Curvature of the pseudocritical line in (2+1)-flavor QCD with HISQ fermions

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in collaboration with: Paolo Cea, Alessandro Papa

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Outline

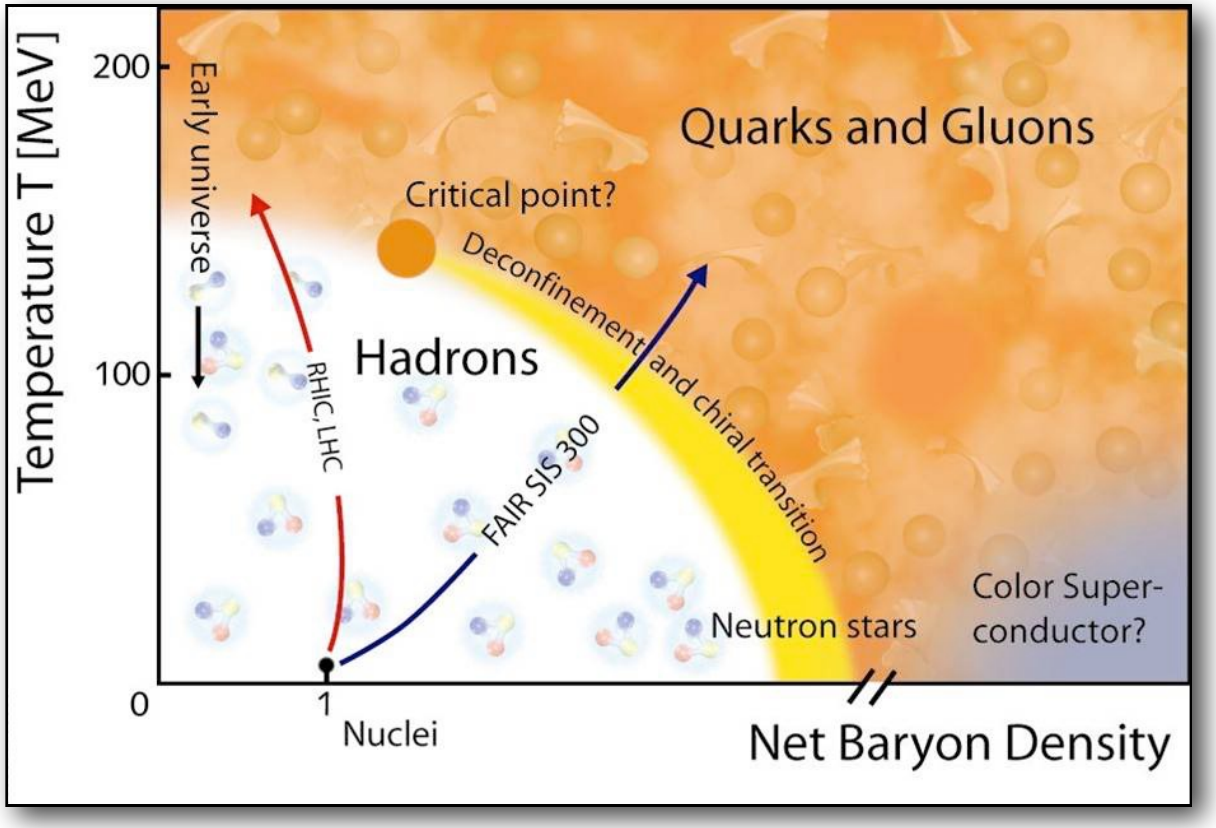
- Introduction
- Lattice setup and numerical simulations
- Results
- Conclusions

Acknowledgements

- This work has been partially supported by the INFN **SUMA** (**SU**per **MA**ssive Computing) Project
- Simulations have been performed on BlueGene/Q "Fermi" at **CINECA** (ISCRA-B HP10BZOO8Q and CINECA-INFN agreement).
- This work was in part based on the **MILC** collaboration's public lattice gauge theory code. See `http://physics.utah.edu/~detar/milc.html`

Introduction

- Lattice QCD simulations at non zero temperature and baryon chemical potential to locate the QCD (pseudo)critical line.



- "***Sign problem***" : possible way out ***analytic continuation*** from ***imaginary*** chemical potential (other methods: reweighting from the ensemble at $\mu_B=0$, the Taylor expansion method, the canonical approach, the density of states method).

- The QCD (pseudo)critical line can be parameterized by a lowest order Taylor expansion in the baryon chemical potential:

$$\frac{T(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T(\mu_B)} \right)^2$$

- Our aim: Estimate the curvature of the (pseudo)critical line of (2+1) flavor QCD using the method of analytic continuation.

Lattice setup and numerical simulation

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- We work on a ***line of constant physics (LCP)*** determined (*) by fixing the ***strange quark mass*** to its physical value m_s at each value of the gauge coupling β. The ***light-quark mass*** has been fixed at $m_l = m_s/20$. ($M_\pi = 160$ MeV)

  (*) as determined in A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012))

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- To perform numerical simulations we used the **MILC** code suitably modified in order to introduce an imaginary quark chemical potential $\mu=\mu_B/3$.

  That has been done by multiplying ***all forward and backward temporal links*** entering the discretized Dirac operator by ***exp(iaμ)*** and ***exp(–iaμ)***, respectively.

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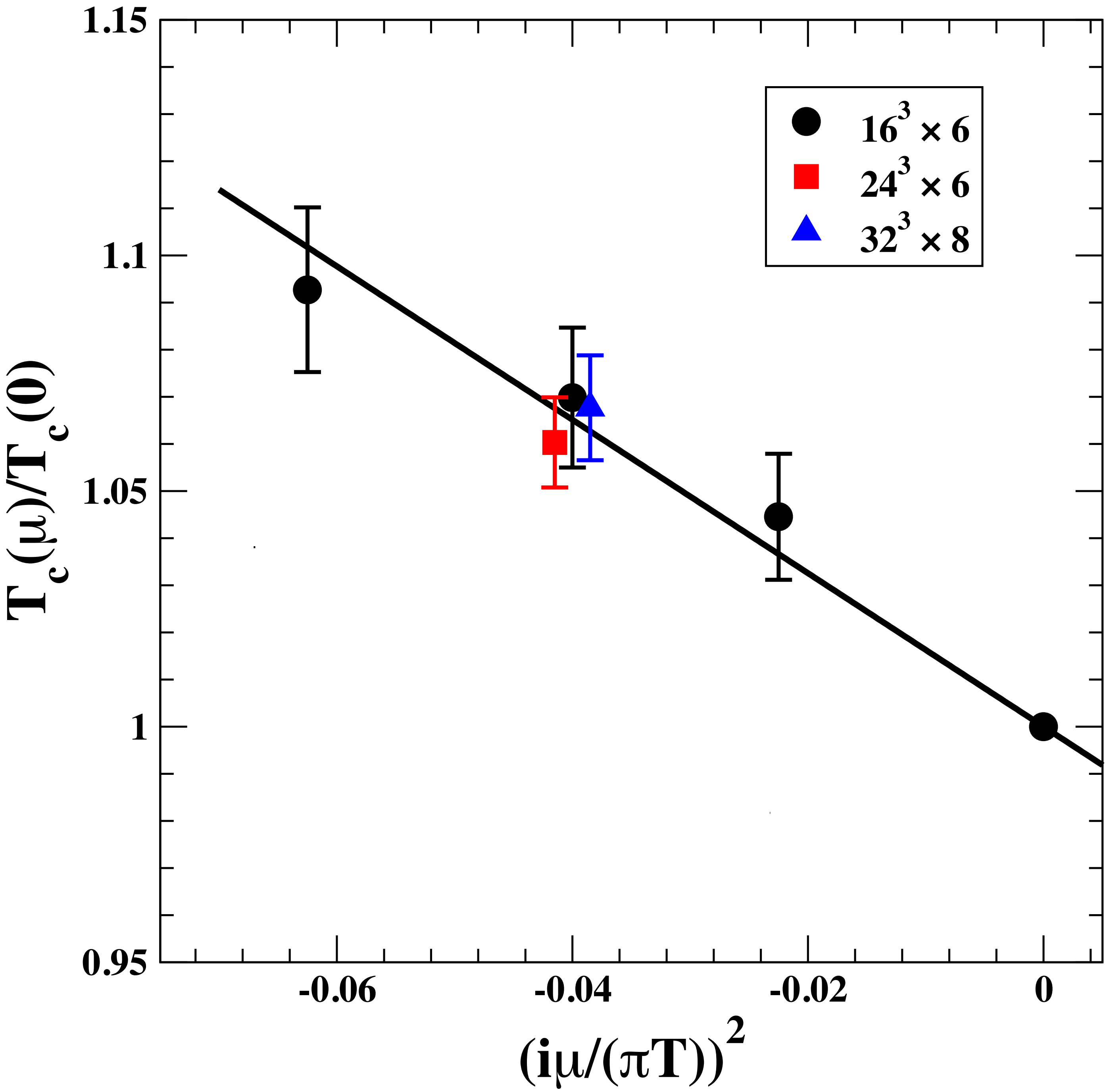
- All simulations make use of the rational hybrid Monte Carlo (RHMC) algorithm. (The length of each RHMC trajectory has been set to 1.0 in molecular dynamics time units.)

Our previous result:

