The strange contribution to $a_\mu^{\mathrm{HVP,LO}}$ with physical quark masses using Möbius domain wall fermions

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Motivation

Magnetic moment:

$$\boldsymbol{\mu} = g \frac{e}{2m} \mathbf{S}; \;\; U = -\boldsymbol{\mu} \cdot \mathbf{B}; \;\; a_\mu = \frac{g_\mu - 2}{2}$$

Contributions to a_μ

Contribution	$a_\mu \times 10^{11}$	Uncertainty
QED (5-loop)	116584718.95	0.08
Electroweak (2-loop)	153.6	1.0
LO hadronic (HVP)	6923	42.1
NLO hadronic	7	26
Total	116591803	49.4
Experimental	116592091	63.2

[PDG, 2014]

- 3.6σ discrepancy between theory and experiment.

Motivation

Magnetic moment:

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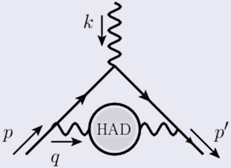
[PDG, 2014]

- 3.6σ discrepancy between theory and experiment.
- Greatest uncertainty comes from HVP.

Hadronic Vaccuum Polarization

HVP in Euclidean Space [T. Blum, 2002]



1. $a_\mu^{\mathrm{s}} = 4\alpha^2 \int_0^\infty \mathrm{d}\hat{q}^2 f(\hat{q}^2)\hat{\Pi}(\hat{q}^2)$
2. $\hat{\Pi}(\hat{q}^2) = \Pi(\hat{q}^2) - \Pi(0)$
3. $\Pi_{\mu\nu}(\hat{q}) = (\delta_{\mu\nu}\hat{q}^2 - \hat{q}_\mu\hat{q}_\nu)\Pi\left(\hat{q}^2\right)$

Computation:

$$\Pi_{\mu\nu}\left(\hat{q}\right) = Z_V \sum_{f,x} Q_f^2 \mathrm{e}^{\mathrm{i}q\cdot x} \left\langle \mathcal{V}_\mu^f\left(x\right) V_\nu^f\left(0\right)\right\rangle;\ \hat{q} = \frac{2}{a}\sin\left(\frac{aq}{2}\right)$$

Challenges:

- Integrand highly peaked near $\hat{q}^2 = m_\mu^2/4$.
- Lattice imposes lower bound on non-zero momenta $(q_\mu = \frac{2\pi n_\mu}{N_\mu})$.
- HVP cannot be directly computed at $\hat{q}^2 = 0$.

HVP Computation

Correlator computation:

- Conserved current at sink
- Z2 wall source - Ward Identity $q^\mu \Pi_{\mu\nu} = 0$ in large hit limit

Zero-mode Subtraction

Reduce statistical noise at low $\hat{q}^2$ by subtracting $q_t = 0$ component:

$$\Pi^{\mathrm{s}}_{\mu\nu}(\hat{q}) = \sum_t \mathrm{e}^{\mathrm{i}\boldsymbol{q_t}\cdot \boldsymbol{t}} C^{\mathrm{s}}_{\mu\nu}(t) - \sum_t C^{\mathrm{s}}_{\mu\nu}(t)$$

[Bernecker and Meyer, 2011; C. Lehner and T. Izubuchi, 2014]

Restriction to diagonal of HVP tensor (remove longitudinal part and reduce cut-off effects):

$$\Pi\left(\hat{q}^2\right) = \frac{1}{3}\sum_i \frac{\Pi_{ii}\left(\hat{q}\right)}{\hat{q}^2};\ \hat{q}_\mu = 0$$

Simulations

Ensembles

RBC/UKQCD 2+1f domain wall fermion ensembles with physical pion masses [RBC/UKQCD, 2014]:

Parameter	48I	64I
$L^3 \times T \times L_s$	$48^3 \times 96 \times 12$	$64^3 \times 128 \times 24$
m_π	139.2(4) MeV	139.2(5) MeV
m_K	499.0(12) MeV	507.6(16) MeV
a^{-1}	1.730(4) GeV	2.359(7) GeV
$m_\pi L$	3.863(6)	3.778(8)

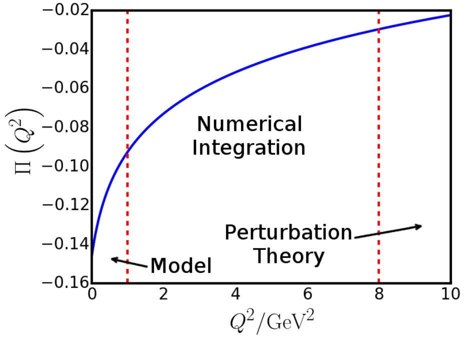
Measurements

Unitary and physical/partially quenched strange masses to account for m_s mistuning.

