Effective Polyakov loop models for QCD-like theories at finite chemical potential

Philipp Scior

with Lorenz von Smekal

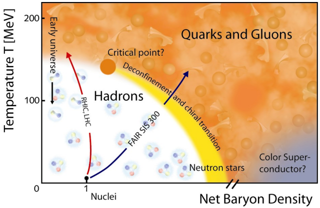
Theoriezentrum, Institut für Kernphysik, TU Darmstadt

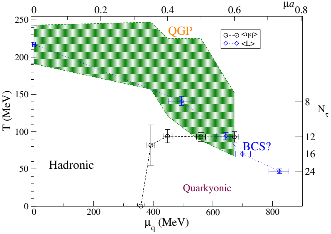
Lattice 2015, Kobe

Motivation

- Exploration of the QCD phase diagram at finite μ
- Use effective Polyakov loop theory to circumvent the sign problem
- Here: Use effective theory for QCD-like theories

  $\rightarrow$ qualitative and quantitative comparison with full theory at finite chemical potential





Boz, Cotter, Fister, Mehta, Skullerud

EPJ A 49, 11 (2013)

Effective Polyakov Loop Theory

- 3d SU(N) spin model sharing universal behavior at deconfinement transition with underlying gauge theory
- Less computational cost, especially with dynamical fermions
- Finite density $\longrightarrow$ Complex Langevin
- Can be derived from combined strong coupling and hopping expansion by integrating out spacial links

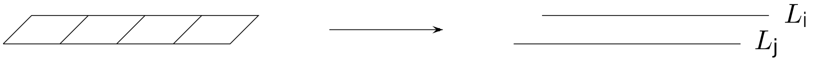
  Fromm, Langelage, Lottini, Neumann, Philipsen, PRL 110 (2013) 12

  Langelage, Neumann, Philipsen, JHEP 1409 (2014) 131

Effective Action

Use strong coupling and hopping expansion: Contributions to the effective action are graphs winding around the lattice in time direction

Gauge Action:



$$\Rightarrow S^g_{\text{eff}} = \lambda \sum_{\langle ij \rangle} L_i L_j + \ldots$$

Low temperature, strong coupling: $\lambda \leq 10^{-16} \longrightarrow$ fermionic partition function

Heavy Fermions and Hopping Expansion

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$$\det[D] = \det[1 - \kappa H] = \exp(-\sum_{i=1}^{\infty} \frac{1}{i} \kappa^i \, \mathrm{Tr}[H^i])$$

Leading order:

$$-S_{\mathrm{eff}} = N_f \sum_{\vec{x}} \log(1 + h \mathrm{Tr} W_{\vec{x}} + h^2)^2$$

Heavy Fermions and Hopping Expansion

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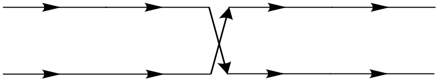
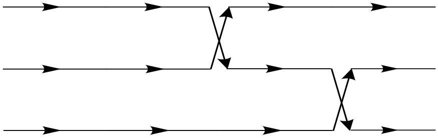
Order κ^2:



$$-S_{\text{eff}} = N_f \sum_{\vec{x}} \log(1 + h\text{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \text{Tr} \frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \text{Tr} \frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

Heavy Fermions and Hopping Expansion

Order κ^4:



Heavy Fermions and Hopping Expansion

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$$-S_{\text{eff}} = N_f \sum_{\vec{x}} \log(1 + h\text{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \text{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \text{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

$$+ 2N_f^2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \text{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \text{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2}$$

$$+ N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \text{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \text{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \text{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}}$$

$$+ 2N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \text{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \text{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \text{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}}$$

$$+ N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \text{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \text{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \text{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}}$$

Heavy Fermions and Hopping Expansion

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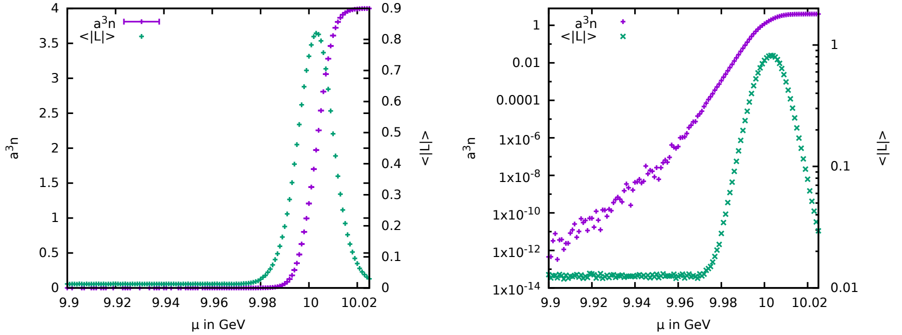
$$
\begin{aligned}
-S_{\text{eff}} = {} & N_f \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
& + 2N_f^2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2} \\
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
& + 2N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + N_f(2N_f - 1)\kappa^4 N_\tau^2 \sum_{x,i} \frac{h^4}{(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)(1 + h\mathrm{Tr}\, W_{\vec{x}+i} + h^2)} \; .
\end{aligned}
$$