P. Cea, L. C., A. Papa, Phys. Rev. D 89, 074512 (2014) [arXiv:1403.0821 [hep-lat]]



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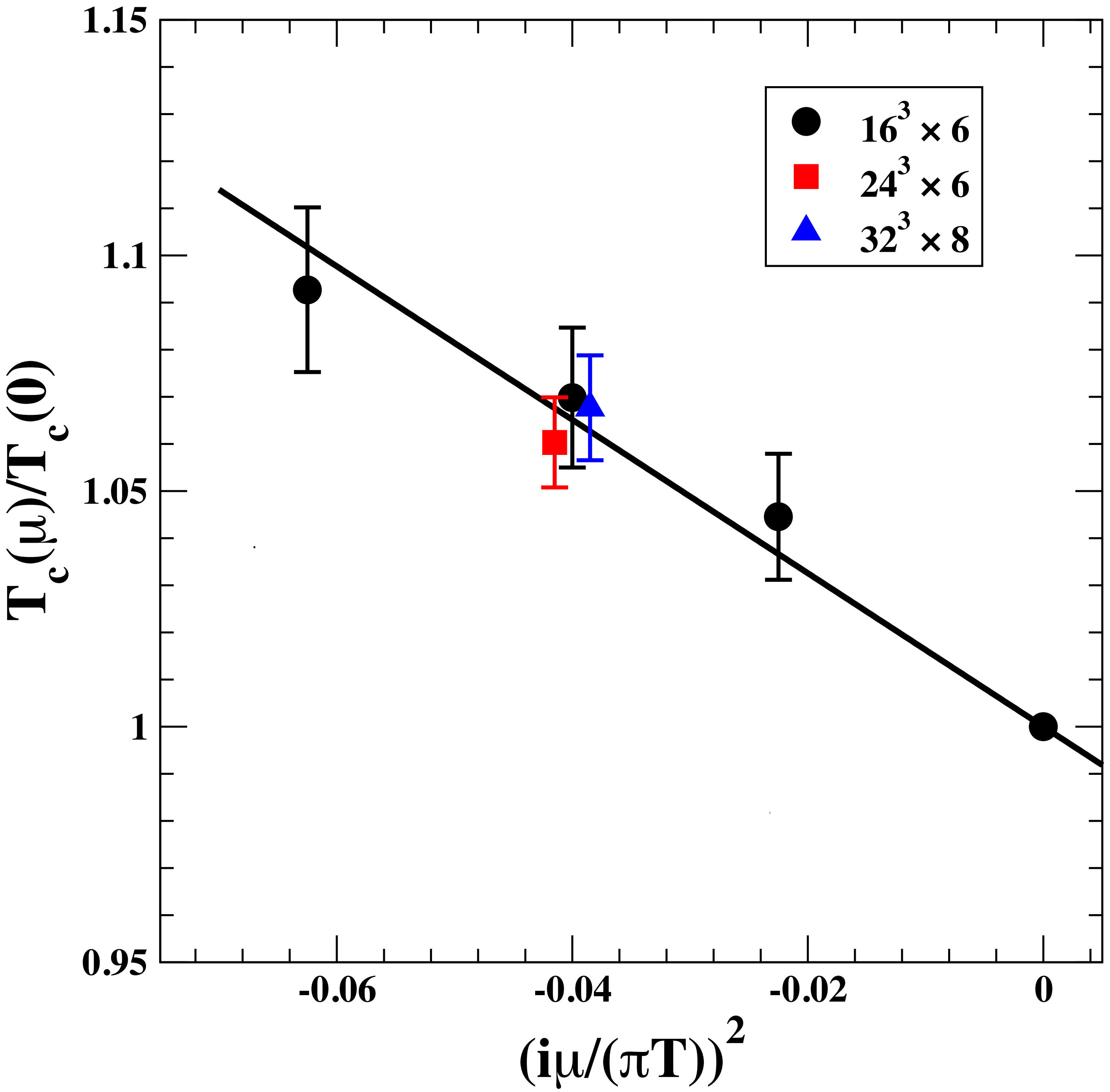

Linear fit (in μ²) to the data

$$\frac{T_c(\mu)}{T_c(0)} = 1 + R_q \left(\frac{i\mu}{\pi T_c(\mu)} \right)^2$$

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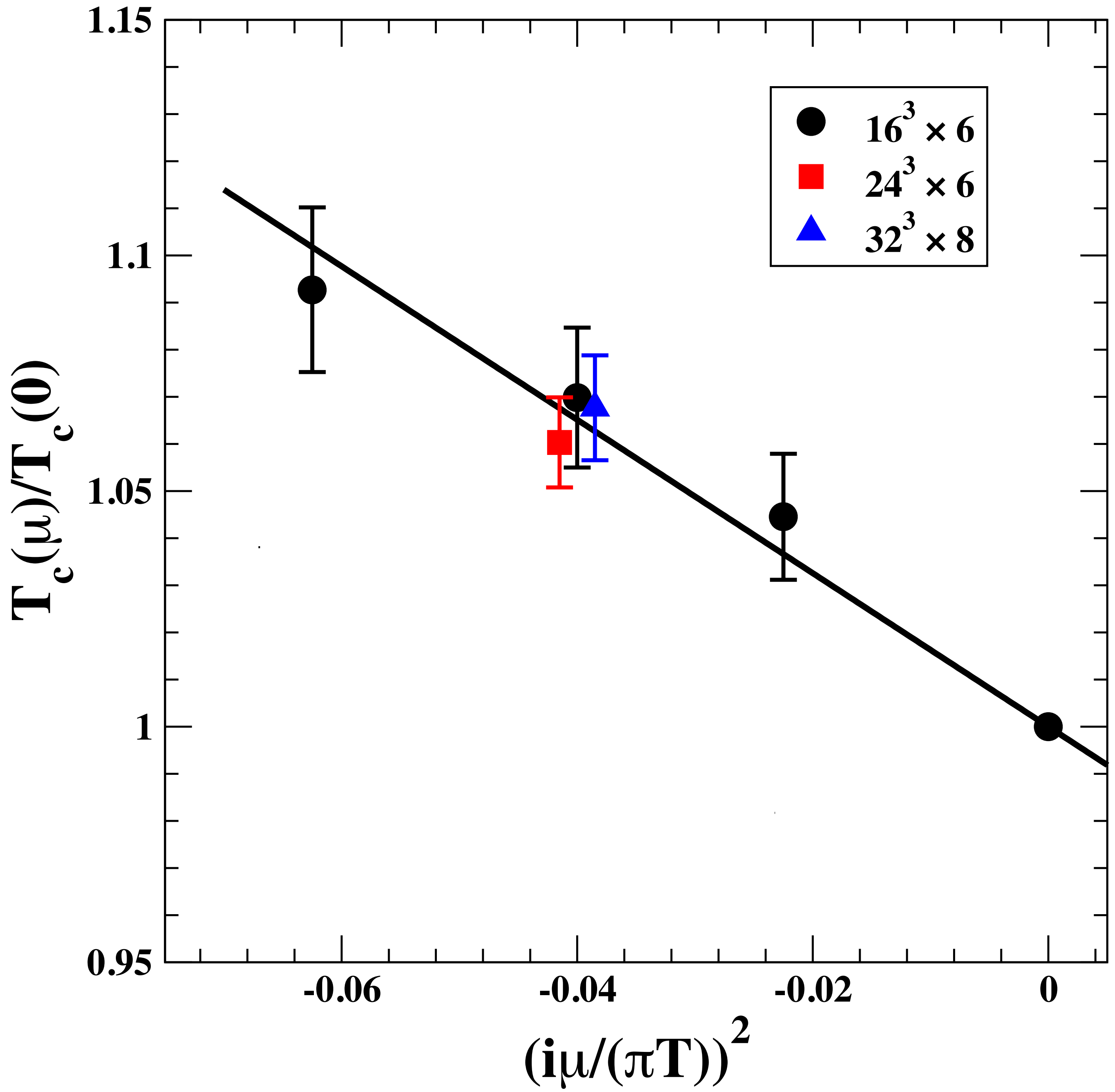

Linear fit (in μ^2) to the data

$$\frac{T_c(\mu)}{T_c(0)} = 1 + R_q \left(\frac{i\mu}{\pi T_c(\mu)} \right)^2$$

curvature of the pseudocritical line:

$$\kappa = -\frac{R_q}{9\pi^2} = 0.018(4)$$

- Table of simulations performed

lattice	$\mu/(\pi T)$
$16^3 \times 6$	$0.15i$
	$0.2i$
	$0.25i$
$24^3 \times 6$	$0.2i$
$32^3 \times 8$	$0.15i$
	$0.2i$
	$0.25i$
$40^3 \times 10$	$0.15i$
	$0.2i$
	$0.25i$
$48^3 \times 12$	$0.20i$
	$0.25i$

- We have typically discarded not less than 1000 trajectories for each run and have collected from 4000 to 8000 trajectories for measurements.

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- To determine the pseudocritical line we consider the **disconnected susceptibility of the light quark chiral condensate**

$$\frac{1}{Z_m^2}\frac{\chi_{l,disc}}{T^2}$$

$$Z_m(\beta) = \frac{m_l(\beta)}{m_l(\beta^\star)} \qquad T = \frac{1}{a(\beta)L_t}$$

$$\chi_{q,disc} = \frac{n_f^2}{16 N_\sigma^3 N_\tau}\left\{\langle\left(\mathrm{Tr}D_q^{-1}\right)^2\rangle - \langle\mathrm{Tr}D_q^{-1}\rangle^2\right\}$$

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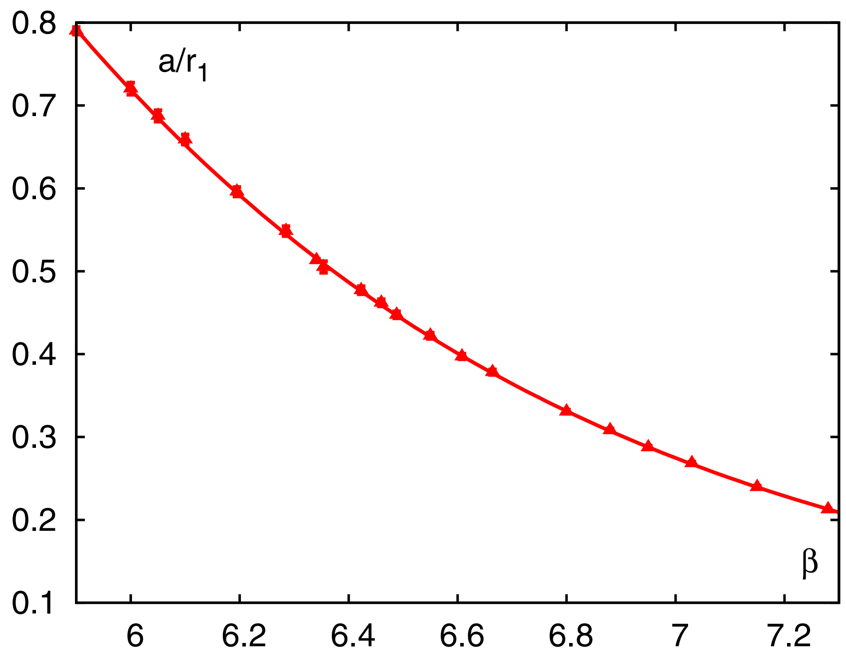
- We need to set the lattice scale in order to get the temperature at a given gauge coupling.

