E.M.T. renormalization constants with the Wilson Flow

Francesco Capponi (speaker)
Plymouth University

Luigi Del Debbio
Edinburgh University

Agostino Patella
CERN & Plymouth University

Antonio Rago
Plymouth University

Kobe: 18 July 2015

renormalized E.M.T. on the lattice:why?

- Thermodynamic quantities

$$\langle \epsilon - 3p \rangle_T = -\left\langle \hat{\mathcal{T}}_{\mu\mu} \right\rangle_T \quad ; \quad \langle s \rangle_T = \left(-\left\langle \hat{\mathcal{T}}_{00} \right\rangle_T + \sum_{i=1}^{3} \left\langle \hat{\mathcal{T}}_{ii} \right\rangle_T \right) / T$$

- Transport coefficients

$$\eta = \pi \lim_{\omega \to 0} \mathrm{Im} \left\{ \left[i \int_0^\infty dt e^{i\omega t} \int d^3x \left\langle \hat{\mathcal{T}}_{12}(t,x) \hat{\mathcal{T}}^{12}(0,0) \right\rangle_T \right] \right\}$$

- Study of conformal field theories ($\left\langle \mathcal{T}_\mu^\mu(x) \right\rangle$ as order parameter)

E.M.T. IN YANG MILLS THEORY.

Two different strategies based on Wilson Flow

- T.W.I. for probe observables at positive flow time
  Del Debbio, Patella, Rago, arXiv:1306.1173 [hep-th]
- Small flow time expansion
  Suzuki, arXiv:1304.0533 [hep-lat]
  Asakawa, Hatsuda, Iritani, Itou, Kitazawa, Suzuki, arXiv:1412.4508 [hep-lat]

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Other strategies

- Shifted Boundary Conditions
  Robaina, Meyer, arXiv:1310.6075 [hep-lat]
  Giusti, Pepe, arXiv:1503.07042 [hep-lat]
  Giusti, Meyer, arXiv:1310.7818 [hep-lat]

Our method

FULLY RENORMALIZED EMT

A possible way for probing T.W.I. using flowed observables have been proposed by Del Debbio, Patella and Rago.
Define $V(t, y)$ built with gauge fields evolved according to the Wilson Flow, then

$$\langle \partial_\mu T_{\mu\nu}(x) V(t, y) \rangle = -\langle \delta_{x,\nu} V(t, y) \rangle$$

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ADVANTAGES

1. $V(t, y)$ is UV finite
   Luescher, Weisz, arXiv:1101.0963

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2. No contact terms ($x \to y$) arise
3. $[\delta_{x,\nu} V(t, y)]_R = Z_\delta \delta_{x,\nu} V(t, y)$, where Z_δ is finite and probe independent.

SETUP

1. SU(3) Yang Mills theory using OQCD
2. hypercubic lattice, periodic B.C.(βs kindly given by L.Giusti and M.Pepe)

L/a	6	8	10	12	16
β	5.8506	6.0056	6.1429	6.2670	6.4822
N_{meas}	87750	49924	20119	10070	7082

MIXING

$$[\hat{T}_{\mu\nu}]_R = Z_1\,[T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu}\rangle_0] + Z_3\,T^{[3]}_{\mu\nu} + Z_6\,T^{[6]}_{\mu\nu}$$

$$\hat{T}^{[1]}_{\mu\nu} = -\,\delta_{\mu\nu}\,\frac{1}{2g_0^2}\sum_{\sigma\tau}\mathrm{tr}\,\hat{F}_{\tau\sigma}\hat{F}_{\tau\sigma}$$

$$\hat{T}^{[3]}_{\mu\nu} = -\,\delta_{\mu\nu}\,\frac{2}{g_0^2}\sum_{\sigma}\mathrm{tr}\left\{\hat{F}_{\sigma\mu}\hat{F}_{\sigma\mu} - \frac{1}{4}\sum_{\tau}\hat{F}_{\sigma\tau}\hat{F}_{\sigma\tau}\right\}$$

$$\hat{T}^{[6]}_{\mu\nu} = -\,(1-\delta_{\mu\nu})\,\frac{2}{g_0^2}\sum_{\sigma}\mathrm{tr}\,\hat{F}_{\mu\sigma}\hat{F}_{\nu\sigma}$$

$$\left\langle \partial_\mu T_{\mu\nu}(x) V_\nu^{[\alpha]}(t,y) \right\rangle = -\left\langle \delta_{x,\nu} V_\nu^{[\alpha]}(t,y) \right\rangle$$