The Hybrid Method

Motivation

- Systematic error of the model at low $\hat{q}^2$ grows with cut.
- Perturbation theory only valid at large $\hat{q}^2$.
- How are these reconciled? [Golterman, Maltman and Peris, 2014]



Variations

- Models
- Cuts
- Techniques to constrain models (fits, moments)

Low q^2 Models

Padé Approximants

Motivated by the spectral decomposition of the HVP [Aubin, Blum, Golterman and Peris, 2012]:

$$R_{mn}\left(\hat{q}^2\right) = \Pi_0 + \hat{q}^2 \left(\sum_{i=0}^{m-1} \frac{a_i^2}{b_i^2 + \hat{q}^2} + \delta_{mn} c^2 \right); \; n = m, \; m+1.$$

Conformal Polynomials

Map domain of analyticity onto region within unit disc. Better convergence properties [Golterman, Maltman and Peris, 2014].

$$P_n\left(\hat{q}^2\right) = \Pi_0 + \sum_{i=1}^{n} p_i w^i,$$

$$w = \frac{1 - \sqrt{1+z}}{1 + \sqrt{1+z}}, \; z = \frac{\hat{q}^2}{E^2}.$$

Time Moments

1. Tensor decomposition:
$$\sum_t \mathrm{e}^{-\mathrm{i}q_t t} C_{\mu\mu}(t) = \hat{q}_t^2 \Pi\left(\hat{q}_t^2\right)$$
2. Differentiate w.r.t. q_t:
$$(-1)^n \sum_t t^{2n} C_{\mu\mu}(t) = \frac{\partial^{2n}}{\partial q_t^{2n}} \left(\hat{q}_t^2 \Pi\left(\hat{q}_t^2\right)\right)\Bigg|_{q_t=0}$$
3. Plug in a model for $\Pi\left(\hat{q}^2\right)$ and solve for parameters. [HPQCD, 2014]

Notes

- Infinite time assumption.
- Expansion around $\hat{q}^2 = 0$, data here carry more weight.
- Model parameterization independent of cut.

Continuous Momenta

- Fourier transform to arbitrary momenta [Bernecker and Meyer, 2011; Feng et al., 2013]:

$$\Pi^{\mathrm{s}}_{\mu\nu}(\hat{q}) = \sum_t \mathrm{e}^{\mathrm{i}q_t \cdot t} C^{\mathrm{s}}_{\mu\nu}(t) - \sum_t C^{\mathrm{s}}_{\mu\nu}(t)$$

$$q_t = \frac{2\pi n_t}{T}, \; n_t \in [-T/2, T/2)$$

- Compute HVP directly at arbitrary $\hat{q}$.
- No hybrid method, model independent.
- Can show interpolation effects are $\mathcal{O}(\exp(-M_\pi L))$ [Portelli and Del Debbio, Tuesday 15:20].

Analysis Matrix

Hybrid Method

- Model determination: diagonal fit, time moments.
- Models:
  - Padés: $R_{0,1}$, $R_{1,1}$, $R_{1,2}$.
  - Conformal polynomials: P_2, P_3, P_4; $E = 500\mathrm{MeV}, 600\mathrm{MeV}$.
- Low cuts: 0.5, 0.7, 0.9 GeV^2, $\left(\frac{2\pi}{aN_t}\right)^2 \approx 0.013\,\mathrm{GeV}^2$.
- High cuts: 4.5, 5.0, 5.5 GeV^2.

Sine Cardinal Reconstruction

- $\Delta n_t = 0.005$
- High cuts: 4.0, 5.0, 6.0 GeV^2.

Extrapolations

- Strange quark mistuning: $\sim$1% on 48I, $\sim$5% on 64I.
- Partially quenched measurements.
- Continuum limit.
- Strange mass extrapolation.

Ansatz

$$a_\mu^{\mathrm{s}}\left(a^2, am_s\right) = a_{\mu,0}^{\mathrm{s}} + \alpha a^2 + \beta \frac{am_s - am_s^{\mathrm{phys}}}{am_s^{\mathrm{phys}} + am_{\mathrm{res}}}$$

Systematic Effects

Accounted For So Far

- $\hat{q}^2$ cuts.
- Low $\hat{q}^2$ model.