Setting the lattice scale

The lattice spacing can be determined using the slope of the static quark-antiquark potential on zero-temperature lattices or the value of the decay constant f_K (we use results of HotQCD collaboration (*)).



$$a(\beta)|_{m_l=0.05m_s} = r_1 \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}$$

$$r_1 = 0.3106 \text{ fm}$$
$$c_0 = 44.06$$
$$c_2 = 272102$$
$$d_2 = 4281$$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$$

b_0, b_1 coefficients of the universal two-loop beta function

(*) as discussed in Appendix B of A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

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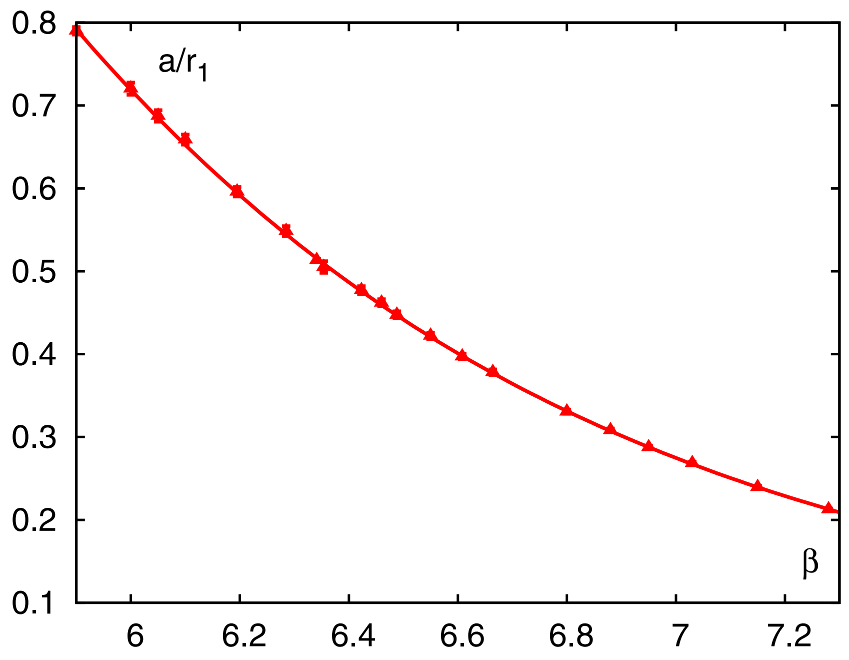

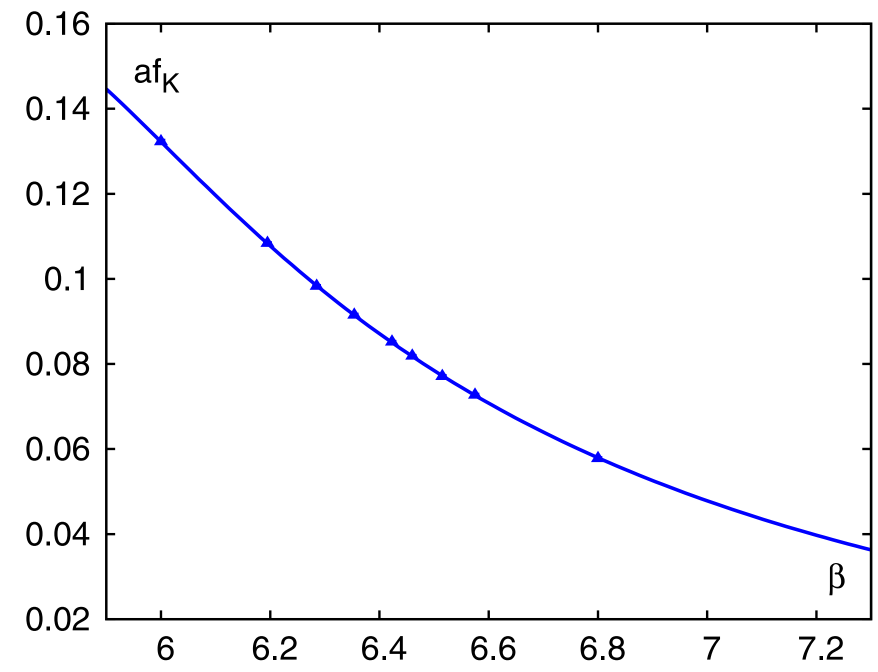

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$r_1 = 0.3106$ fm
$c_0 = 44.06$
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$$a f_K(\beta)_{m_l=0.05m_s} = \frac{c_0^K f(\beta) + c_2^K(10/\beta) f^3(\beta)}{1 + d_2^K(10/\beta) f^2(\beta)}$$

$r_1 f_K = 0.1738$
$c_0^K = 7.66$
$c_2^K = 32911$
$d_2^K = 2388$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$$

b_0, b_1 coefficients of the universal two-loop beta function

(*) as discussed in Appendix B of A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

Numerical results: $T_c(\mu)/T_c(0)$

- The (pseudo)critical line $T_c(\mu)$ has been determined as the value for which the **renormalized disconnected susceptibility of the light quark chiral condensate** exhibits a peak
- **To localize the peak**, a Lorentzian fit has been used: $$\frac{a_1}{1 + a_2(T - T_c)^2}$$

$$\frac{1}{Z_m^2}\frac{\chi_{\text{light}}}{T^2}$$

$$Z_m = \frac{m_{\text{light}}(\beta)}{m_{\text{light}}(\beta^\star)}$$

$$T = \frac{1}{a(\beta)L_t}$$

$$\frac{r_1}{a(\beta^\star)} = 2.37$$

$$\beta^\star = 6.54706 \;\; (r_1 \text{ scale})$$

$$\beta^\star = 6.56778 \;\; (f_K \text{ scale})$$

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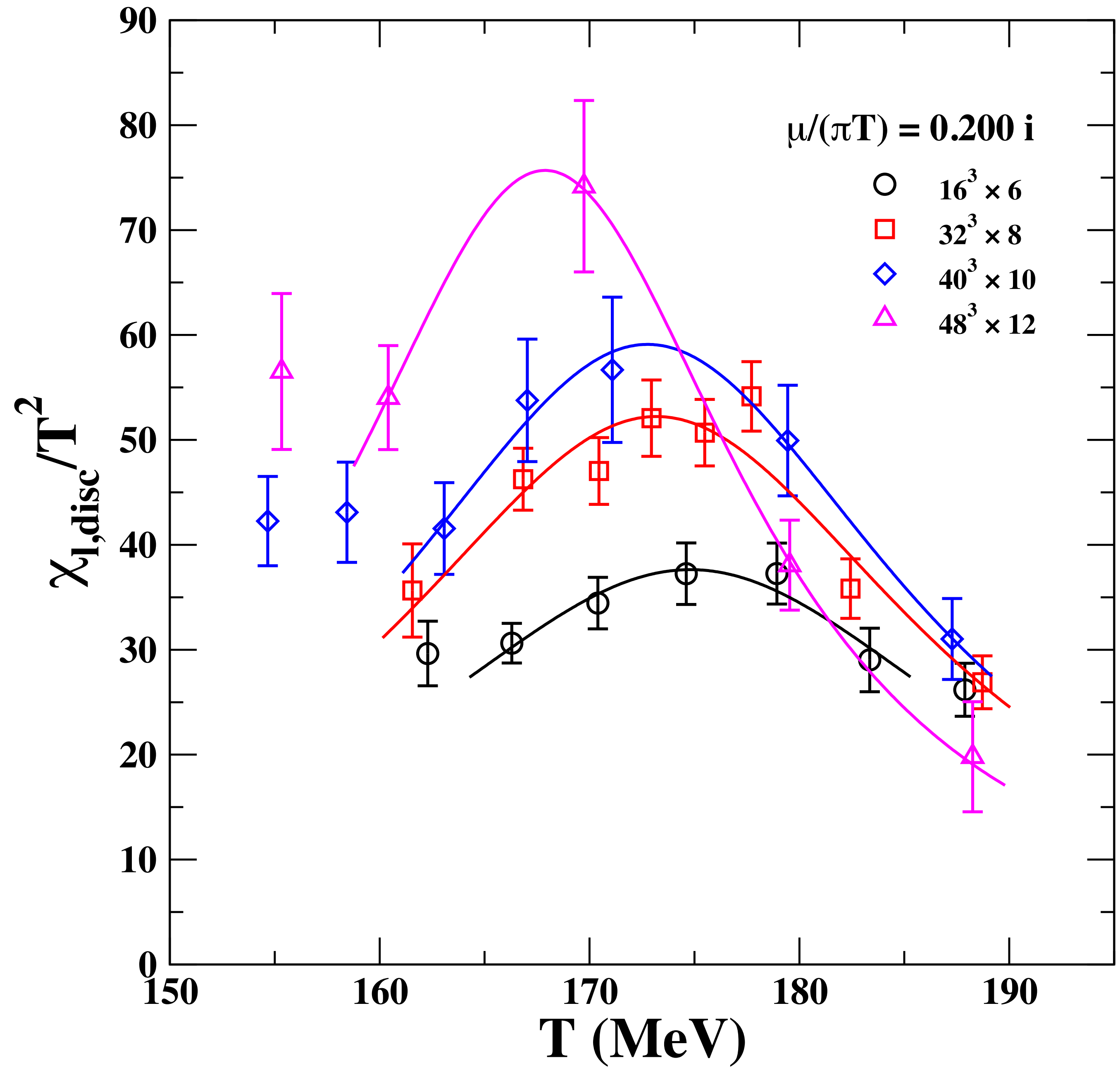

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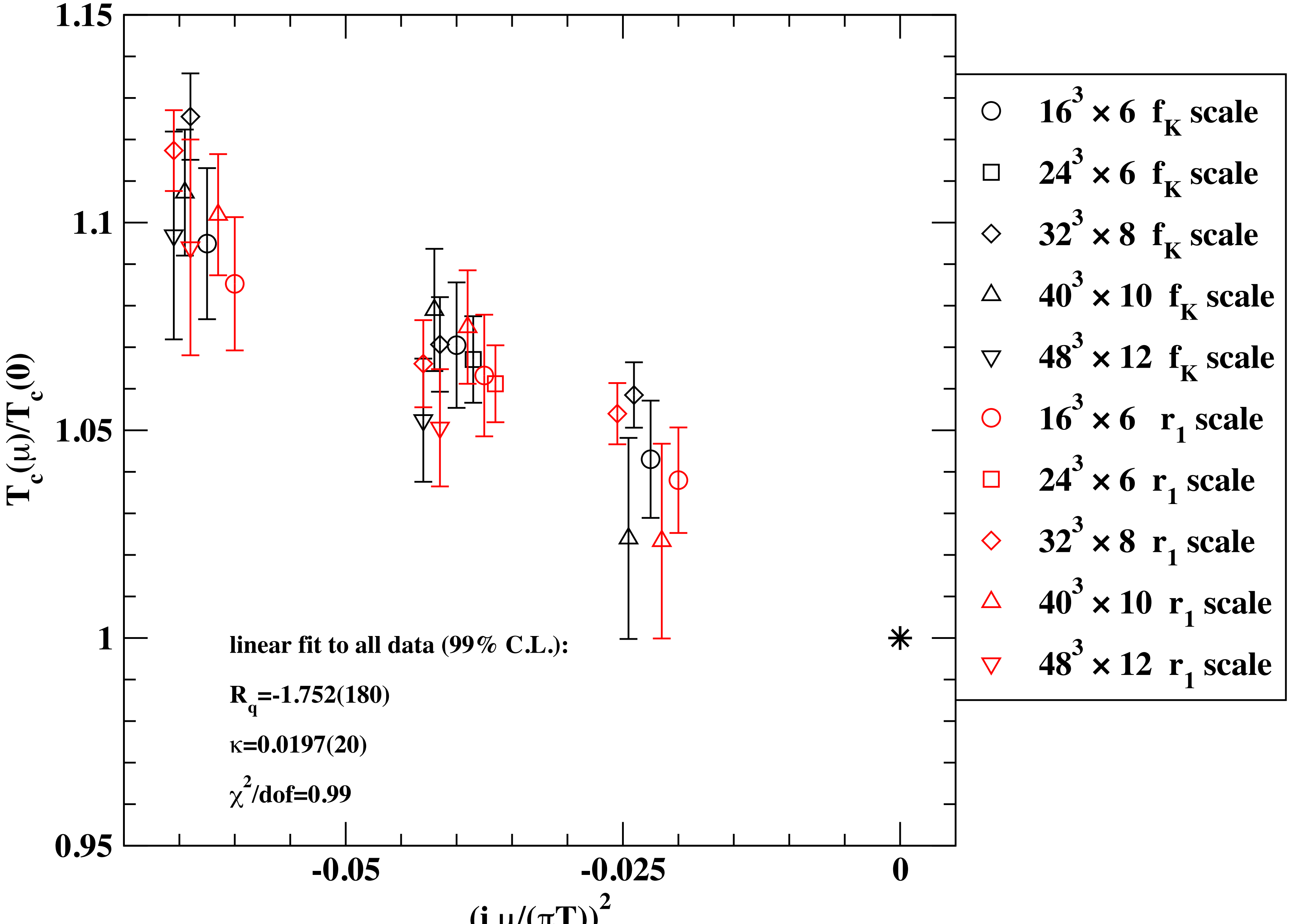
$16^3 \times 6$	$0.15i$	1.038(13)	1.043(14)
	$0.2i$	1.063(15)	1.070(15)
	$0.25i$	1.085(16)	1.095(18)
$24^3 \times 6$	$0.2i$	1.061(9)	1.067(10)
$32^3 \times 8$	$0.15i$	1.054(7)	1.059(8)
	$0.2i$	1.066(10)	1.071(11)
	$0.25i$	1.117(10)	1.126(10)
$40^3 \times 10$	$0.15i$	1.023(23)	1.024(24)
	$0.2i$	1.075(14)	1.079(15)
	$0.25i$	1.102(15)	1.107(15)
$48^3 \times 12$	$0.20i$	1.051(14)	1.052(15)
	$0.25i$	1.094(26)	1.097(25)

