Prospects and status of quark mass renormalization in three-flavour QCD

Patrick Fritzsch
<p.fritzsch@csic.es>

in collaboration with
I.Campos, C.Pena, D.Preti, A.Ramos, A.Vladikas

Instituto de Física Teórica UAM-CSIC

ALPHA Collaboration

EXCELENCIA SEVERO OCHOA

33rd International Symposium on Lattice Field Theory
Kobe International Conference Center, Kobe, Japan
July 14-18, 2015

Renormalization Group Invariant (RGI) parameters

RG equations for running coupling and quark mass (mass-independent scheme)

$$\mu\frac{\partial}{\partial q}\bar{g}(q) = \beta(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^3(b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots)$$

$$\mu\frac{\partial}{\partial q}\overline{m}(q) = \tau(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^2(d_0 + d_1\bar{g}^2 + \ldots)$$

b_0, b_1, d_0 are universal and $b_{i>1}, d_{j>0}$ scheme-dependent coeff.s

Renormalization Group Invariant (RGI) parameters

RG equations for running coupling and quark mass (mass-independent scheme)

$$\mu\frac{\partial}{\partial q}\bar{g}(q) = \beta(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^3(b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots)$$

$$\mu\frac{\partial}{\partial q}\overline{m}(q) = \tau(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^2(d_0 + d_1\bar{g}^2 + \ldots)$$

b_0, b_1, d_0 are universal and $b_{i>1}, d_{j>0}$ scheme-dependent coeff.s

integrated over renormalization scale $q \in [\mu, \infty]$

$$\Lambda \equiv \mu\left[b_0\bar{g}^2(\mu)\right]^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}^2(\mu))} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

$$M_i \equiv \overline{m}_i(\mu)\left[2b_0\bar{g}^2(\mu)\right]^{-d_0/(2b_0)} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}g\left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}$$

Renormalization Group Invariant (RGI) parameters

RG equations for running coupling and quark mass (mass-independent scheme)

$$\mu \frac{\partial}{\partial q} \bar{g}(q) = \beta(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^3 (b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots)$$

$$\mu \frac{\partial}{\partial q} \overline{m}(q) = \tau(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^2 (d_0 + d_1 \bar{g}^2 + \ldots)$$

b_0, b_1, d_0 are universal and $b_{i>1}, d_{j>0}$ scheme-dependent coeff.s

integrated over renormalization scale $q \in [\mu, \infty]$

$$\Lambda \equiv \mu \left[b_0 \bar{g}^2(\mu) \right]^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0 \bar{g}^2(\mu))} \exp\left\{ -\int_0^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

$$M_i \equiv \overline{m}_i(\mu) \left[2b_0 \bar{g}^2(\mu) \right]^{-d_0/(2b_0)} \exp\left\{ -\int_0^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g} \right] \right\}$$

encode information about fundamental parameters of QCD

- defined without relying on perturbation theory
- RGI mass independent of renormalisation scheme, (Λ trivial dep.)
- allow for easy conversion (at high μ) between renorm. mass/coupling in diff. schemes

General strategy

Capitani et.al.[1]

1. compute bare current quark mass at some hadronic scale $\mu_{\rm had}$, renormalize & take CL

$$\overline{m}_i(\mu_{\rm had}) = \lim_{a\to 0}\left[\frac{Z_{\rm A}(g_0^2)}{Z_{\rm P}(g_0^2, a\mu_{\rm had})}\frac{m_i}{f_{\rm had}}\right] \times f_{\rm had}^{\rm phys}$$

using some scale-setting observable $f_{\rm had} \in \{f_{\rm K}, \ldots\}$

2. connect to RGI mass

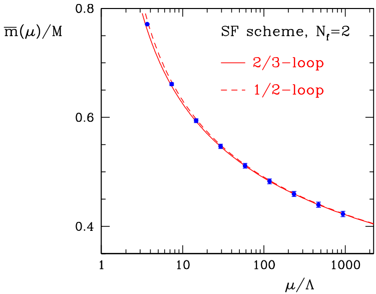
$$M_i = \frac{M}{\overline{m}(\mu_{\rm had})} \times \overline{m}_i(\mu_{\rm had})$$

