Neutron Electric Dipole Moment from quark Chromoelectric Dipole Moment

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Introduction

CP violation and nEDM

CP violation needed in the universe.
Observed baryon asymmetry: $n_B/n_\gamma = 6.1^{+0.3}_{-0.2} \times 10^{-10}$.

WMAP + COBE 2003

Without CP violation, freezeout ratio: $n_B/n_\gamma \approx 10^{-20}$.

Kolb and Turner, *Front. Phys.* **69** (1990) 1.

Either asymmetric initial conditions or baryogenesis!
Sufficiently asymmetric initial conditions kills inflation.

Sakharov Conditions — Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32.

- Baryon Number violation
- C, CP and T violation
- Out of equilibrium evolution

Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar arXiv:hep-ph/0008248.

- CP-violating mass term and effective $\Theta G\tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Crewther *et al.*, *Phys. Lett.* **B88** (1979) 123.

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM

Introduction
Form Factors

Vector form-factors

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\begin{aligned}\langle N|V_\mu(q)|N\rangle &= \overline{u}_N \Bigg[\gamma_\mu\, F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu\, \frac{F_2(q^2)}{2m_N} \\ &\qquad + (2i\, m_N \gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\, \frac{F_A(q^2)}{m_N^2} \\ &\qquad + \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5\, \frac{F_3(q^2)}{2m_N}\Bigg] u_N\end{aligned}$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A and F_3 violate P; F_3 violates CP.

Introduction
Projection

The three point function we calculate is

$$N \equiv \bar{d}^c \gamma_5 \frac{1+\gamma_4}{2} u \; d$$

$$\langle \Omega | N(\vec{0}, 0) V_\mu(\vec{q}, t) N^\dagger(\vec{p}, T) | \Omega \rangle = u_N e^{-m_N t} \langle N | V_\mu(q) | N' \rangle \, e^{-E_{N'}(T-t)} \overline{u}_N$$

We project onto only one component of the neutron spinor with

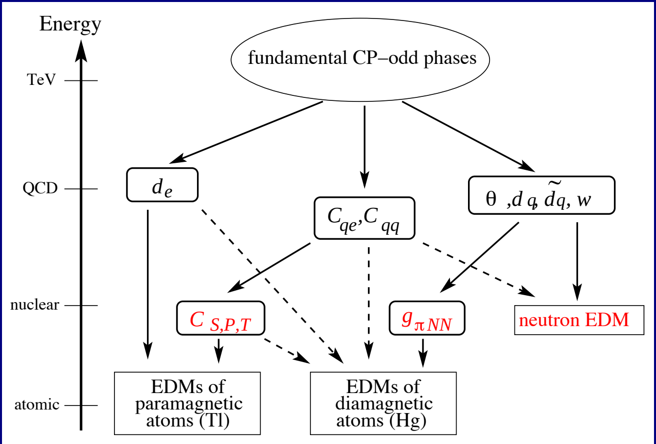
$$\mathcal{P} = \frac{1}{2}(1+\gamma_4)(1+i\gamma_5\gamma_3)$$

Noting that in presence of CP violation $u_N \overline{u}_N = e^{i\alpha_N \gamma_5}(i\not{p} + m_N) e^{i\alpha_N \gamma_5}$ and assuming $N' = N$, we can extract:

$$\begin{aligned} \mathrm{Tr}\, \mathcal{P} \langle \Omega | N V_3 N^\dagger | \Omega \rangle \quad \propto \quad & i m_N q_3 G_E \\ & + \alpha_N m_N (E_N - m_N) F_1 + \alpha_N [m_N(E_N - m_N) + \frac{q_3^2}{2}] F_2 \\ & - 2i\,(q_1^2 + q_2^2) F_A - \frac{q_3^2}{2} F_3 \end{aligned}$$

Introduction

Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\rm EW}/M_{\rm BSM}^2$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M_{\rm BSM}^2$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Renormalization and Mixing

Vacuum Alignment and Phase Choice

CP and chiral symmetry do not commute. Outer automorphism: $\mathrm{CP}_\chi \equiv \chi^{-1}\mathrm{CP}\chi$ also a CP.