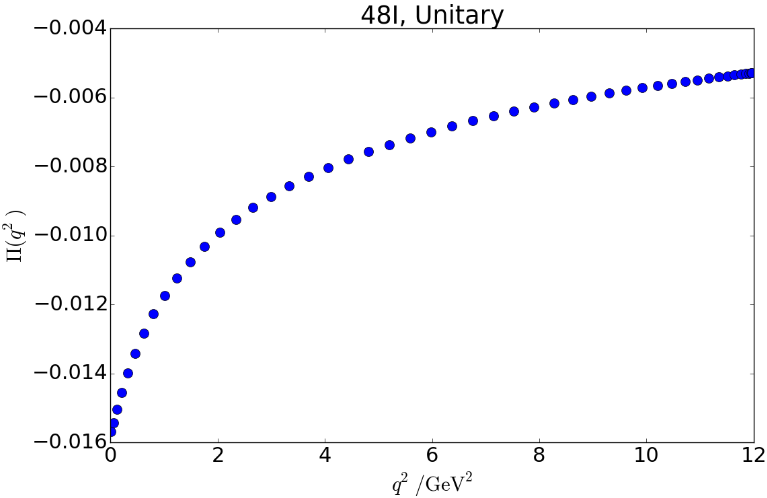
Short Term

- Finite volume effects.
- Non-unitarity.

Long Term

- Disconnected diagrams.
- Charm quark in the sea.
- Isospin breaking effects, including EM effects.

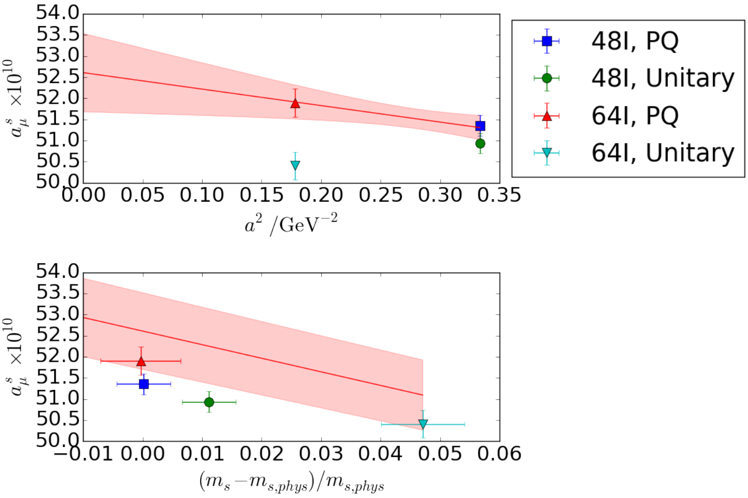
HVP



48I, Unitary
−0.004
−0.006
−0.008
−0.010
−0.012
−0.014
−0.016
Π(q²)
0
2
4
6
8
10
12
q² /GeV²