- RG running factor to $\mu = \infty$ (continuum, flavour-independent)
- hadronic computation

employing the Schrödinger functional (SF) as intermediate renormalization scheme, $\mu = 1/L$

$N_\mathrm{f} = 2$: (plaquette gauge action)

ALPHA [2, 3]



$$\frac{M}{\overline{m}(\mu)} = 1.308(16)$$

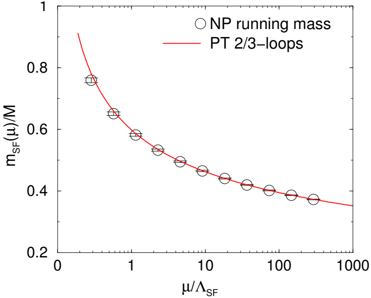
M_ud

$$M_\mathrm{s} = 138(3)(1)\,\mathrm{MeV}$$

$f_\mathrm{had} \equiv f_\mathrm{K} = 155\,\mathrm{MeV}$

$N_\mathrm{f} = 3$: (Iwasaki gauge action)

PACS-CS, CP-PACS/JLQCD [4, 5]



$$\frac{M}{\overline{m}(\mu)}$$

$$M_\mathrm{ud} = 3.49(34)\,\mathrm{MeV}$$

$$M_\mathrm{s} = 109.0(3)\,\mathrm{MeV}$$

$f_\mathrm{had} \equiv m_{\hat{\mathrm{K}}} = 495.7\,\mathrm{MeV}$

Previous continuum results

A closer look, error budget

$N_\mathrm{f} = 2$ (plaquette gauge action)

$$M_\mathrm{s} = \frac{M}{\overline{m}(\mu_\mathrm{had})} \times \overline{m}_\mathrm{s}(\mu_\mathrm{had}) \qquad \begin{cases} \frac{M}{\overline{m}(\mu)} = 1.308(16)\ , & \delta_{RG} = 1.2\% \\ M_\mathrm{s} = 138(3)(1)\ \mathrm{MeV}\ , & \delta_M = 2.3\% \end{cases}$$

Previous continuum results

A closer look, error budget

$N_\mathrm{f} = 2$ (plaquette gauge action)

$$M_\mathrm{s} = \frac{M}{\overline{m}(\mu_\mathrm{had})} \times \overline{m}_\mathrm{s}(\mu_\mathrm{had}) \qquad \begin{cases} \frac{M}{\overline{m}(\mu)} = 1.308(16)\ , & \delta_{RG} = 1.2\% \\ M_\mathrm{s} = 138(3)(1)\,\mathrm{MeV}\ , & \delta_M = 2.3\% \end{cases}$$

$$\Downarrow$$

$$\overline{m}_\mathrm{s}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 102(3)(1)\,\mathrm{MeV}\ , \qquad \delta = 3.1\%$$

PDG'14 : $93.5(2.5)\,\mathrm{MeV}\ ,$ $\qquad \delta = 2.7\%$

$N_\mathrm{f} = 2$ (plaquette gauge action)

$$M_\mathrm{s} = \frac{M}{\overline{m}(\mu_\mathrm{had})} \times \overline{m}_\mathrm{s}(\mu_\mathrm{had}) \qquad \begin{cases} \frac{M}{\overline{m}(\mu)} = 1.308(16)\ , & \delta_{RG} = 1.2\% \\ M_\mathrm{s} = 138(3)(1)\,\mathrm{MeV}\ , & \delta_M = 2.3\% \end{cases}$$

$$\Downarrow$$

$$\overline{m}_\mathrm{s}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 102(3)(1)\,\mathrm{MeV}\ , \qquad \delta = 3.1\%$$

PDG'14 : $93.5(2.5)\,\mathrm{MeV}\ ,$ $\delta = 2.7\%$

AIM FOR $\delta = 2\%$ FOR NEW 3-FLAVOUR COMPUTATION.