(*) To estimate $T_c(0)$ on lattices $24^3\times6$, $32^3\times8$, $40^3\times10$, $48^3\times12$ we take data for disconnected light chiral susceptibility form Table X, XI, XII of A.Bazavov et al (HotQCD Collaboration) arXiv:1111.1710 Phys. Rev. D 85, 054503 (2012) and from Table XI of A.Bazavov et al (HotQCD Collaboration) arXiv:1407.6387 Phys. Rev. D 90, 094503 (2014)



1.15
1.1
1.05
1
0.95
-0.05
-0.025
0
$T_c(\mu)/T_c(0)$
$(i\,\mu/(\pi T))^2$
linear fit to all data (99% C.L.):
R_q=-1.752(180)
κ=0.0197(20)
χ^2/dof=0.99
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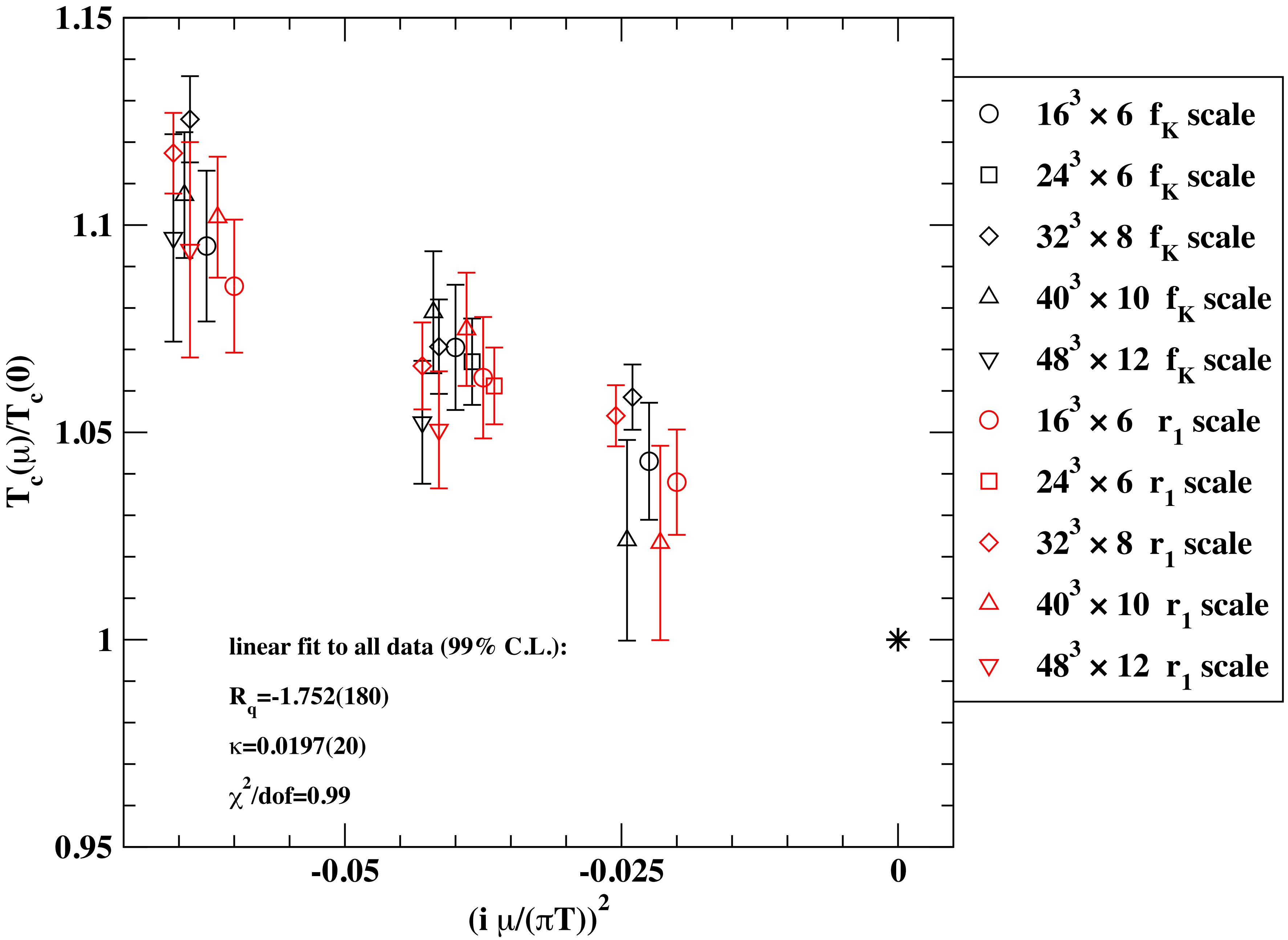

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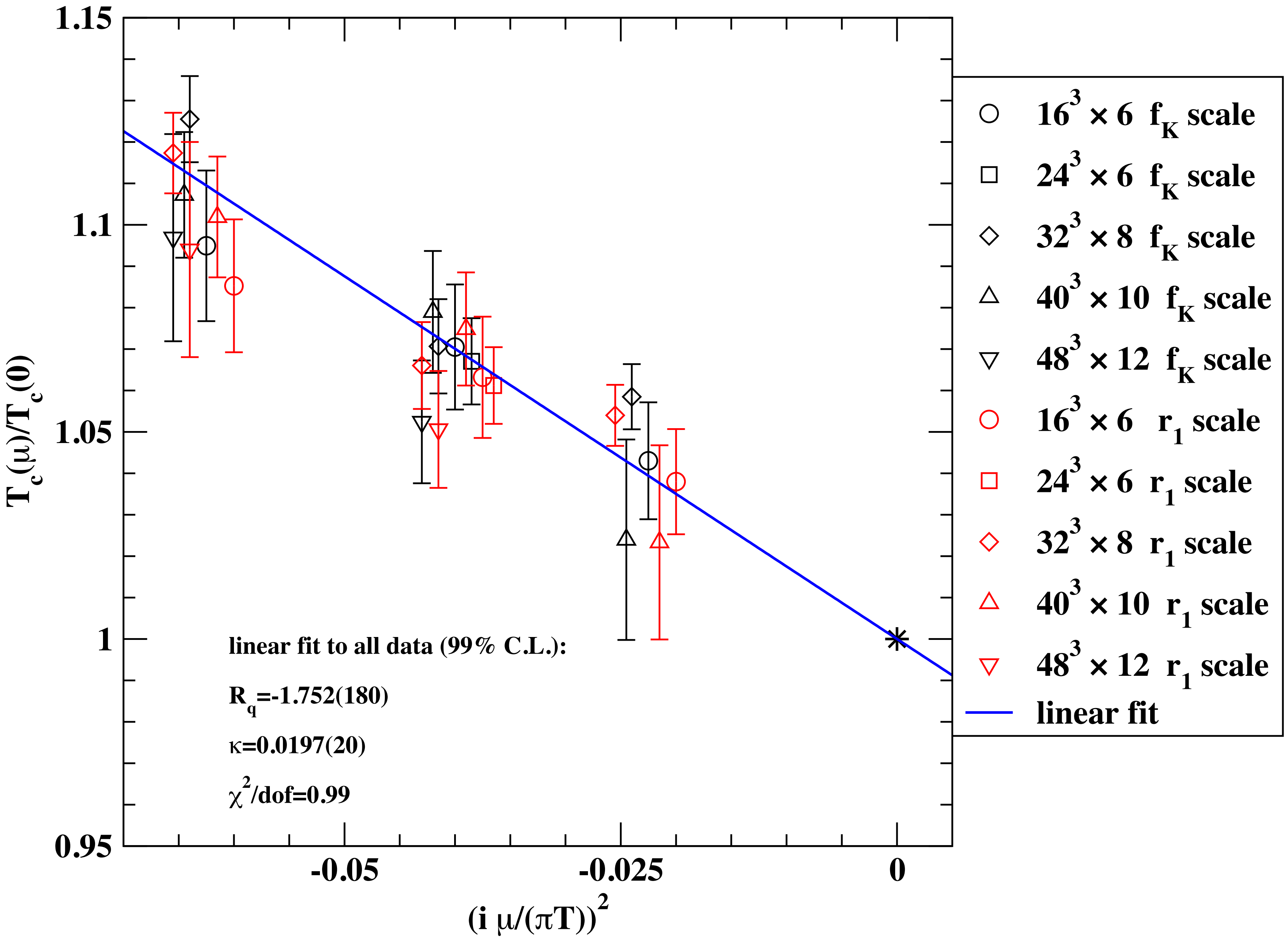
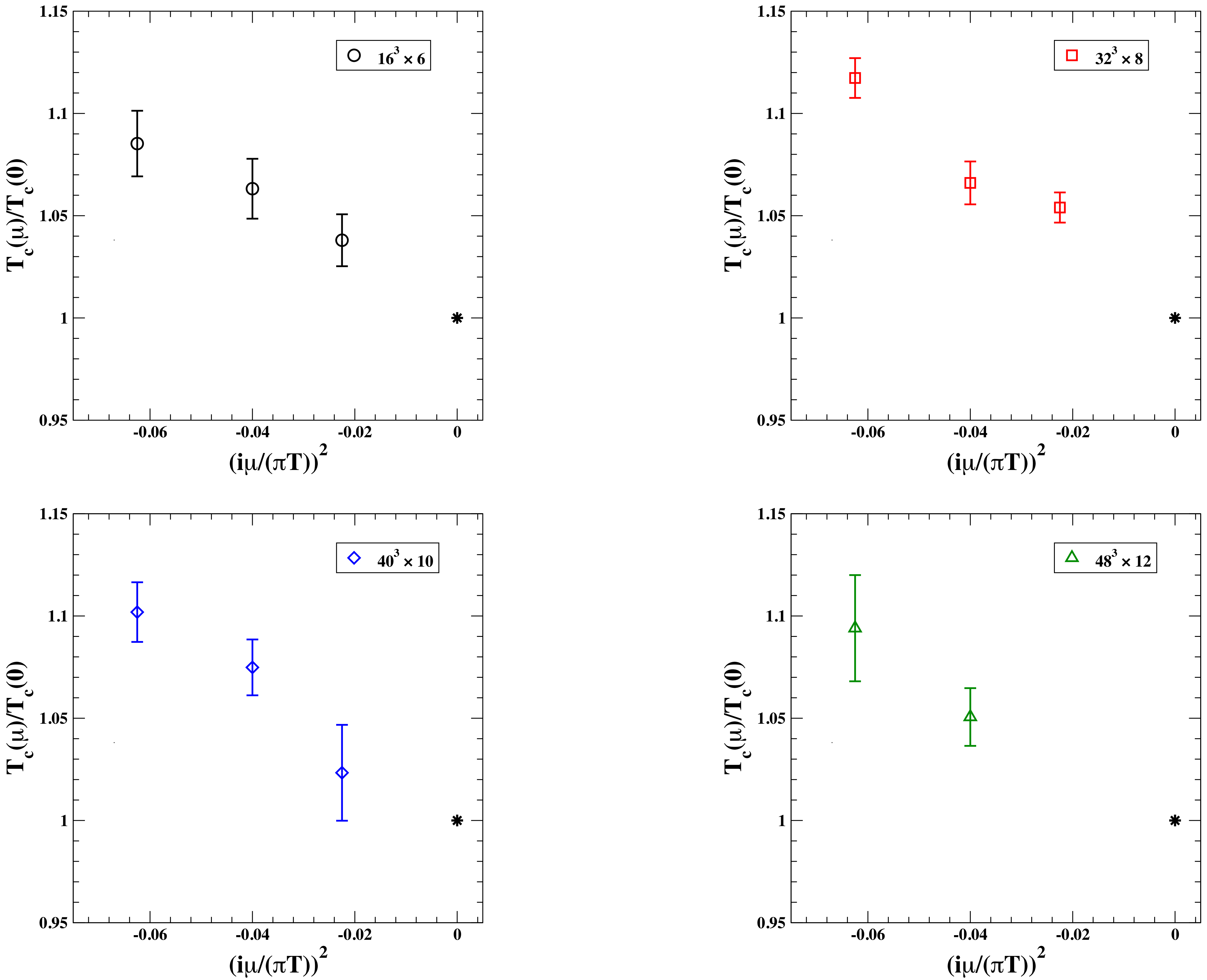
$$R_q = -1.7518(1805)$$