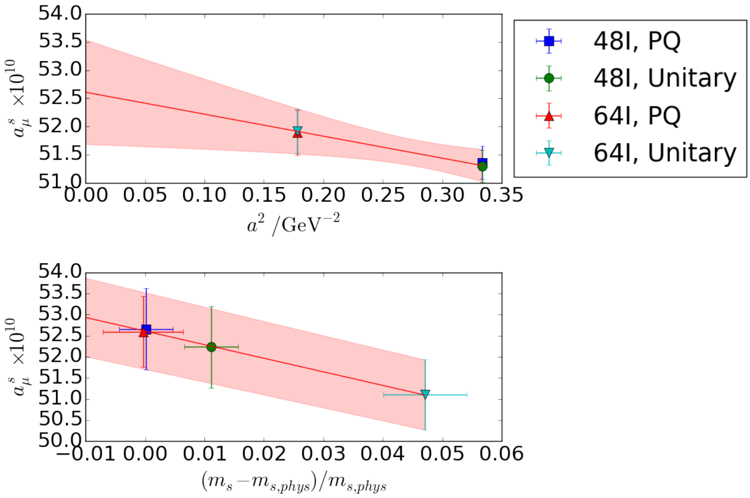
Extrapolations

Fit, R_{01}, low cut = 0.5 GeV^2, high cut = 4.5 GeV^2

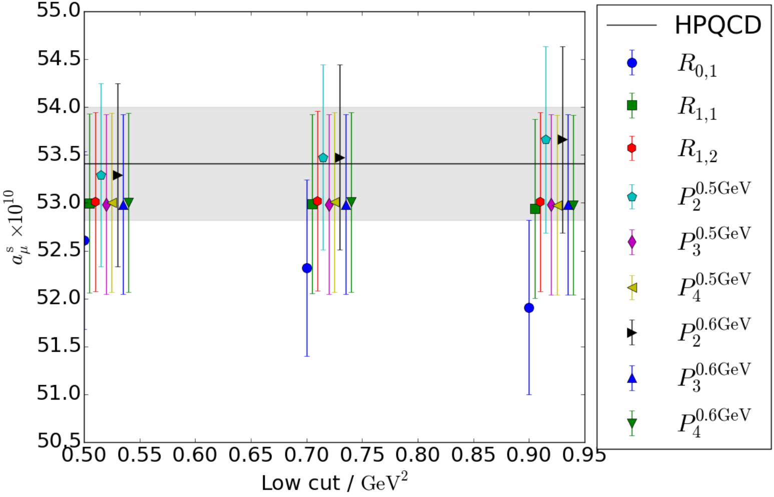


Extrapolations

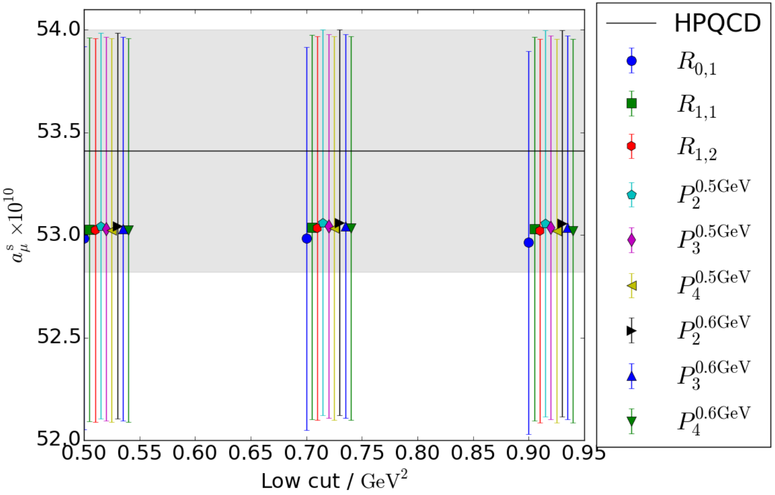
Fit, R_{01}, low cut = 0.5 GeV^2, high cut = 4.5 GeV^2



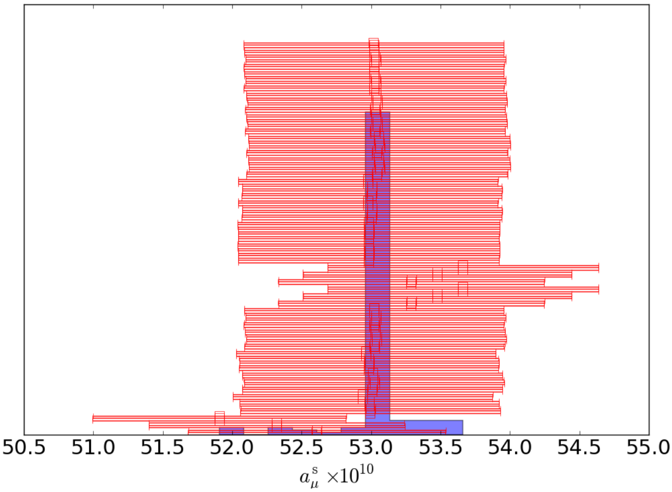
Results: Fits



Results: Moments



Results: a_μ



Summary

Conclusions

- a_μ^{s} computed using Möbius domain wall fermions with 2+1f.
- Extensive systematic study of analysis techniques.
- Final value of a_μ^{s} largely insensitive to analysis method.
- Results consistent with other studies (HPQCD, ETMC).
- a_μ^{s} analysis provides solid foundation for future work.

Outlook

- Finalize systematics.
- Noise reduction in light contribution.
- Disconnected diagrams.
- Light-by-light contribution.

Ensembles

RBC/UKQCD ensembles using 2+1f domain wall fermions with a physical pion mass.