$\Rightarrow \delta_M < 2\%$

- we expect improvement on the hadronic part, $N_\mathrm{f} = 3$ CLS ensembles (open b.c.), but hard to quantify today

$\Downarrow$

can systematically improve on the RG running part

Computing $M/\overline{m}(\mu_{\rm had})$

- Compute

$$\frac{M}{\overline{m}(\mu_{\rm had})} \propto \left[\prod_i^N \sigma_{\rm P}(u_i)\right]^{-1}$$

- *Step-scaling function:*

$$\sigma_{\rm P}(u) = \exp\left[-\int_{\bar{g}(\mu)}^{\bar{g}(\mu/2)} {\rm d}g\, \frac{\tau(g)}{\beta(g)}\right]_{\bar{g}^2(\mu)=u} = \lim_{a\to 0} \Sigma_{\rm P}(u, a/L)$$

- *Lattice step-scaling function:*

$$\Sigma_{\rm P}(u, a/L) = \frac{Z_{\rm P}(g_0, 2L/a)}{Z_{\rm P}(g_0, L/a)}$$

can be 1-loop improved (plaq. action):

$$\Sigma_{\rm P}(u, a/L) \;\to\; \Sigma_{\rm P}^{(1)}(u, a/L) = \frac{\Sigma_{\rm P}(u, a/L)}{1 + \delta_{\rm P}(a/L)u}$$

- *Renormalization condition:*

$$\left[Z_{\rm P}(g_0, L/a)\frac{f_{\rm P}(L/2)}{\sqrt{3f_1}}\right]^{\theta}_{m=0} = c_3(\theta, a/L)$$

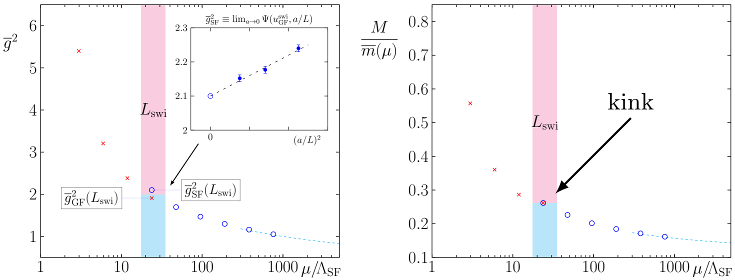
at fixed values of $u \in \{u_{\rm SF}, u_{\rm GF}\}$ but vanishing boundary field, $T/L = 1$, $\theta = 0.5$

But things become more complicated

- more accurate results demand even better control of systematics such as
  - boundary and bulk $\mathrm{O}(a)$ improvement
  - tuning to critical mass and fixed coupling
  - ...

But things become more complicated

- more accurate results demand even better control of systematics such as
  - boundary and bulk $\mathrm{O}(a)$ improvement
  - tuning to critical mass and fixed coupling
  - ...
- strategy of running coupling (ALPHA'14 [6]) imposes (computational) constraints:



and requires precise determination of scheme-switching scale $\mu_{\mathrm{swi}} = 1/L_{\mathrm{swi}}$

GF SF SF-PT

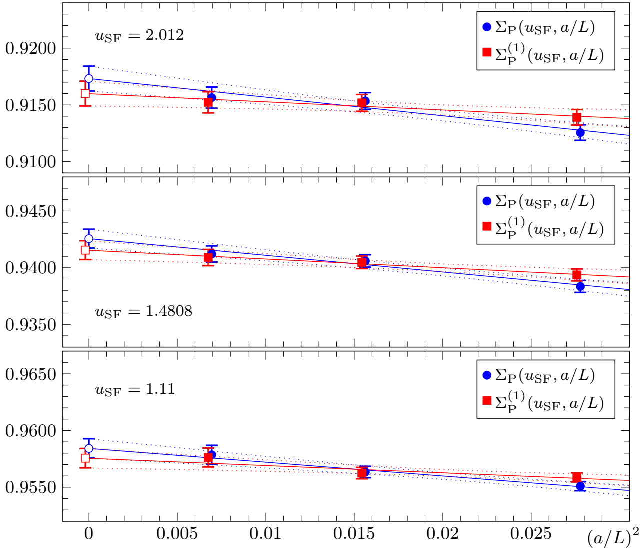
$$\frac{M}{\bar{m}(\mu_{\mathrm{had}})} = \frac{\bar{m}(\mu_{\mathrm{swi}})}{\bar{m}(\mu_{\mathrm{had}})} \times \frac{\bar{m}(\mu_{\mathrm{pert}})}{\bar{m}(\mu_{\mathrm{swi}})} \times \frac{M}{\bar{m}(\mu_{\mathrm{pert}})}$$