- $\psi_L^{CP} = i\gamma_4 C\bar\psi_L^T$ and $\psi_R^{CP} = i\gamma_4 C\bar\psi_R^T$.
- $\psi_L^\chi = e^{i\chi}\psi_L$ and $\psi_R^\chi = e^{-i\chi}\psi_R$
- $\psi_L^{CP_\chi} = e^{-2i\chi} i\gamma_4 C\bar\psi_L^T$ and $\psi_R^{CP_\chi} = e^{+2i\chi} i\gamma_4 C\bar\psi_R^T$

Consider the chiral and CP violating parts of the action

$$\mathcal{L} \supset d_i^\alpha O_i^\alpha$$

where i is flavor and α is operator index.
Consider only one chiral symmetric CP violating term: $\Theta G\tilde{G}$

Convert to polar basis

$$d_i \equiv |d_i| e^{i\phi_i} \equiv \frac{\sum_\alpha d_i^\alpha \langle \Omega | \mathcal{I}m\, O_i^\alpha | \pi \rangle}{\sum_\alpha \langle \Omega | \mathcal{I}m\, O_i^\alpha | \pi \rangle}$$

Then CP violation is proportional to:

$$\bar{d}\bar{\Theta}\, \mathcal{R}e\, \frac{d_i^\alpha}{d_i} - |d_i|\, \mathcal{I}m\, \frac{d_i^\alpha}{d_i}$$

with

$$\frac{1}{\bar{d}} \equiv \sum_i \frac{1}{d_i} \qquad \bar{\Theta} = \Theta - \sum_i \phi_i$$

CP violation depends on $\bar{\Theta}$ and on a *mismatch* of phases between d_i^α and d_i.

Renormalization and Mixing

Operator Basis

$ig\bar\psi\tilde\sigma^{\mu\nu}G_{\mu\nu}t^a\psi$ $\quad\partial^2\left(\bar\psi i\gamma_5 t^a\psi\right)$

$\frac{ie}{2}\bar\psi\tilde\sigma^{\mu\nu}F_{\mu\nu}\left\{Q,t^a\right\}\psi$

$\mathrm{Tr}\left[MQ^2t^a\right]\frac{1}{2}\tilde F_{\mu\nu}F^{\mu\nu}$ $\quad\mathrm{Tr}\left[Mt^a\right]\frac{1}{2}\tilde G^a_{\mu\nu}G^{\mu\nu a}$

$\mathrm{Tr}\left[Mt^a\right]\partial_\mu\left(\bar\psi\gamma^\mu\gamma_5\psi\right)$ $\quad\frac{1}{2}\partial_\mu\left(\bar\psi\gamma^\mu\gamma_5\left\{M,t^a\right\}\psi\right)\Big|_{\text{traceless}}$

$\frac{1}{2}\bar\psi i\gamma_5\left\{M^2,t^a\right\}\psi$ $\quad\mathrm{Tr}\left[M^2\right]\bar\psi i\gamma_5 t^a\psi$

$\mathrm{Tr}\left[Mt^a\right]\bar\psi i\gamma_5 M\psi$

$i\bar\psi_E\gamma_5 t^a\psi_E$ $\quad\mathrm{Re}\,\partial_\mu\left[\bar\psi_E\gamma^\mu\gamma_5 t^a\psi\right]$

$\mathrm{Re}\,\bar\psi\gamma_5\not\partial t^a\psi_E$ $\quad\mathrm{Re}\,\frac{ie}{2}\bar\psi\left\{Q,t^a\right\}\not A^{(\gamma)}\gamma_5\psi_E$

Renormalization and Mixing

RI-$\tilde{\text{S}}$MOM scheme

$$\begin{pmatrix} O \\ N \end{pmatrix}_{\text{ren}} = \begin{pmatrix} Z_O & Z_{ON} \\ 0 & Z_N \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_{\text{bare}}$$

O: Gauge-invariant operators, does not vanish by equation of motion.

N: Gauge-dependent operators, restricted by BRST, vanish by equation of motion.