Parameter	48I	64I
$L^3 \times T \times L_s$	$48^3 \times 96 \times 24$	$64^3 \times 128 \times 12$
am_l	0.00078	0.000678
am_s	0.0362	0.02661
am_s^{phys}	0.03580(16)	0.02539(17)
a^{-1} / GeV	1.730(4)	2.359(7)
L / fm	5.476(12)	5.354(16)
$m_\pi L$	3.863(6)	3.778(8)
am_{res}	0.0006012	0.0003116

Strange quark mistuning: partially quenched runs

Analysis Strategy: χ^2 Fit

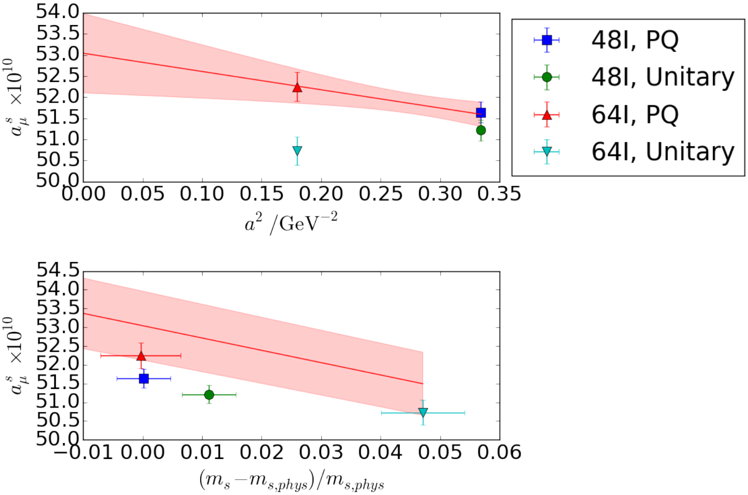
χ^2 Fit

- Standard χ^2 minimization - covariance approximated by diagonal.

$$\chi^2 = \sum_{\hat{q}^2} \left(\frac{\Pi\left(\hat{q}^2\right) - f\left(\hat{q}^2\right)}{\delta\Pi\left(\hat{q}^2\right)} \right)^2$$

- Fits can be unstable - Z2 wall reduces d.o.f.
- Fit biased towards large $\hat{q}^2$.
- Parameters dependent on cut.

Moments, $P_2^{0.5\mathrm{GeV}}$, low cut = 0.5 GeV^2, high cut = 4.5 GeV^2



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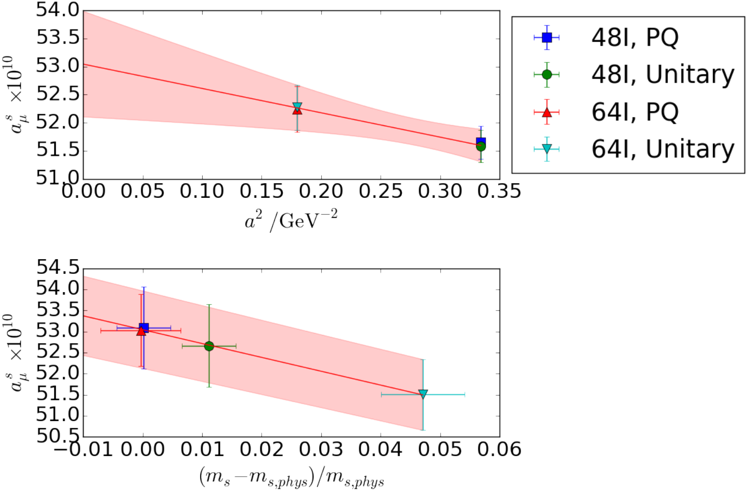
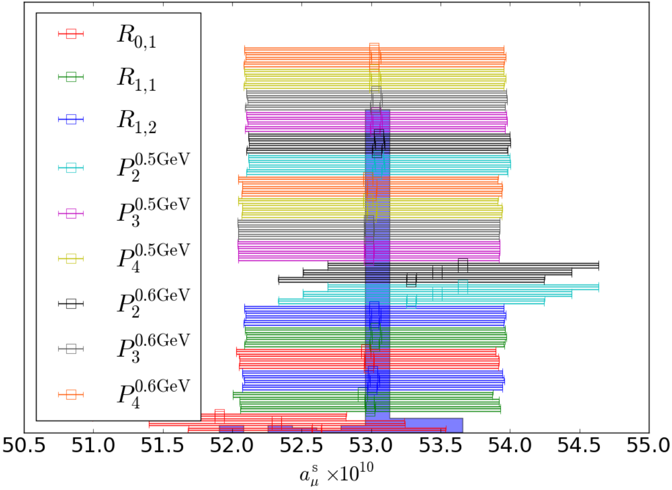
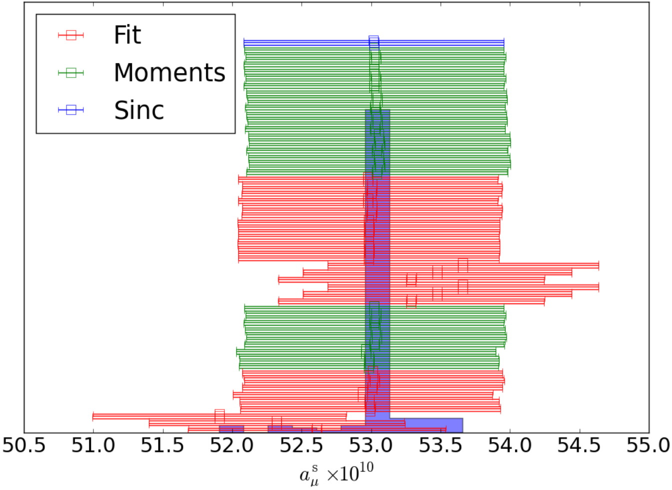


