarrow$ $\sim \mathcal{O}(100)$ MeV cancellation
- • t-dep. HAL method works well!!
- ○ wall src. — weak t-dep.



□ smeared src.



■ wall src.



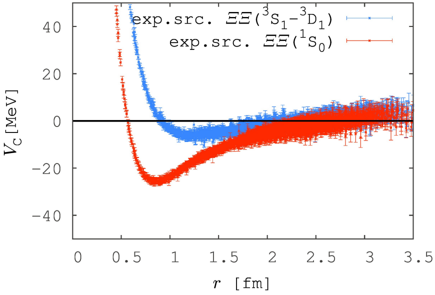
ΞΞ Potential from HAL QCD method

- ΞΞ(^{1}S$_0$) — attractive pocket + repulsive core, cf. NN(^{1}S$_0$)
- ΞΞ(^{3}S$_1$-^{3}D$_1$) — weak attractive

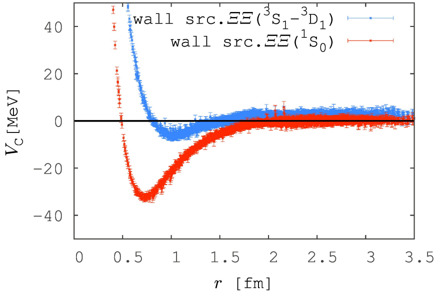
potentials are qualitatively consistent with each other

- difference — NLO term ? $U(\vec{r}, \vec{r'}) = [V_0(\vec{r}) + V_1(\vec{r})\nabla^2]\delta(\vec{r} - \vec{r'})$

□ smeared source

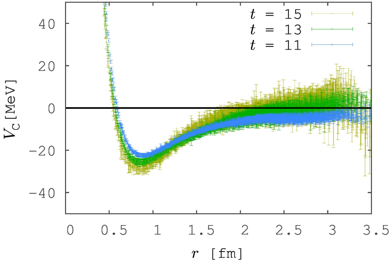


■ wall source

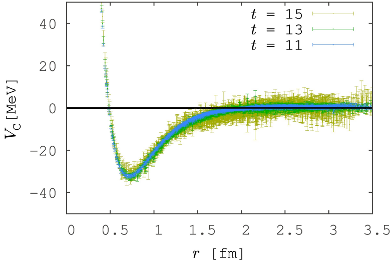


Convergence of ΞΞ potential

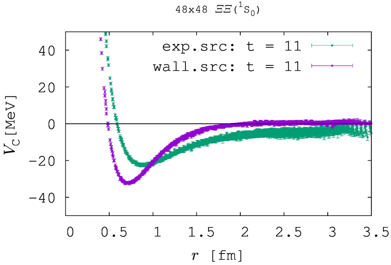
□ smeared src. shows t-dep.

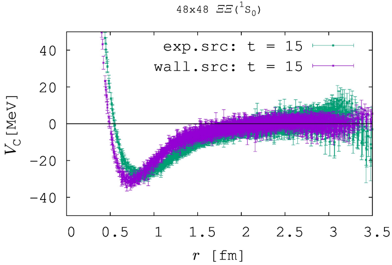


■ wall src. "stable"



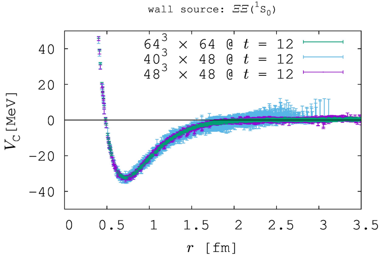
By increasing t, "smeared src." results seem to converge "wall src."





