Monte Carlo studies of dynamical compactification of extra dimensions in a model of nonperturbative string theory

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1. Introduction

Difficulties in putting complex partition functions on computers.

$$Z = \int dA \exp(-S_0 + i\Gamma), \quad Z_0 = \int dA e^{-S_0}$$

e.g. lattice QCD,
matrix models for string theory

1. Sign problem:
   The reweighting $\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. $\exp[O(N^2)]$

   $\langle * \rangle_0$ = (V.E.V. for phase-quenched Z_0)

2. Overlap problem:
   Discrepancy of important configs. between Z_0 and Z.

2. Factorization method

Method to sample important configs. for Z.
[J. Nishimura and K.N. Anagnostopoulos, hep-th/0108041
K.N. Anagnostopoulos, T.A. and J. Nishimura, arXiv:1009.4504]

Constrain the observables

$$\Sigma = \{ \mathcal{O}_k | k = 1, 2, \cdots, n \}$$

correlated with the phase Γ.

Normalization $\tilde{\mathcal{O}}_k = \mathcal{O}_k / \langle \mathcal{O}_k \rangle_0$

Factorization of the distribution function ρ.

$$\rho(x_1, \cdots, x_n) \stackrel{\text{def}}{=} \left\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathcal{O}}_k) \right\rangle$$

$$= \underbrace{\frac{1}{\langle e^{i\Gamma} \rangle_0}}_{=1/C} \times \underbrace{\left\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathcal{O}}_k) \right\rangle_0}_{\stackrel{\text{def}}{=} \rho^{(0)}(x_1, \cdots, x_n)} \times \underbrace{\frac{\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathcal{O}}_k) e^{i\Gamma} \rangle_0}{\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathcal{O}}_k) \rangle_0}}_{\stackrel{\text{def}}{=} w(x_1, \cdots, x_n)}$$

$$w(x_1, \cdots, x_n) = \langle e^{i\Gamma} \rangle_x$$

$$\left(\langle * \rangle_x = \left\{ \text{V.E.V. for } Z_x = \int dA e^{-S_0} \prod_{k=1}^{n} \delta(x_k - \tilde{\mathcal{O}}_k) \right\} \right)$$

Simulation of Z_x with a proper choice of the set Σ ⇒sample the important region for Z.

Evaluation of the observables $\langle \tilde{\mathcal{O}}_k \rangle$
Peak of ρ at V=(system size)→∞.
= Minimum of the free energy $\mathcal{F} = -\frac{1}{N^2} \log \rho$
⇒Solve the saddle-point equation

$$\frac{1}{N^2} \frac{\partial}{\partial x_n} \log \rho^{(0)} = -\frac{\partial}{\partial x_n} \frac{1}{N^2} \log w$$

Applicable to general systems with sign problem.

3. The IKKT model

Promising candidate for nonperturbative string theory [N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{-\frac{N}{4} \text{tr}[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2} \text{tr} \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta]}_{=S_F}$$

- *Euclidean* case after the Wick rotation $A_0 \to iA_{10}$, $\Gamma^0 \to -i\Gamma_{10}$.
- A_μ, Ψ_α ⇒N×N Hemitian matrices (μ=1,2,…,d=10, α,β=1,2,…,16)
- Eigenvalues of A_μ ⇒spacetime coordinate
- Spontaneous Symmetry Breaking (SSB) of SO(10) ⇒dynamical emergence of spacetime.

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Result of Gaussian Expansion Method (GEM)

Order parameter of the SSB of SO(10).

$$\lambda_n (\lambda_1 \geq \cdots \geq \lambda_{10}) : \text{eigenvalues of } T_{\mu\nu} = \frac{1}{N} \text{tr}(A_\mu A_\nu)$$

Extended d-dim. and shrunken (10-d) dim.
at N→∞ ⇒ SSB SO(10)→SO(d)

Main Results of GEM

[J. Nishimura, T. Okubo and F. Sugino, arXiv:1108.1293]

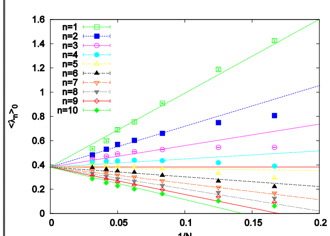
- Universal compactification scale $r^2 \cong 0.15$ for SO(d) ansatz (d=2,3,…7).
- Constant volume property except d=2 $V = R^d \times r^{10-d} = l^{10}$, $l^2 \cong 0.38$
- SSB SO(10)→SO(3).