$$\kappa = -\frac{R_q}{9\pi^2} = 0.0197(20)$$



Linear fit (in μ^2) to data at fixed L_t



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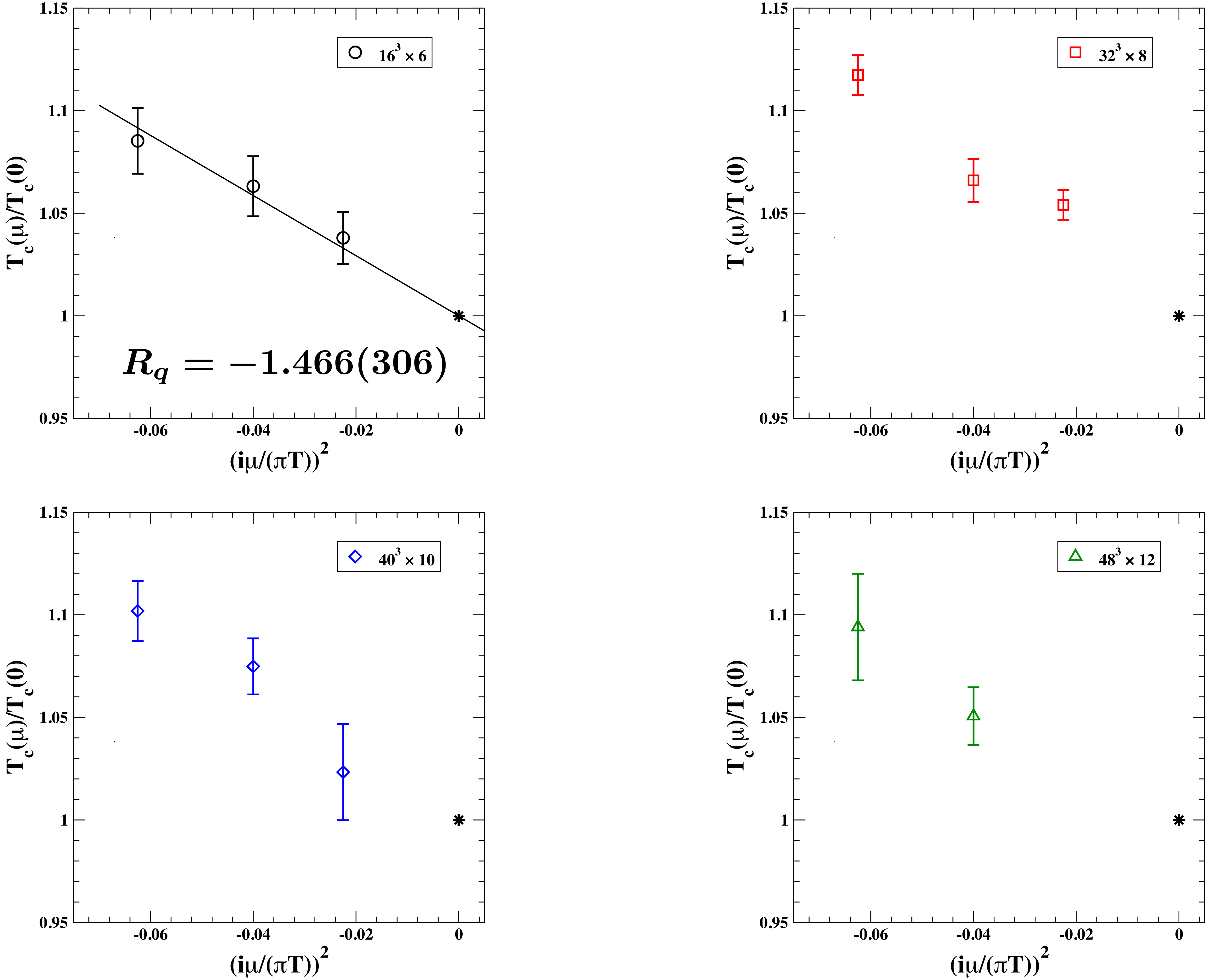

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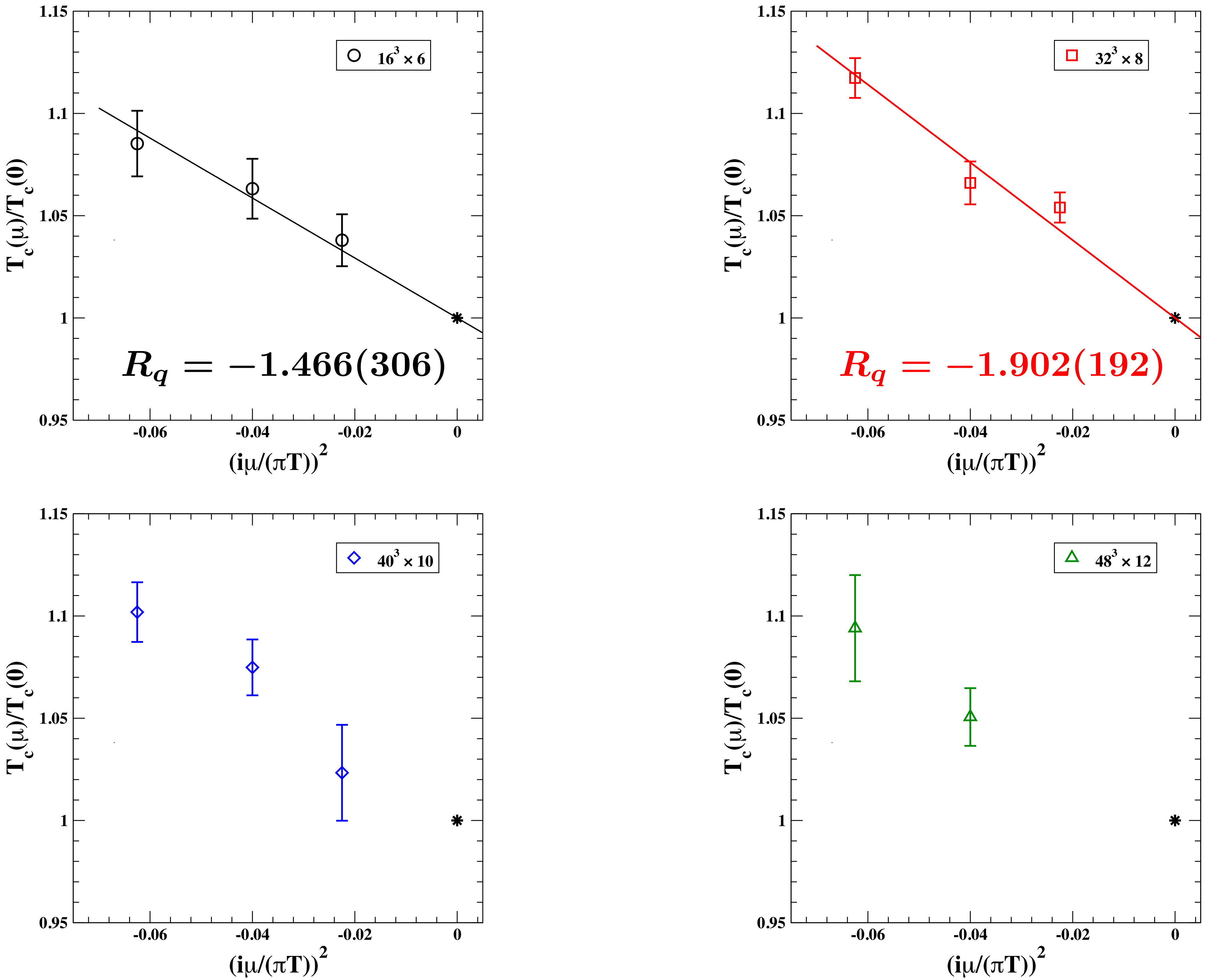