Use translational invariance and the following identity

$$\partial_\nu V_\nu^{[\alpha]}(t,y) = \int d^4 z \left[\delta_{z,\nu} V_\nu^{[\alpha]}(t,y) \right]$$

to obtain

$$\left\langle \delta_{y,\nu} V_\nu^{[\alpha]}(t,x) \right\rangle = \left\langle T_{\mu\nu}(y) \int d^4 z \left[\delta_{z,\mu} V_\nu^{[\alpha]}(t,x) \right] \right\rangle$$

Choose $V_\mu(t,x)$

$$V_\mu^{[\alpha]}(t,x) = \partial_\nu T_{\nu\mu}^{[\alpha]}(t,x) \quad ; \quad \alpha = 1,3,6$$

Next step: expand $[T_{\nu\mu}]_R$

$$\sum_{\beta=1,3,6} \left(\frac{\left\langle \hat{T}^{[\beta]}_{\mu\nu}(y) \sum_z \delta_{z,\mu} V^{[\alpha]}_{\nu}(t,x) \right\rangle}{\left\langle \hat{\delta}_{y,\nu} V^{[\alpha]}_{\nu}(t,x) \right\rangle} \right) C_\beta = 1$$

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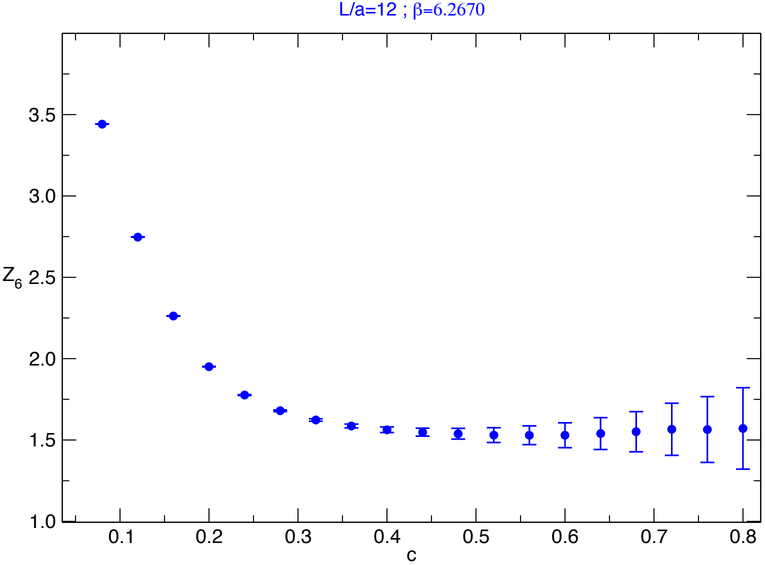
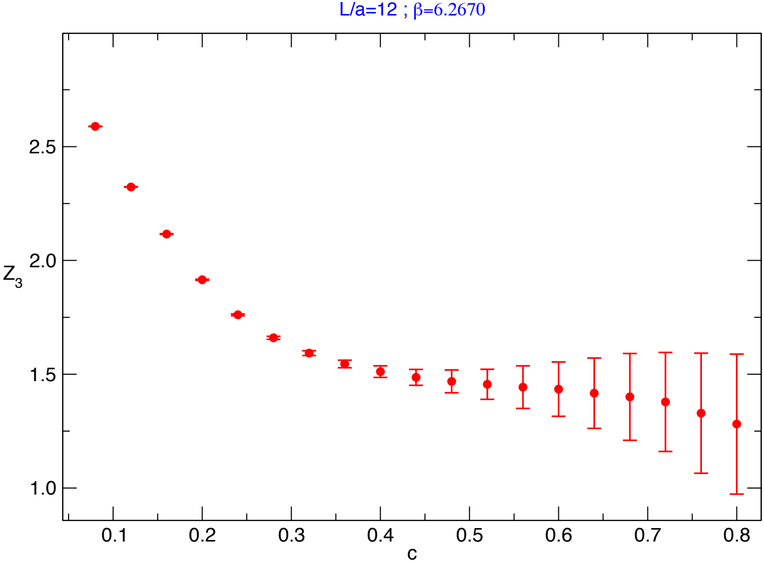
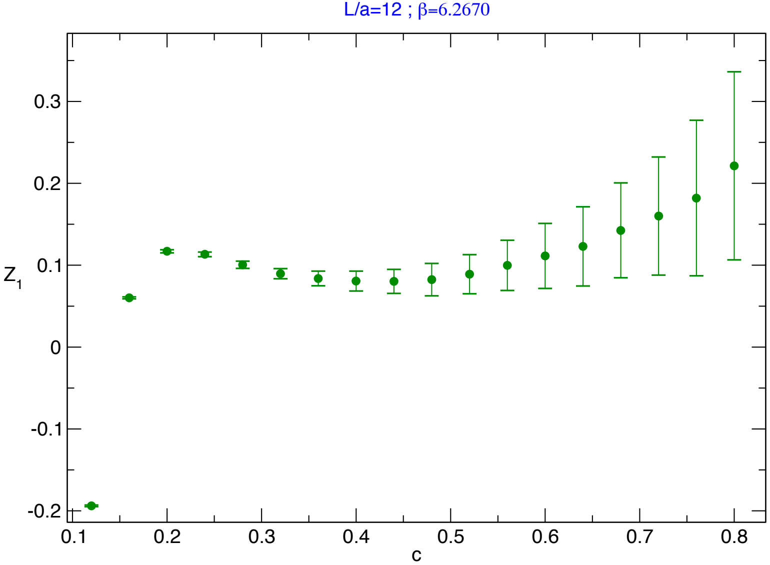
$$\sum_{\beta=1,3,6} M_{\alpha\beta} C_\beta = 1$$