Mechanism of SSB in Euclidean IKKT model

Partition function of the model:

$$Z = \int dA e^{-S_B} \underbrace{\left(\int d\psi e^{-S_F} \right)}_{=\text{Pf}\mathcal{M} = |\text{Pf}\mathcal{M}| e^{i\Gamma}} = \int dA \underbrace{e^{-S_0}}_{=e^{-S_B} |\text{Pf}\mathcal{M}|} e^{i\Gamma}$$

The Pfaffian PfM is complex in the Euclidean case.
Complex phase Γ is crucial for the SSB of SO(10).
[J. Nishimura and G. Vernizzi hep-th/0003223]



No SSB in the phase-quenched partition function.

$$Z_0 = \int dA e^{-S_0}$$

$\langle * \rangle_0$=V.E.V. for Z_0

4. Result of Monte Carlo simulation

It turns out sufficient to constrain only one eigenvalue λ_{d+1}

$\Sigma = \{ \lambda_{d+1} \text{ only} \}$ ⇒ SO(d) vacuum

$$\langle \lambda_1 \rangle = \cdots = \langle \lambda_d \rangle (= R^2) \gg \langle \lambda_{d+1} \rangle = \cdots = \langle \lambda_{10} \rangle (= r^2)$$

$\tilde{\lambda}_n \stackrel{\text{def}}{=} \lambda_n / \langle \lambda_n \rangle_0$ ⇒$(r/l)^2$[≃0.15/0.38=0.40 (GEM)]

- We study the SO(d) symmetric vacua (d=2,3,4) $x_1 = \ldots = x_d > 1 > x_{d+1}, \ldots, x_{10}$
- The large eigenvalues $\lambda_1, \ldots \lambda_d$ do not affect much the fluctuation of the phase.

Solve the saddle-point equation for n=d+1.
Simulation by Rational Hybrid Monte Carlo (RHMC) algorithm.

$$\frac{1}{N^2} f_n^{(0)}(x) = -\frac{d}{dx} \frac{1}{N^2} \log w_n(x) \quad \text{where}$$

$$f_n^{(0)}(x) \stackrel{\text{def}}{=} \frac{d}{dx} \log \langle \delta(x - \tilde{\lambda}_n) \rangle_0, \quad w_n(x) \stackrel{\text{def}}{=} \langle e^{i\Gamma} \rangle_{n,x} = \langle \cos \Gamma \rangle_{n,x}$$

$$\langle * \rangle_{n,x} = \left\{ \text{V.E.V. for } Z_{n,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_n) \right\}$$

Solution ⇒$\bar{x}_n = \langle \tilde{\lambda}_{d+1} \rangle_{\text{SO(d)}}$ in the SO(d) vacuum.

The phase $w_n(x)$ scales at large N as

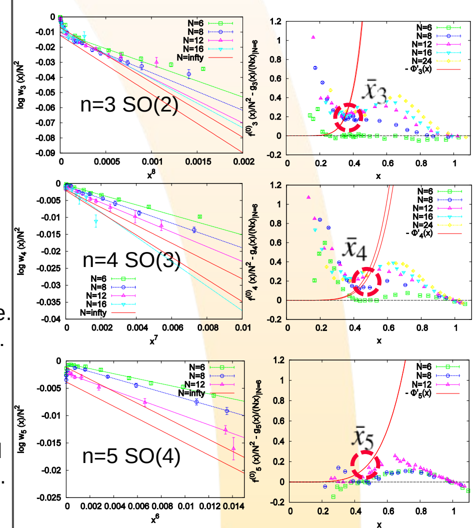
$$\Phi_n(x) = \lim_{N \to \infty} \frac{1}{N^2} \log w_n(x) \simeq -a_n x^{10-(n-1)} - b_n \quad (x < 1)$$

- Around x≅1: $f_n^{(0)}(x)/N$ scales at large N:

$$\frac{x}{N} f_n^{(0)}(x) \simeq g_n(x) = c_{1,n}(x-1) + c_{2,n}(x-1)^2$$

- Around x<0.4: $f_n^{(0)}(x)/N^2$ scales at large N
  → existence of the hardcore potential.

Preliminary Monte Carlo results:
$\bar{x}_{3,4,5}$ are close to the GEM result $\bar{x}_n = \langle \tilde{\lambda}_{d+1} \rangle_{\text{SO(d)}}$ ≃0.40.

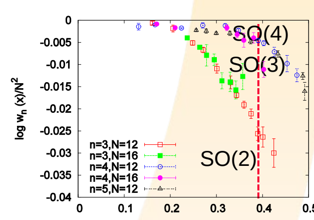


Comparison of the free energy

Free energy for the SO(d) vacuum:

$$\mathcal{F}_{\text{SO(d)}} = \int_{\bar{x}_n}^{1} \frac{1}{N^2} f_n^{(0)}(x) dx - \frac{1}{N^2} \log w_n(\bar{x}_n), \text{ where } n = d+1$$

→0 at large N



The SO(2) vacuum is disfavored.

$\mathcal{F}_{\text{SO(3,4)}} \ll \mathcal{F}_{\text{SO(2)}}$

5. Summary

We have studied the dynamical compactification of the spacetime in the Euclidean IKKT model.

Monte Carlo simulation via factorization method ⇒We have obtained the results consistent with GEM:

- Universal compactification scale for SO(2,3,4) vacuum.
- SO(2) vacuum is disfavored.