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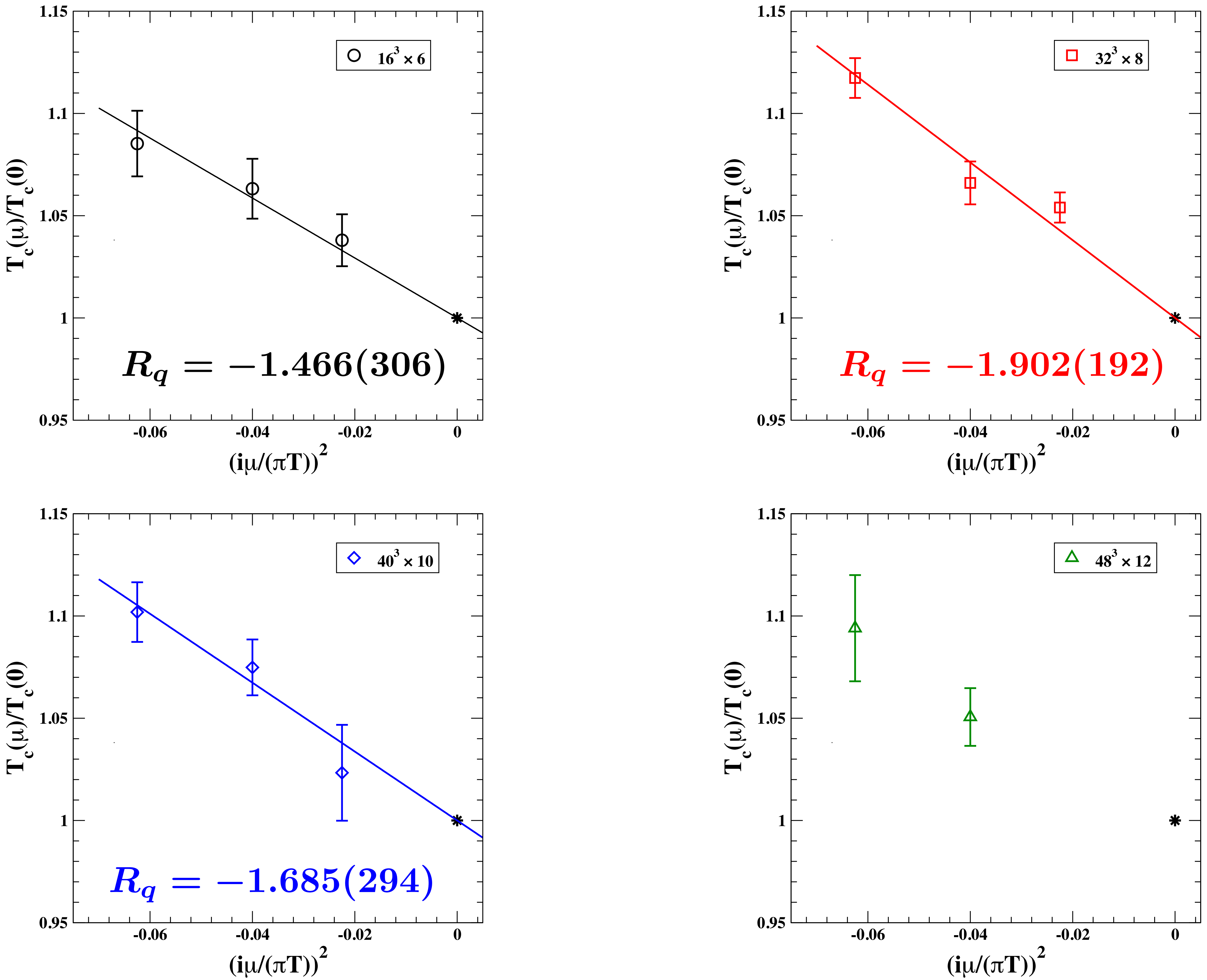

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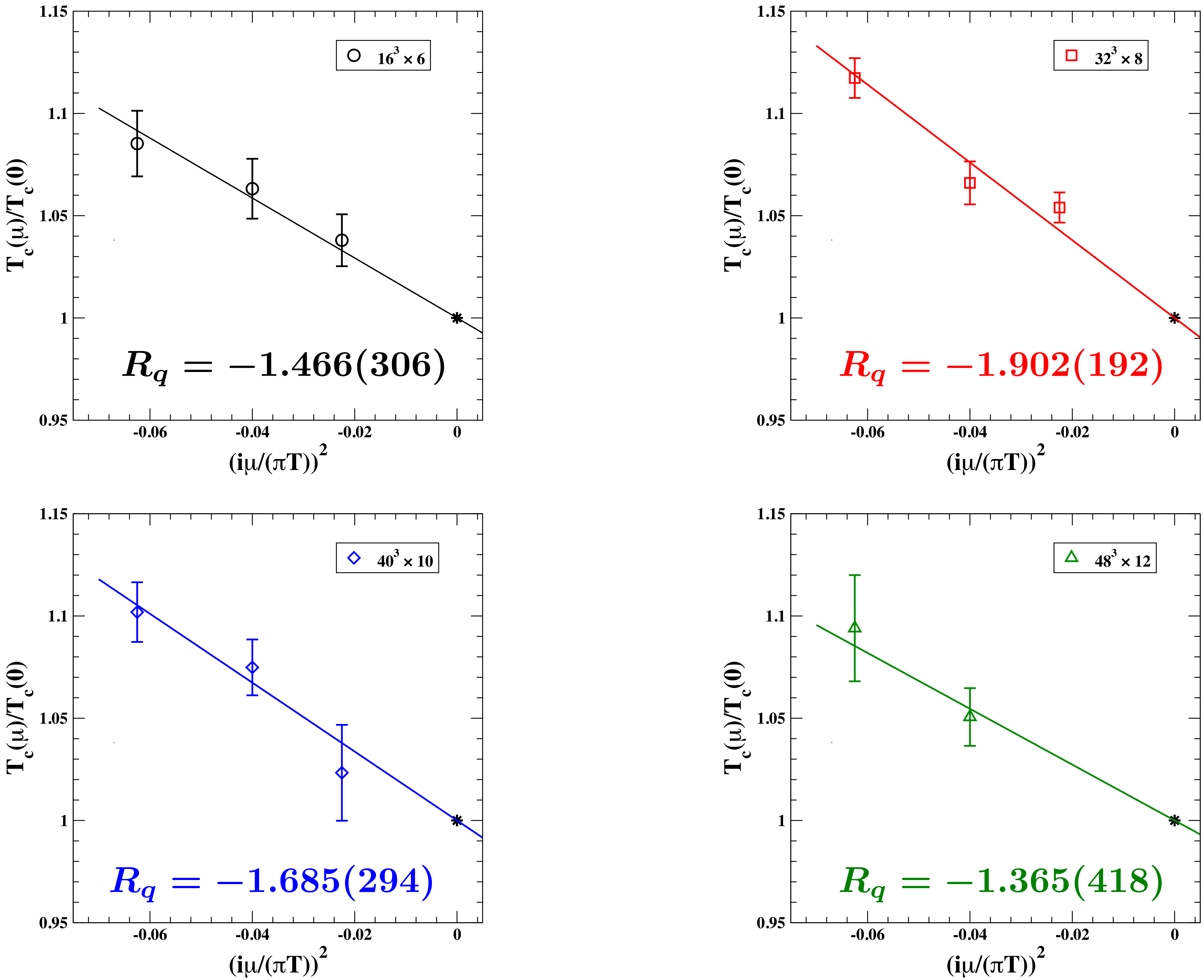
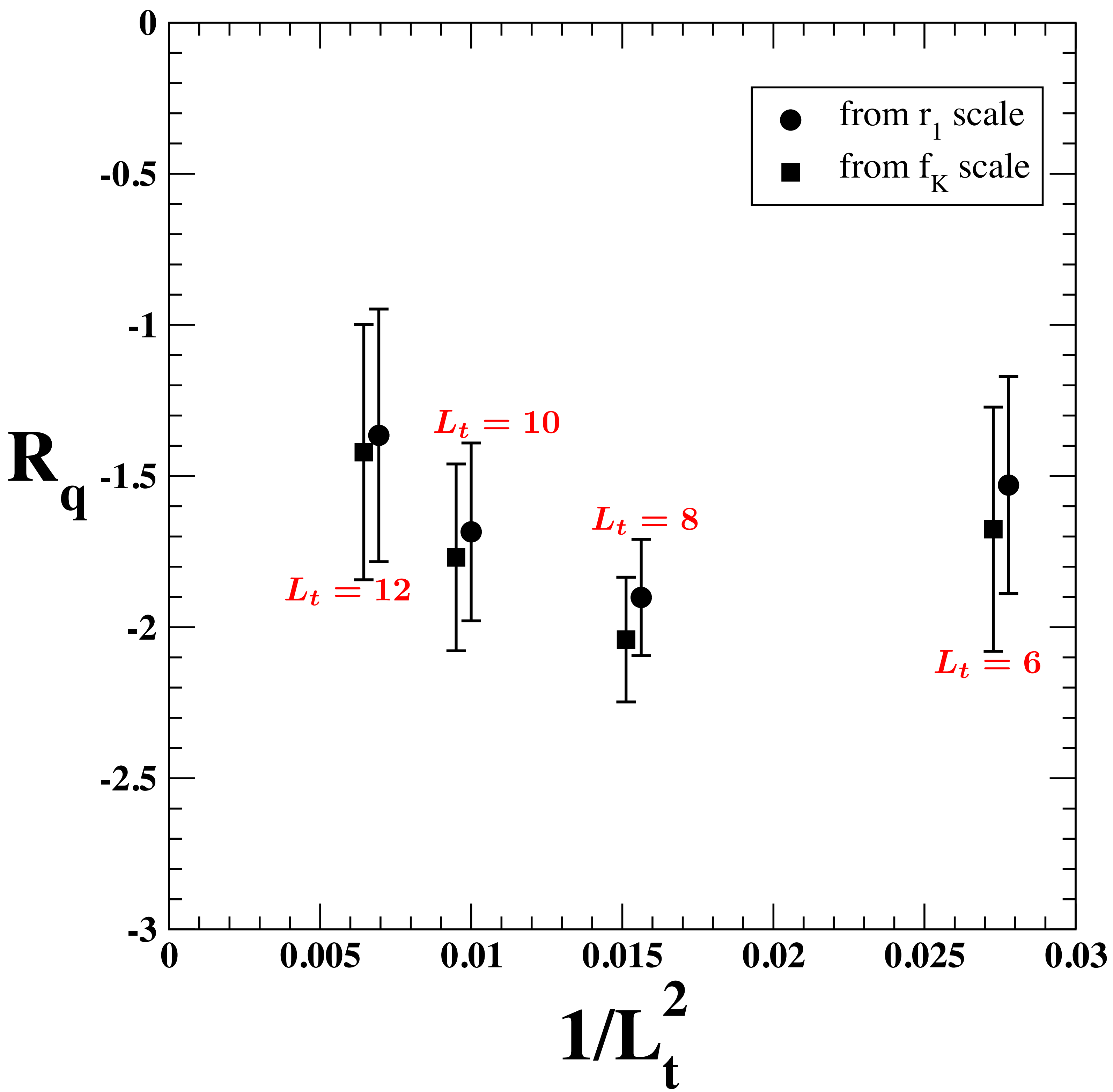