- $u_{\rm GF} : \kappa_{\rm crit}^{\rm GF}(\beta)$, $u_{\rm GF}$ in progress (see previous talk by Stefan Sint)
- $u_{\rm SF} : u_{\rm SF} \equiv u_{\rm SF}^{\rm fit}(\beta) \quad \Rightarrow \quad \beta \quad \Rightarrow \kappa_{\rm crit}^{\rm SF} \equiv \kappa_{\rm crit}^{\rm fit}(\beta)$

  fixed coupling $u_{\rm SF} \in \{1.110, 1.18446, 1.26569, 1.3627, 1.4808, 1.6173, 1.7943, 2.012\}$

$u_{\rm SF}$	L/a	β	$\kappa_{\rm crit}^{\rm SF}$	$u_{\rm SF}^{\rm fit}$
1.1100	6	8.5403(55)	0.1323361(12)	1.1100(12)
1.1100	8	8.7325(72)	0.1321338(13)	1.1100(15)
1.1100	12	8.995(11)	0.1318617(10)	1.1100(24)
...				
1.4808	6	7.2618(28)	0.1339337(13)	1.4808(11)
1.4808	8	7.4424(38)	0.1336745(11)	1.4808(15)
1.4808	12	7.7299(89)	0.13326299(69)	1.4808(35)
...				
2.0120	6	6.2735(44)	0.1355713(17)	2.0120(32)
2.0120	8	6.4680(51)	0.1352362(15)	2.0120(39)
2.0120	12	6.7299(68)	0.1347591(10)	2.0120(49)

¡PRELIMINARY!



$\sigma_P = 0.9173(11)(19)_{sys}$

$\sigma_P = 0.9160(11)(8)_{sys}$

$\sigma_P = 0.9426(8)(17)_{sys}$

$\sigma_P = 0.9416(8)(9)_{sys}$

$\sigma_P = 0.9584(9)(17)_{sys}$

$\sigma_P = 0.9576(9)(9)_{sys}$

We will add the step $L/a = 16 \rightarrow 32$ to neglect coarsest point.

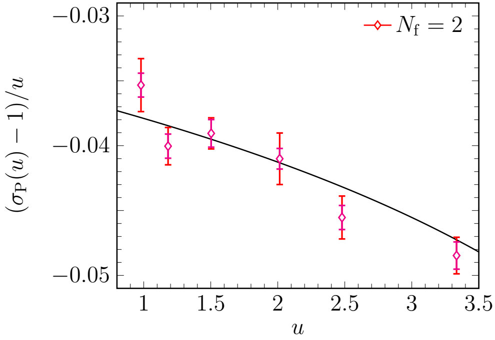
Step-scaling function II

¡PRELIMINARY!

- statistical accuracy in continuum limit: $\Delta[\sigma_{\mathrm{P}}] \lesssim 1‰$
- statistical error from tuning not propagated yet, but systematic deviation w.r.t. 2-pt weighted avg.: $\delta_{\mathrm{sys}} \lesssim 1‰$

Step-scaling function II

- statistical accuracy in continuum limit: $\Delta[\sigma_{\rm P}] \lesssim 1‰$
- statistical error from tuning not propagated yet, but systematic deviation w.r.t. 2-pt weighted avg.: $\delta_{\rm sys} \lesssim 1‰$

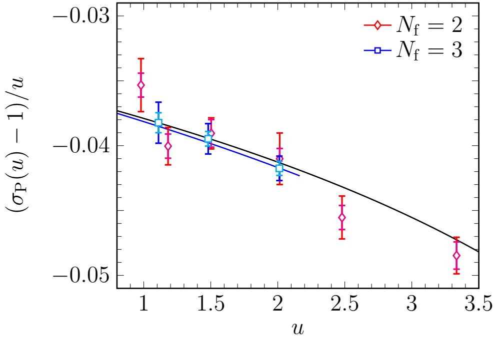


SSF: $N_{\rm f} = 2$ vs. $N_{\rm f} = 3$

- $N_{\rm f} = 2$ data from ALPHA'12 [2]
- innermost error = statistical
- outermost error = stat.+sys.
- solid line = 2/3-loop PT