Cold and Dense QC_2D with Heavy Quarks

- Very heavy quarks: m_q = 10.0014 GeV
- Diquark mass is fixed to m_d = 20 GeV $\rightarrow$ small binding energy
- Deconfinement transition with unphysical lattice saturation



Cold and Dense QC_2D with Heavy Quarks

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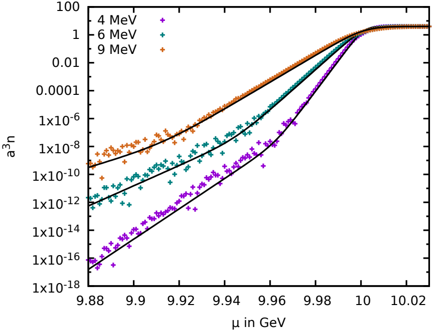
- Behaviour of $a^3 n$ is well described by a LO mean field model

$$a^3 n = 4N_f \frac{1 + Le^{\frac{m_q - \mu}{T}}}{1 + 2Le^{\frac{m_q - \mu}{T}} + e^{2\frac{m_q - \mu}{T}}}$$

- No Fit! All values taken from Simulations or are input
- $\langle L \rangle \approx 5 \cdot 10^{-5} < \langle |L| \rangle = 0.006$

  $\langle L \rangle$ determined by histograms

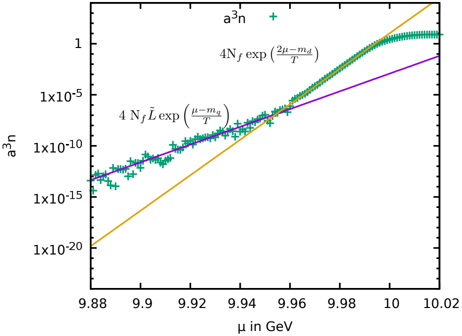


Cold and Dense QC_2D with Heavy Quarks

- Most precise description of the second exponential by

$$a^3 n = 4 N_f \exp\left(\frac{2\mu - m_d}{T}\right)$$

- Bound State?
- Model includes Confinement



Phase Diagram

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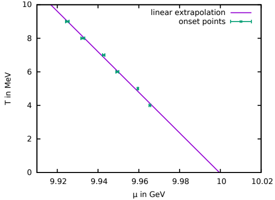
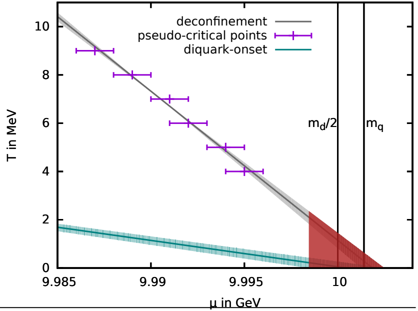
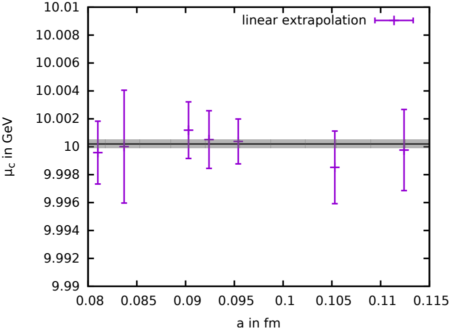


- Diquark onset line is not a phase-boundry!
- Line terminates at $\mu = \frac{m_d}{2}$ according to Silverblaze property
- Deconfinement transiton terminates at $\mu > \frac{m_d}{2}$
- Hints for BEC at lower Temperatures?



Extrapolation to $T = 0$

- $T = 0$ endpoint is in perfect agreement with $\mu = \frac{m_d}{2}$
- Endpoint is independent of lattice spacing



G2 QCD

- Smallest exceptional Lie Group
- All representation are real (β = 4)
- No sign problem: real and positive for N_f = 1
- YM theory has 1st order phase transition
- Spectrum: bosonic and fermionic baryons

$$\{7\} \otimes \{7\} = \{1\} \oplus \{7\} \oplus \{14\} \oplus \{27\}$$
$$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus 4 \cdot \{7\} \oplus 2 \cdot \{14\} \oplus \ldots$$
$$\{14\} \otimes \{14\} = \{1\} \oplus \{14\} \oplus \{27\} \oplus \ldots,$$
$$\{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \{7\} \oplus 5 \cdot \{14\} \oplus \ldots,$$
$$\{7\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \ldots$$

diquarks (baryon number 2)