The curvature of the pseudocritical line



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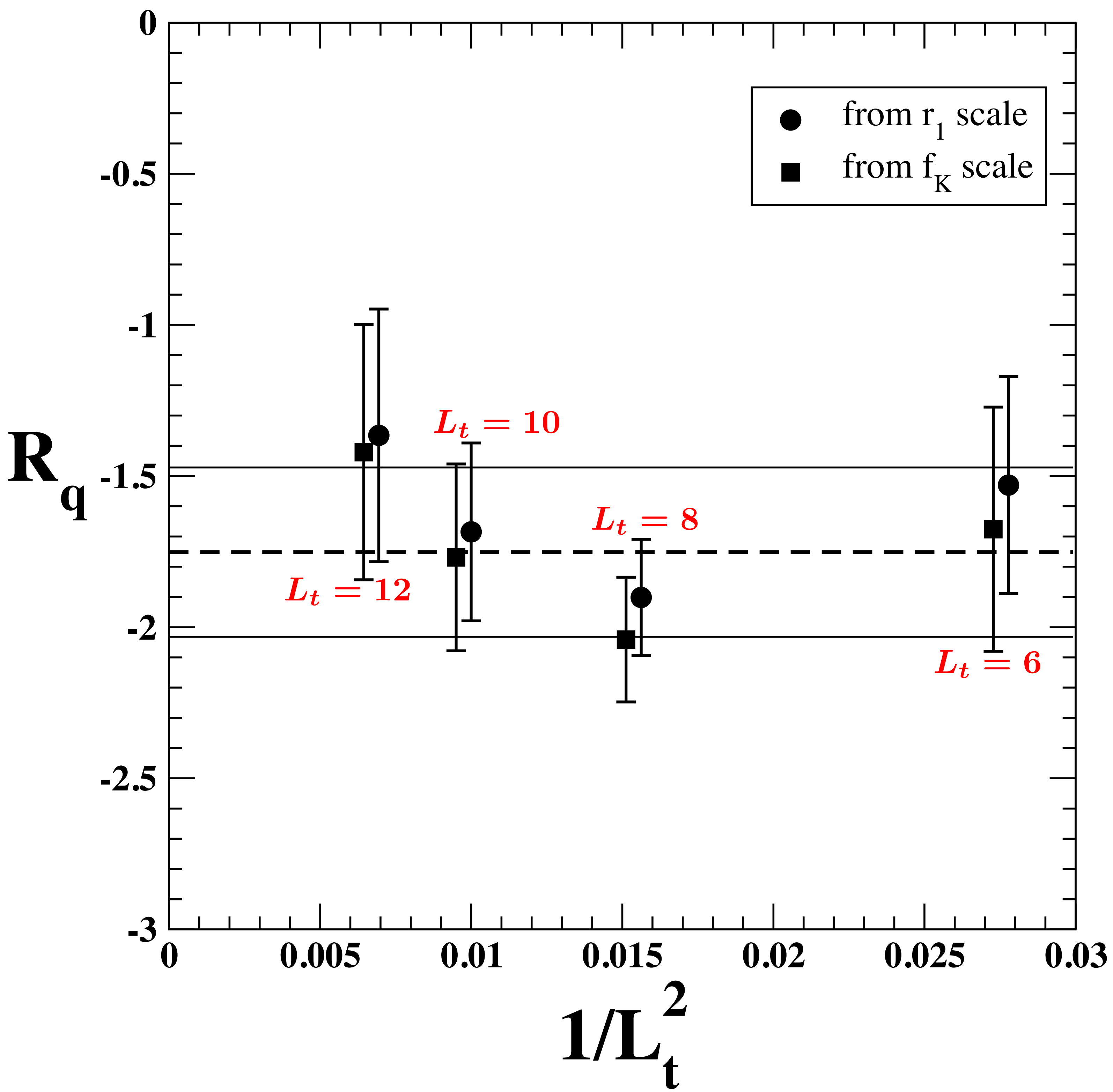
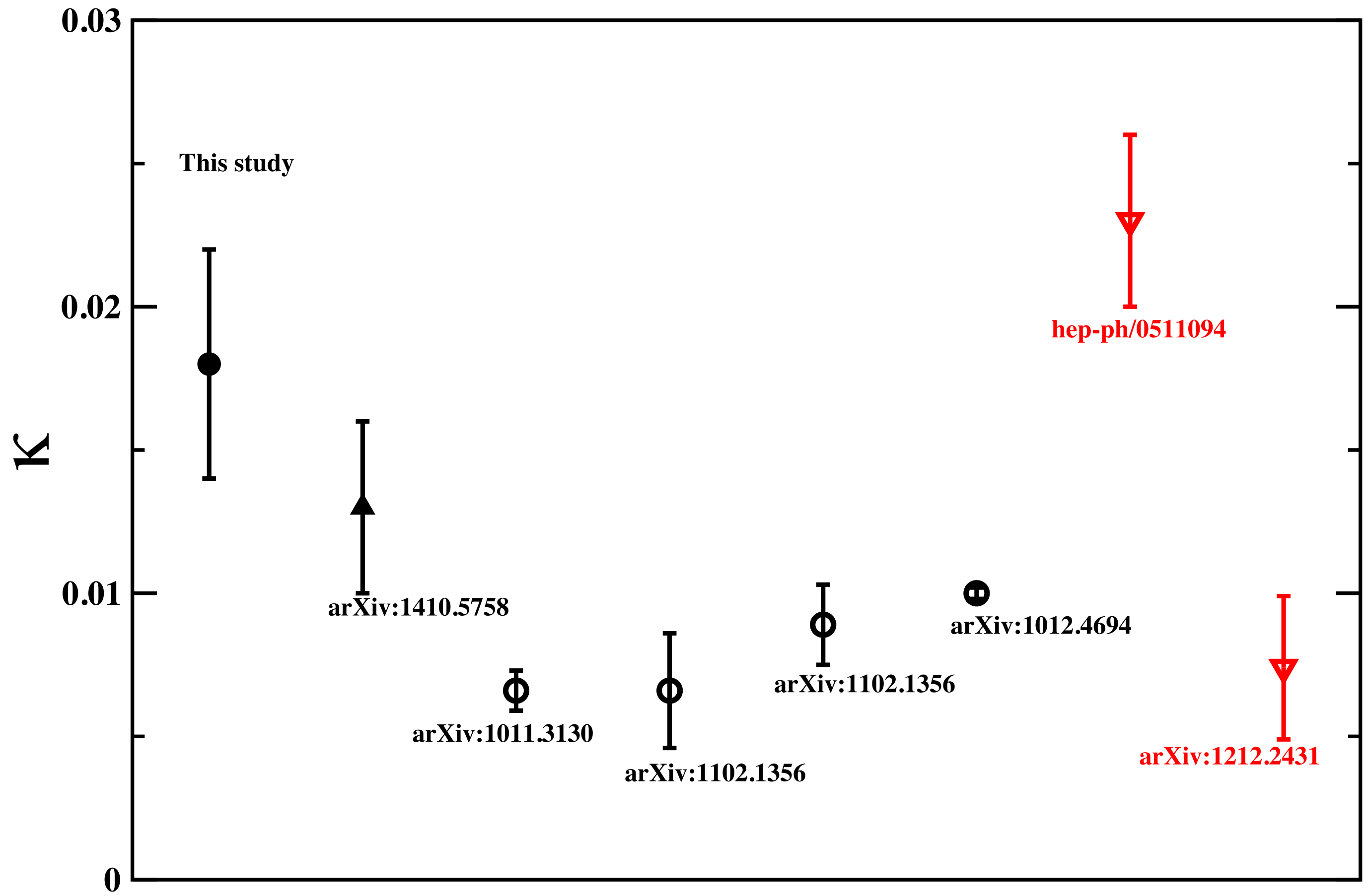


$$R_q = -1.7518(2802)$$

$$\kappa = -\frac{R_q}{9\pi^2} = 0.0197(32)$$

Comparison with other results for the curvature κ



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- P. Cea, L. Cosmai, A. Papa, Phys. Rev. D 89, 074512 (arXiv:1403.0821)
- new data

analytic continuation, ***HISQ/tree action***,
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arXiv:1410.5758 C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, F. Sanfilippo, Phys.Rev. D90 (2014) 11, 114025

analytic continuation, stout-smeared staggered quarks, $\mu_s=0$, chiral condensate, chiral susceptibility

arXiv:1011.3130 O. Kaczmarek, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt, W. Soeldner, and W. Unger, Phys.Rev. D83 (2011) 014504

Taylor expansion, p4-action, chiral susceptibility

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freeze-out curvature, analysis based on the standard statistical hadronization model

arXiv:1212.2341 F. Becattini, M. Bleicher, Th. Kollegger, T. Schuster, Jan Steinheimer, and Reinhard Stock, Phys.Rev.Lett. 111 (2013) 082302

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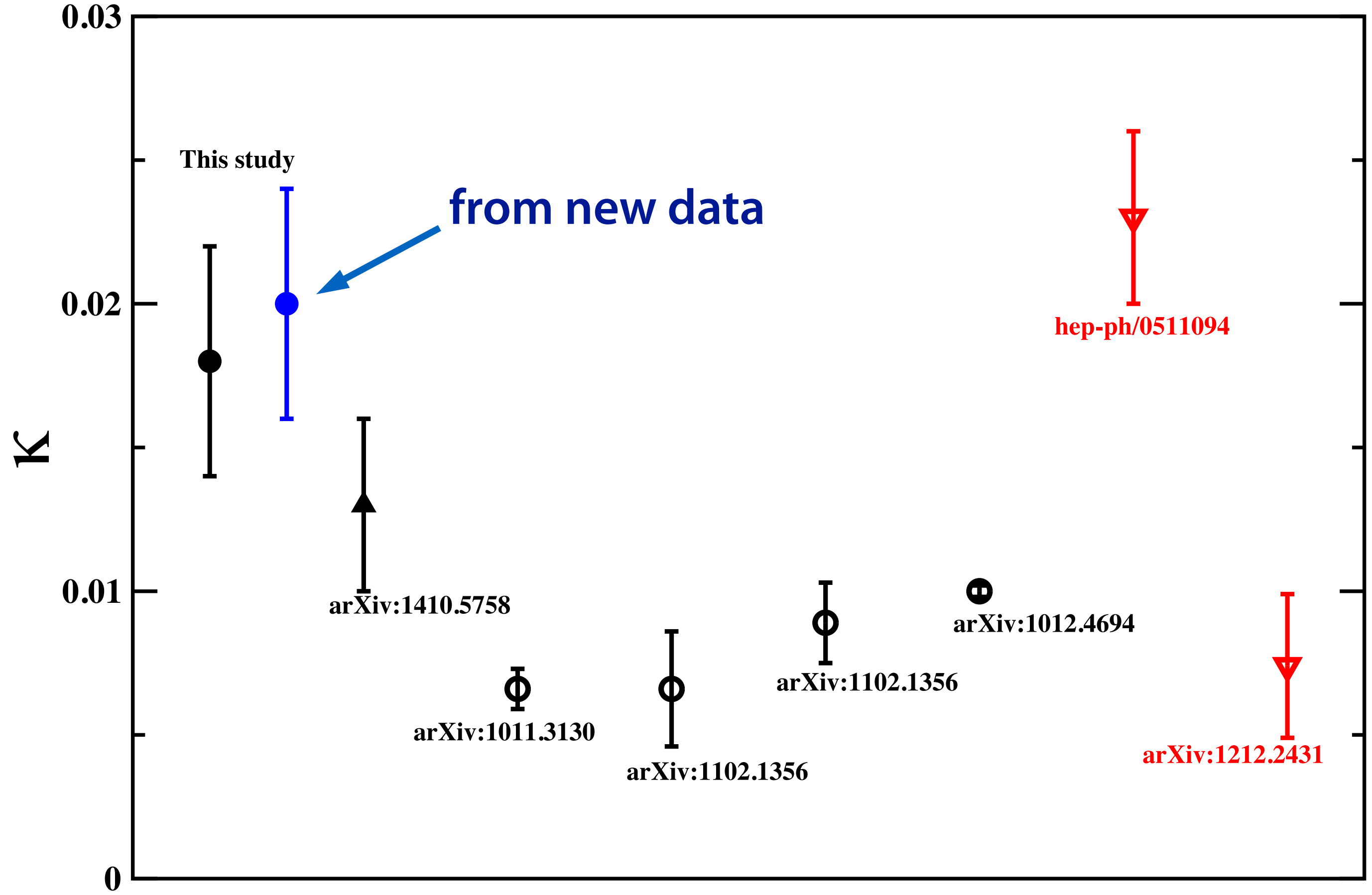