Step-scaling function II

- statistical accuracy in continuum limit: $\Delta[\sigma_{\rm P}] \lesssim 1‰$
- statistical error from tuning not propagated yet, but systematic deviation w.r.t. 2-pt weighted avg.: $\delta_{\rm sys} \lesssim 1‰$



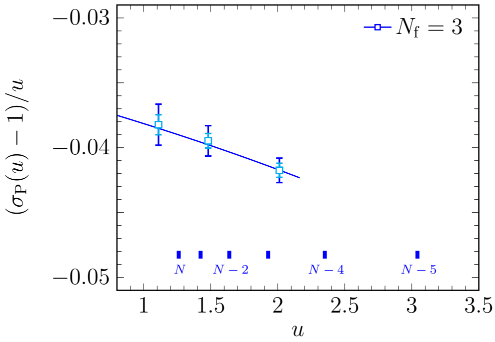
SSF: $N_{\rm f} = 2$ vs. $N_{\rm f} = 3$

- $N_{\rm f} = 2$ data from ALPHA'12 [2]
- innermost error = statistical
- outermost error = stat.+sys.
- solid line = 2/3-loop PT
- $N_{\rm f} = 3$ data, see previous slide
  - systematic errors seem better controlled than for $N_{\rm f} = 2$
  - 5 more data points on the way

Step-scaling function II

¡PRELIMINARY!

- statistical accuracy in continuum limit: $\Delta[\sigma_P] \lesssim 1‰$
- statistical error from tuning not propagated yet, but systematic deviation w.r.t. 2-pt weighted avg.: $\delta_{sys} \lesssim 1‰$



SSF: $N_f = 2$ vs. $N_f = 3$

- $N_f = 2$ data from ALPHA'12 [2]
- innermost error = statistical
- outermost error = stat.+sys.
- solid line = 2/3-loop PT
- $N_f = 3$ data, see previous slide
  - systematic errors seem better controlled than for $N_f = 2$
  - 5 more data points on the way

assuming $\Delta[\sigma_P(u_i)]_{total} = 2‰ \; (i = 1, \ldots, N)$ we can naively/conservatively expect:

$$\Delta[M/\overline{m}(\mu_{had})] \simeq \prod_i^N \Delta[\sigma_P(u_i)] \simeq \sqrt{N} \cdot 2‰ \simeq \begin{cases} 0.49\% & \text{for } N = 6 \quad \text{(as for } N_f = 2) \\ 0.63\% & \text{for } N = 10 \end{cases}$$

Recap & next steps

RG running mass in SF-coupling scheme:

- current results for $\sigma_{\mathrm{P}}(u_{\mathrm{SF}}^{i})$ (small-coupling regime) look very promising
- 5 remaining values from $L/a = 6, 8, 12$ within next 2-3 months
- better estimate of systematic errors after including $L/a = 16$ in the long term

. . . total error in this scheme may become negligble compared to large-couplung regime.

Recap & next steps

RG running mass in SF-coupling scheme:

- current results for $\sigma_{\rm P}(u^{i}_{\rm SF})$ (small-coupling regime) look very promising
- 5 remaining values from $L/a = 6, 8, 12$ within next 2-3 months
- better estimate of systematic errors after including $L/a = 16$ in the long term

. . . total error in this scheme may become negligible compared to large-couplung regime.

RG running mass in GF-coupling scheme: uncharted waters

- will share gauge configurations with running coupling project

$\rightsquigarrow$ correlations need to be taken into account

but data taking along the way almost for free

Recap & next steps

RG running mass in SF-coupling scheme:

- current results for $\sigma_{\mathrm{P}}(u^{i}_{\mathrm{SF}})$ (small-coupling regime) look very promising
- 5 remaining values from $L/a = 6, 8, 12$ within next 2-3 months
- better estimate of systematic errors after including $L/a = 16$ in the long term

... total error in this scheme may become negligible compared to large-couplung regime.

RG running mass in GF-coupling scheme: uncharted waters

- will share gauge configurations with running coupling project

  $\rightsquigarrow$ correlations need to be taken into account

  but data taking along the way almost for free
- simulations get more expensive towards larger volumes / couplings

  $\rightsquigarrow$ impact on statistics unknown as of yet

Recap & next steps

RG running mass in SF-coupling scheme:

- current results for $\sigma_{\mathrm{P}}(u^{i}_{\mathrm{SF}})$ (small-coupling regime) look very promising
- 5 remaining values from $L/a = 6, 8, 12$ within next 2-3 months
- better estimate of systematic errors after including $L/a = 16$ in the long term

…total error in this scheme may become negligible compared to large-couplung regime.