Name	$\mathcal{O}$	T	J	P	C
$d(0^{++})$	$\bar{u}^C \gamma_5 u + c.c.$	SASS	0	+	+
$d(0^{+-})$	$\bar{u}^C \gamma_5 u - c.c.$	SASS	0	+	-
$d(0^{-+})$	$\bar{u}^C u + c.c.$	SASS	0	-	+
$d(0^{--})$	$\bar{u}^C u - c.c.$	SASS	0	-	-
$d(1^{++})$	$\bar{u}^C \gamma_\mu d - \bar{d}^C \gamma_\mu u + c.c.$	SSSA	1	+	+
$d(1^{+-})$	$\bar{u}^C \gamma_\mu d - \bar{d}^C \gamma_\mu u - c.c.$	SSSA	1	+	-
$d(1^{-+})$	$\bar{u}^C \gamma_5 \gamma_\mu d - \bar{d}^C \gamma_5 \gamma_\mu u + c.c.$	SSSA	1	-	+
$d(1^{--})$	$\bar{u}^C \gamma_5 \gamma_\mu d - \bar{d}^C \gamma_5 \gamma_\mu u - c.c.$	SSSA	1	-	-

mesons (baryon number 0)

Name	$\mathcal{O}$	T	J	P	C
π	$\bar{u}\gamma_5 d$	SASS	0	-	+
η	$\bar{u}\gamma_5 u$	SASS	0	-	+
a	$\bar{u}d$	SASS	0	+	+
f	$\bar{u}u$	SASS	0	+	+
ρ	$\bar{u}\gamma_\mu d$	SSSA	1	-	+
ω	$\bar{u}\gamma_\mu u$	SSSA	1	-	+
b	$\bar{u}\gamma_5\gamma_\mu d$	SSSA	1	+	+
h	$\bar{u}\gamma_5\gamma_\mu u$	SSSA	1	+	+

baryons (baryon number 3)

Name	$\mathcal{O}$	T	J	P	C
N	$T^{abc}(\bar{u}_a^C \gamma_5 d_b) u_c$	SAAA	1/2	$\pm$	$\pm$
Δ	$T^{abc}(\bar{u}_a^C \gamma_\mu u_b) u_c$	SSAS	3/2	$\pm$	$\pm$

T: (x, s, C, F)

Phase diagram

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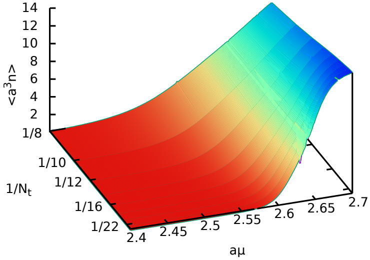
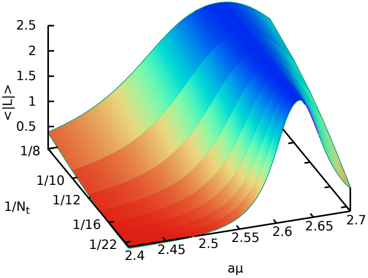
Density



Polyakov loop



- Cold and dens region of the G2 effective theory phase diagram
- $\beta/N_c = 1.4$, $\kappa = 0.0357$, $N_t = 8, ...24$

Cold and Dens Region

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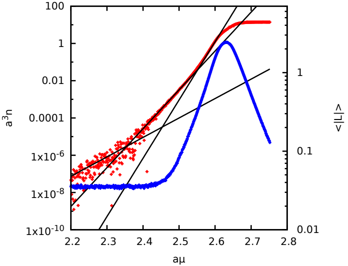
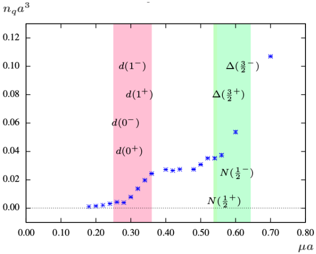
- Deconfinement and unphysical lattice saturation
- three regions with different exponential growth
- diquark onset
- fermionic baryon onset





compare with *Wellegehausen, Maas, Wipf, von Smekal PRD 89 (2014) 4*

with a smaller quark mass $\kappa = 0.147$

Summary Conclusion

- Effective Polyalov loop theory for two-color QCD up to order κ^4
- Thermal two-quark excitation onset at $T > 0$
- Extrapolation $T \to 0$ gives $m_d/2$ according to Silverblaze Property
- However no signal for BEC
- Effective Polyalov loop theory for G2
- Two- and three-quark excitations in agreement with possible spectrum

Analytic relations for Cold and Dense regime

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$$h = \exp\left[N_\tau\left(a\mu + \ln 2\kappa + 6\kappa^2\frac{u - u^{N_\tau}}{1-u}\right)\right] \ ,$$

$$am_d = -2\ln(2\kappa) - 6\kappa^2 - 24\kappa^2\frac{u}{1-u} + 6\kappa^4 + \mathcal{O}(\kappa^4 u^2, \kappa^2 u^5) \ .$$