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Estimate of the (pseudo)critical line

From our estimate of the curvature

$$\kappa = 0.020(4)$$

and

$$T_c(\mu_B) = a - b\mu_B^2$$

$$a = T_c(0)$$

$$b = \frac{\kappa}{T_c(0)}$$

$$T_c(0) = 154(9)\ \mathrm{MeV} \quad (*)$$

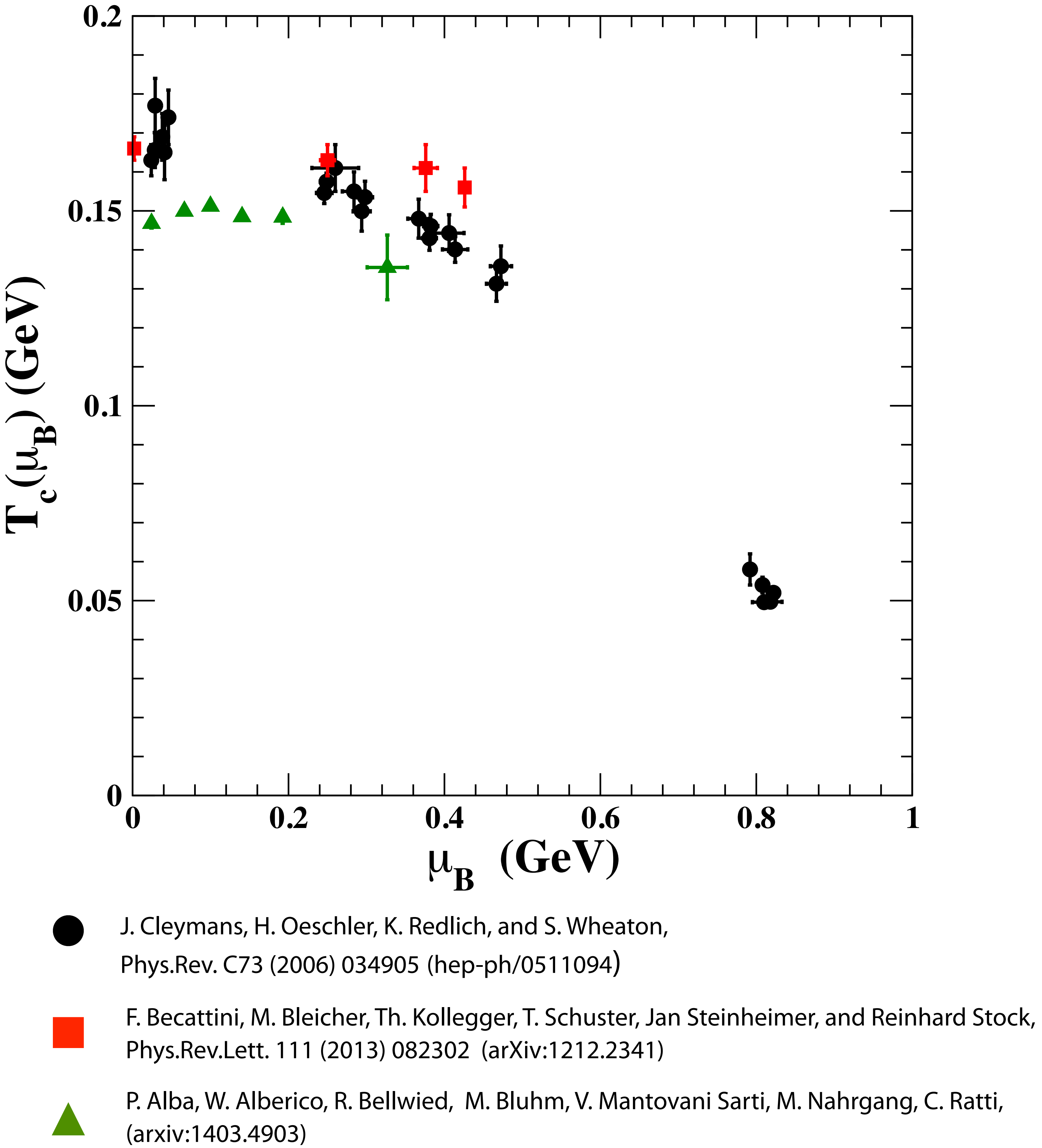
we get:

$$b = 0.128(25)\ \mathrm{GeV}^{-1}$$

to be compared with:

$$b = 0.139(16)\ \mathrm{GeV}^{-1}$$

hep-ph/0511094 J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905



(*) A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

Estimate of the (pseudo)critical line

From our estimate of the curvature

$$\kappa = 0.020(4)$$

and

$$T_c(\mu_B) = a - b\mu_B^2$$

$$a = T_c(0)$$

$$b = \frac{\kappa}{T_c(0)}$$

$$T_c(0) = 154(9)\,\mathrm{MeV} \quad (*)$$

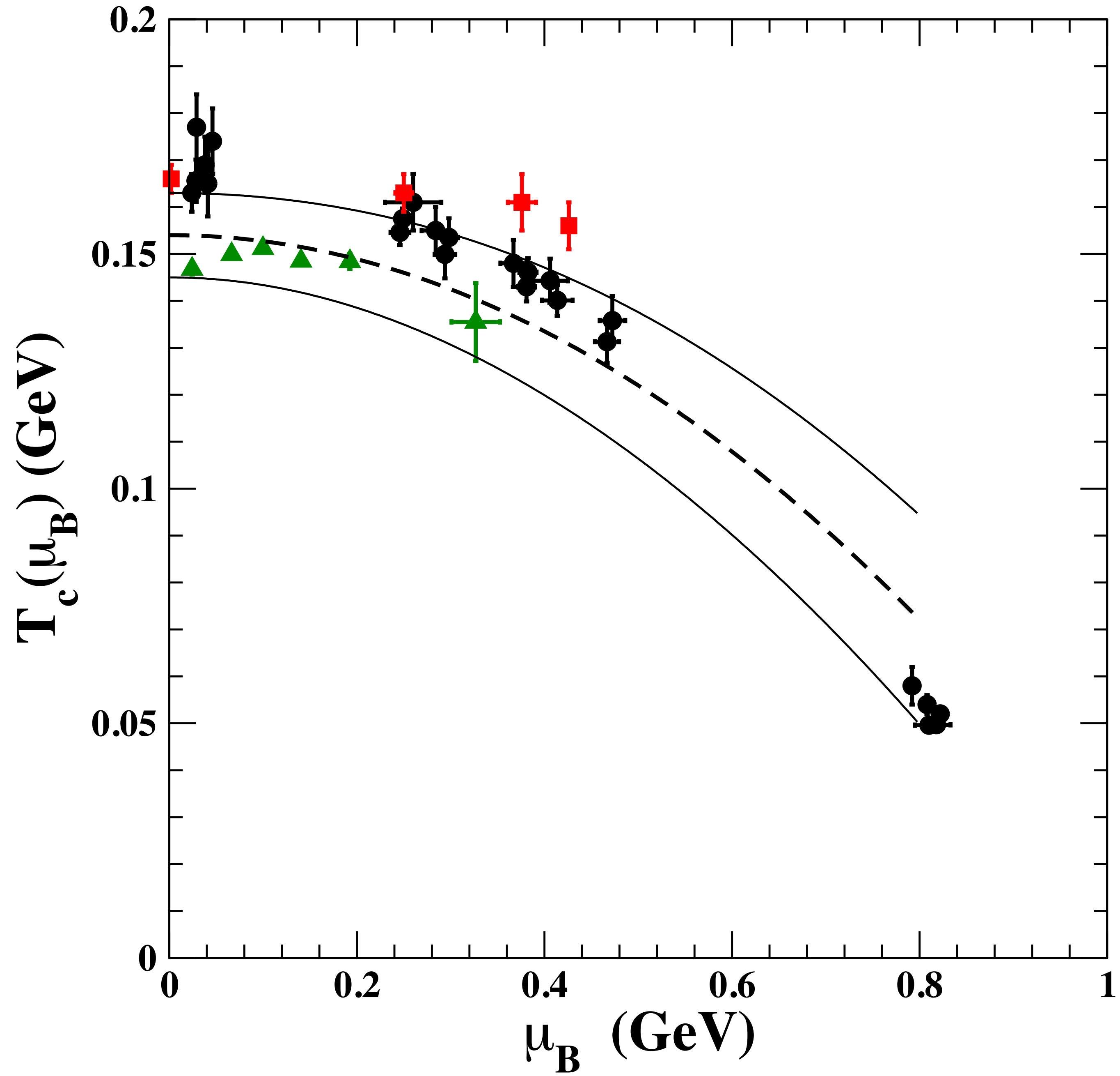
we get:

$$b = 0.128(25)\,\mathrm{GeV}^{-1}$$

to be compared with:

$$b = 0.139(16)\,\mathrm{GeV}^{-1}$$

hep-ph/0511094 J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905



● J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905 (hep-ph/0511094)

■ F. Becattini, M. Bleicher, Th. Kollegger, T. Schuster, Jan Steinheimer, and Reinhard Stock, Phys.Rev.Lett. 111 (2013) 082302 (arXiv:1212.2341)

▲ P. Alba, W. Alberico, R. Bellwied, M. Bluhm, V. Mantovani Sarti, M. Nahrgang, C. Ratti, (arxiv:1403.4903)

(*) A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

Summary & Conclusions

- We have studied QCD with 2+1 flavors discretized in the HISQ/tree staggered fermion formulation and in the presence of an imaginary chemical potential, with a light-to-strange mass ratio $m_l/m_s = 1/20$ and quark chemic potentials $\mu=\mu_l=\mu_s$.

- We have estimated by the method of analytic continuation the curvature of the (pseudo)critical line in the temperature - baryon chemical potential. The observable adopted to identify, for each fixed μ, the crossover temperature is the disconnected part of the (renormalized) susceptibility of the light quark condensate.

- We have found that, within the accuracy of our determinations, cutoff effects on the curvature are negligible (used lattices with temporal size L_t=6,8,10,12):

  $\kappa = 0.020(4)$ this work $\kappa = 0.018(4)$ our previous paper

- The extrapolation of the critical line as determined in this work to the region of real baryon density compares quite well with the freeze-out curves resulting from a few phenomenological analyses of relativistic heavy-ion collisions.