Finite Volume Energy from Potential: Lüscher from HAL

"wall source" at $t = 12$



- Now, we have the potential $V_c^{\Xi\Xi}(\vec{r})$

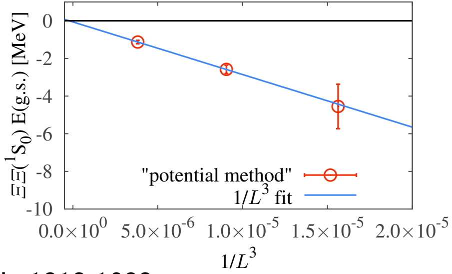
$\Rightarrow$ solve "finite volume" eigen energies[†]

use 40^3, 48^3 and 64^3 wall src. results

- consistent with "wall src." $\Delta E(t)$ fit $-2.25(1.28)$ MeV @ $48^3 \times 48$
- vol. dep. implies scattering states

energy eigenvalues

vol.	E(g.s.) [MeV]	E(1st) [MeV]
40^3	$-4.55(1.18)$	$75.63(1.31)$
48^3	$-2.58(22)$	$52.87(33)$
64^3	$-1.13(9)$	$28.71(9)$



† ref. Charron for HAL QCD Coll. arXiv:1312.1032.

Summary

We have investigated baryon interactions from
both HAL QCD method and Lüscher's finite volume method.

we have shown

- we should be careful with "plateau" in energy shift $\Delta E(t)$.
- time-dependent HAL QCD method works well without ground state saturation.
- we obtain the energy shift ΔE in finite volume for $\Xi\Xi(^1\mathrm{S}_0)$-channel, which is consistent with the potential by HAL QCD method.

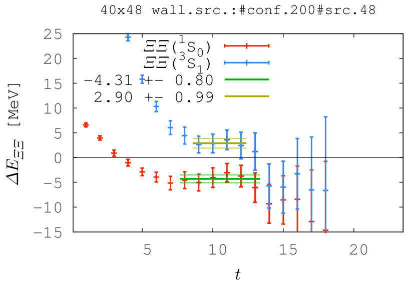
we have to check

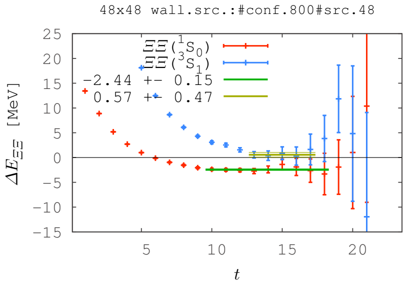
- convergence of smared src. to wall src. results
- contributions of inelastic/elastic excited states in NBS wave function and $\Delta E(t)$ plot
- NLO of velocity expansion in HAL QCD method for smeared source
- ...

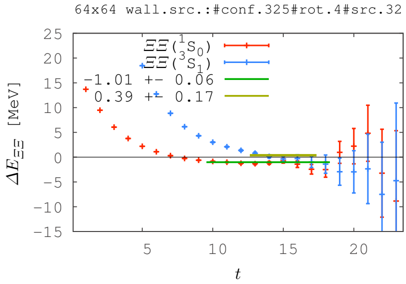
Appendix

Volume Dependence of Energy Shift from Wall Source

- ∘ approximate the energy shift by "plateau" value



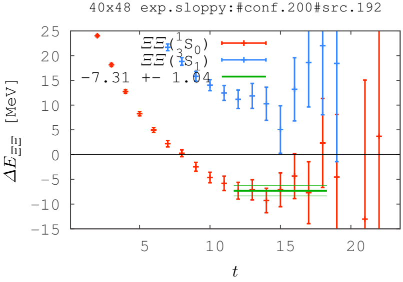


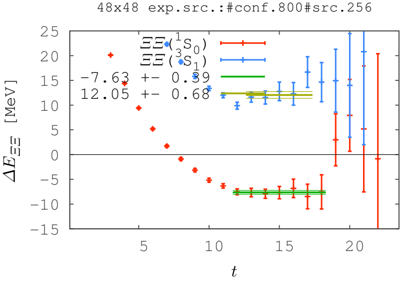


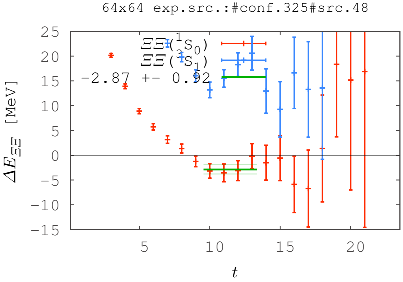
wall. src.

vol.	$\Delta E_{\Xi\Xi}(^1S_0)$
40^3	$-4.31(80)$ MeV
48^3	$-2.45(15)$ MeV
64^3	$-1.01(6)$ MeV

Volume Dep. of Energy Shift from Smeared Src.