3 equations for 3 coefficients.

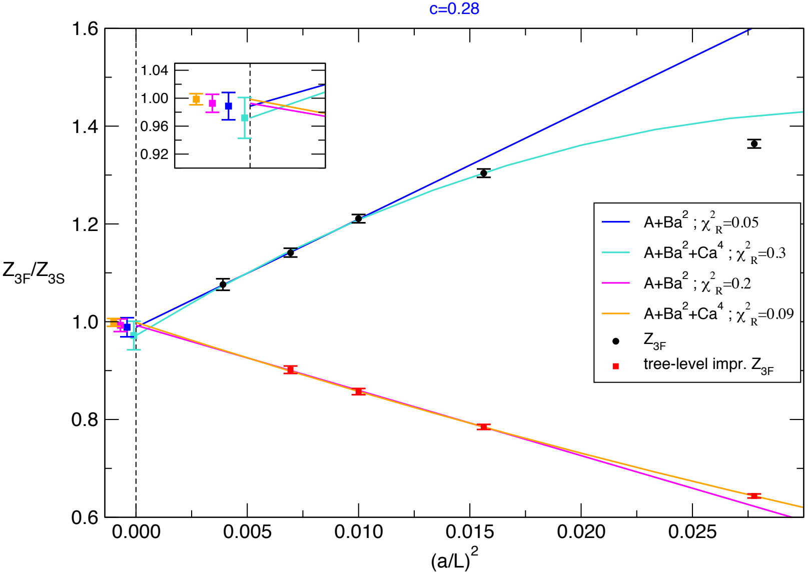
Results




L/a=12 ; β=6.2670
Z3
2.5
2.0
1.5
1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
c

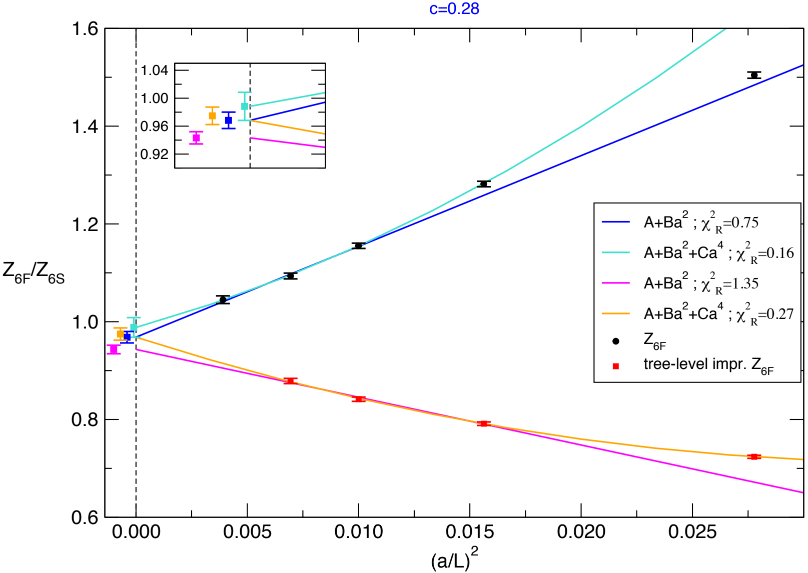

L/a=12 ; β=6.2670
Z_1
c
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Results