Results: a_μ



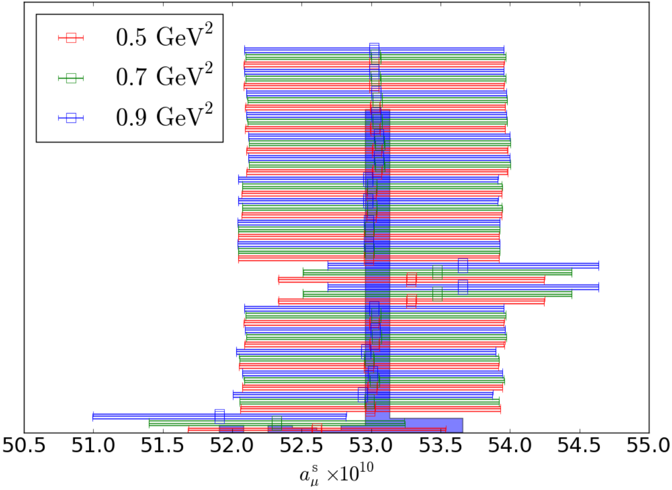
$R_{0,1}$
$R_{1,1}$
$R_{1,2}$
$P_2^{0.5\mathrm{GeV}}$
$P_3^{0.5\mathrm{GeV}}$
$P_4^{0.5\mathrm{GeV}}$
$P_2^{0.6\mathrm{GeV}}$
$P_3^{0.6\mathrm{GeV}}$
$P_4^{0.6\mathrm{GeV}}$
50.5 51.0 51.5 52.0 52.5 53.0 53.5 54.0 54.5 55.0
$a_\mu^{\mathrm{s}} \times 10^{10}$

Results: a_μ



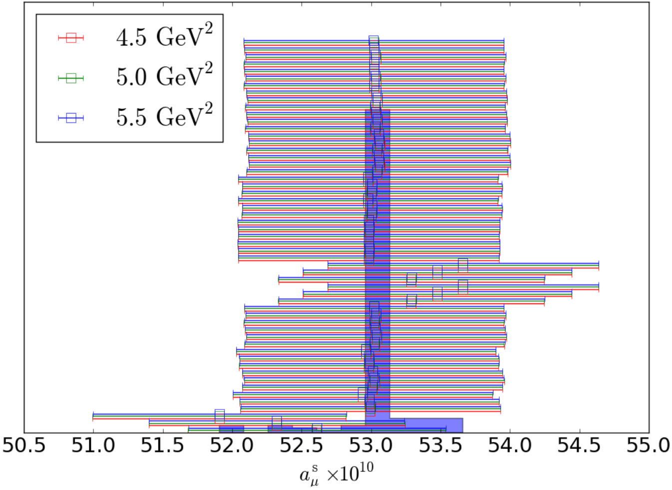
Fit
Moments
Sinc
50.5
51.0
51.5
52.0
52.5
53.0
53.5
54.0
54.5
55.0
$a_\mu^s \times 10^{10}$

Results: a_μ



0.5 GeV^2
0.7 GeV^2
0.9 GeV^2
50.5
51.0
51.5
52.0
52.5
53.0
53.5
54.0
54.5
55.0
$a_\mu^s \times 10^{10}$

Results: a_μ



4.5 GeV^2
5.0 GeV^2
5.5 GeV^2
50.5
51.0
51.5
52.0
52.5
53.0
53.5
54.0
54.5
55.0
$a_\mu^s \times 10^{10}$