RG running mass in GF-coupling scheme: uncharted waters

- will share gauge configurations with running coupling project
  $\rightsquigarrow$ correlations need to be taken into account
  but data taking along the way almost for free
- simulations get more expensive towards larger volumes / couplings
  $\rightsquigarrow$ impact on statistics unknown as of yet
- 1-loop improvement term δ_{P} not known $\rightsquigarrow$ larger systematic errors?

Recap & next steps

RG running mass in SF-coupling scheme:

- current results for $\sigma_{\mathrm{P}}(u^i_{\mathrm{SF}})$ (small-coupling regime) look very promising
- 5 remaining values from $L/a = 6, 8, 12$ within next 2-3 months
- better estimate of systematic errors after including $L/a = 16$ in the long term

. . . total error in this scheme may become negligible compared to large-couplung regime.

RG running mass in GF-coupling scheme: uncharted waters

- will share gauge configurations with running coupling project
  $\rightsquigarrow$ correlations need to be taken into account
  but data taking along the way almost for free
- simulations get more expensive towards larger volumes / couplings
  $\rightsquigarrow$ impact on statistics unknown as of yet
- 1-loop improvement term δ_{P} not known $\rightsquigarrow$ larger systematic errors?

ToDo:

- better understanding of NP scheme-switching step on general grounds, . . .

Summary

presented current status of RG running part to compute RGI quark masses in 3-flavour QCD

- standard techniques used and systematically improved $(\kappa_{\rm crit}(\beta), \ldots)$
- more complicated RG pattern due to scheme-switch at intermediate scale $\mu_{\rm swi}$
- results in SF-scheme at small coupling are very encouraging
- an error reduction by 50% compared to $N_{\rm f} = 2$ seems possible
- other projects, such as HQET $(m_{\rm b})$, will profit directly from accurate estimates of $M/\overline{m}(\mu_{\rm had})$

Summary

presented current status of RG running part to compute RGI quark masses in 3-flavour QCD

- standard techniques used and systematically improved $(\kappa_{\rm crit}(\beta), \ldots)$
- more complicated RG pattern due to scheme-switch at intermediate scale $\mu_{\rm swi}$
- results in SF-scheme at small coupling are very encouraging
- an error reduction by 50% compared to $N_{\rm f} = 2$ seems possible
- other projects, such as HQET $(m_{\rm b})$, will profit directly from accurate estimates of $M/\overline{m}(\mu_{\rm had})$

THANK YOU FOR YOUR ATTENTION!

[1] S. Capitani, M. Lüscher, R. Sommer, and H. Wittig, *Nonperturbative quark mass renormalization in quenched lattice QCD*, *Nucl.Phys.* **B544** (1999) 669–698, [hep-lat/9810063].

[2] M. Della Morte et al., *Non-perturbative quark mass renormalization in two-flavor QCD*, *Nucl.Phys.* **B729** (2005) 117–134, [hep-lat/0507035].

[3] P. Fritzsch, F. Knechtli, B. Leder, M. Marinkovic, S. Schaefer, et al., *The strange quark mass and Lambda parameter of two flavor QCD*, *Nucl.Phys.* **B865** (2012) 397–429, [arXiv:1205.5380].

[4] T. Ishikawa et al., *Light quark masses from unquenched lattice QCD*, *Phys. Rev.* **D78** (2008) 011502, [arXiv:0704.1937].

[5] S. Aoki et al., *Non-perturbative renormalization of quark mass in $N_f = 2 + 1$ QCD with the Schrödinger functional scheme*, *JHEP* **1008** (2010) 101, [arXiv:1006.1164].

[6] M. Dalla Brida, P. Fritzsch, T. Korzec, A. Ramos, S. Sint, et al., *Towards a new determination of the QCD Lambda parameter from running couplings in the three-flavour theory*, *PoS* **LATTICE2014** (2014) 291, [arXiv:1411.7648].