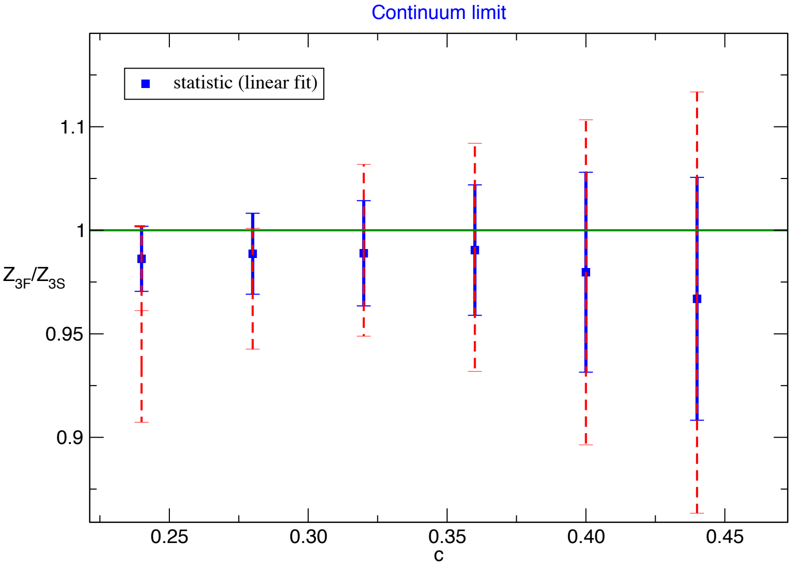
c=0.28
1.6
1.4
1.2
1.0
0.8
0.6
1.04
1.00
0.96
0.92
Z3F/Z3S
0.000
0.005
0.010
0.015
0.020
0.025
(a/L)2
A+Ba2 ; χ2R=0.05
A+Ba2+Ca4 ; χ2R=0.3
A+Ba2 ; χ2R=0.2
A+Ba2+Ca4 ; χ2R=0.09
Z3F
tree-level impr. Z3F

Results



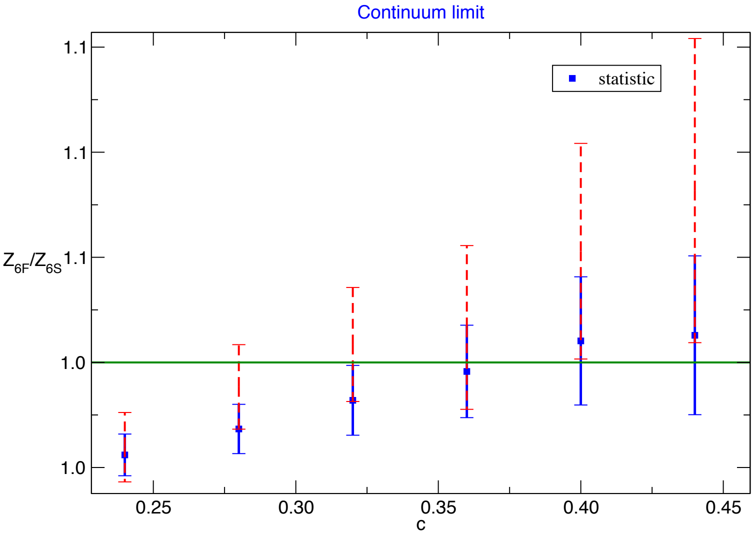
c=0.28
Z_{6F}/Z_{6S}
$(a/L)^2$
1.6
1.4
1.2
1.0
0.8
0.6
0.000
0.005
0.010
0.015
0.020
0.025
1.04
1.00
0.96
0.92
A+Ba^2 ; χ^2_R=0.75
A+Ba^2+Ca^4 ; χ^2_R=0.16
A+Ba^2 ; χ^2_R=1.35
A+Ba^2+Ca^4 ; χ^2_R=0.27
Z_{6F}
tree-level impr. Z_{6F}

Results



Continuum limit
statistic (linear fit)
1.1
1
0.95
0.9
Z_{3F}/Z_{3S}
0.25
0.30
0.35
0.40
0.45
c

Results


Continuum limit
statistic
Z_{6F}/Z_{6S}
1.1
1.1
1.1
1.0
1.0
0.25
0.30
0.35
0.40
0.45
c

CONCLUSION

- The renormalization constants of the E.M.T. can be measured probing TWI with observables at positive flow time
- With the adopted setup, we see that the biggest lattice artifacts contribution arises at tree-level.
- The results that we obtain are in a quite good agreement with the ones obtained by the shifted boundary condition method.
- Bigger lattices needed to have a more precise estimate of the constants
- The method seems to be solid and can be applied to other theories