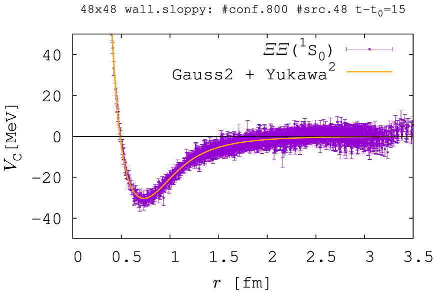


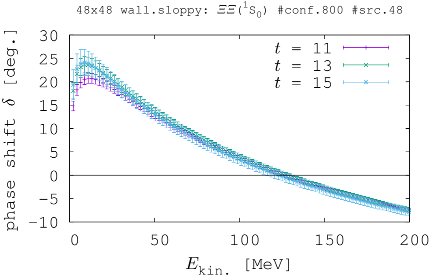
exp. src. (PRELIMINARY)

vol.	$\Delta E_{\Xi\Xi}(^1S_0)$
40^3	$-7.31(1.04)$ MeV
48^3	$-7.63(39)$ MeV
64^3	$-2.83(92)$ MeV

Phase Shift Analysis

wall source results at $48^{\times}48$





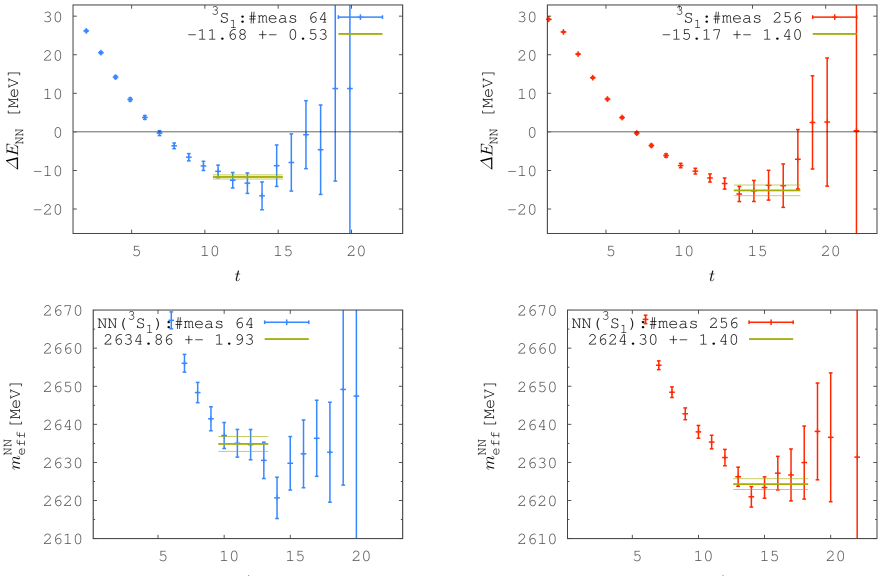
use (2 Gauss + [Yukawa]^2)-type fit

$$V_c(r) = a_0 \exp(-a_1 r^2) + a_2 \exp(-a_2 r^2) + a_4 (1 - \exp(a_5 r^2))^2 \left(\frac{\exp(-a_6 r)}{r} \right)^2$$

NN Energy Shift and Statistics

it is difficult to identify "plateau" in effective mass plot

800 conf. × 64 smeared src. 800 conf. × 256 smeared src.



NN Energy Shift from Wall Source