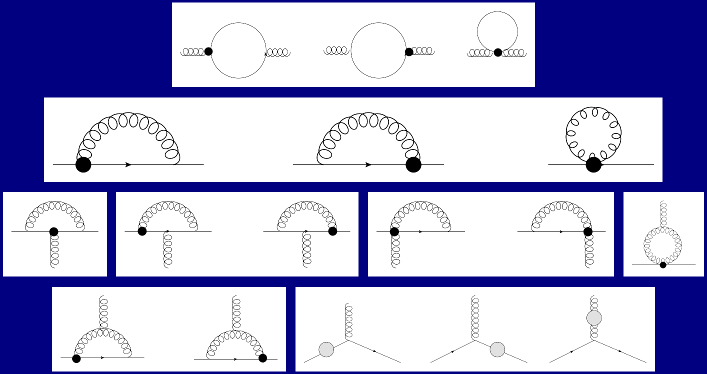
Impose conditions on matrix elements of quarks and gluons:

- Use $\overline{\text{MS}}$ quark masses in the expansion.
- Three point functions at $p^2 = p'^2 = q^2 = -\Lambda^2 \ll 0$ (RI-SMOM).
- Four point functions at $p^2 = p'^2 = k^2 = q^2 = s = u = t/2 = -\Lambda^2$.

This choice eliminates non-1PI contributions. (See arXiv:1502.07325 [hep-ph]).

Renormalization and Mixing

Connection to $\overline{\text{MS}}$ scheme



Lattice Calculation

Technique

The quark chromo-EDM operator is a quark bilinear.
Schwinger source method: Add it to the Dirac operator in the propagator inversion routine:

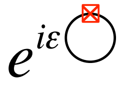
$$\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})$$

The fermion determinant gives a ‘reweighting factor’

$$\begin{aligned} &\frac{\det(\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})}{\det(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})} \\ &\quad = \quad \exp \operatorname{Tr} \ln \left[1 + i\epsilon\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] \\ &\quad \approx \quad \exp\left[i\epsilon \operatorname{Tr} \Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] . \end{aligned}$$

Lattice Calculation

Three-point function



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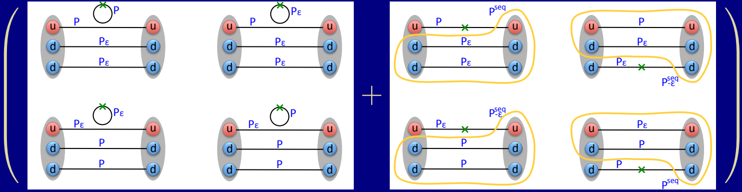


The chromoEDM operator is dimension 5.

Uncontrolled divergences unless $\epsilon \lesssim 4\pi a\Lambda_{\mathrm{QCD}} \sim 1$.

Need to check linearity.

Lattice Calculation

Propagator inversion

Using BiCGStab in Chroma (Clover on HISQ $a \approx 0.12$fm, $m_\pi \approx 310$MeV)

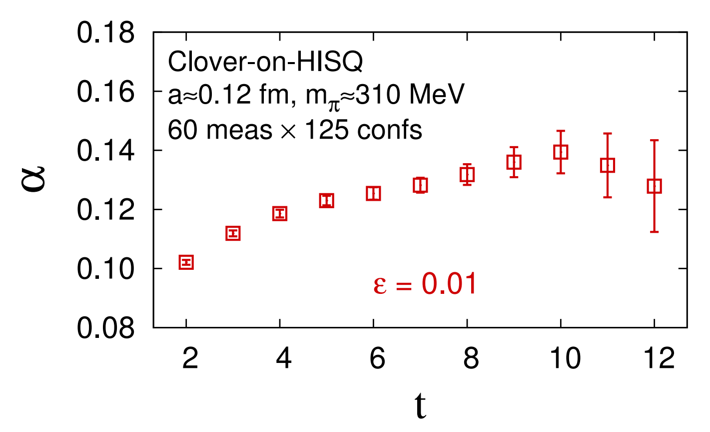
- Cost of $\not{D}$ increases by about 7%.
- Condition number changes by less than 5%.
- Can use $\epsilon = 0$ as initial guess.

Each extra inversion less than the cost of the $\epsilon = 0$ inversion.

Accuracy	$\epsilon = 0.005$	$\epsilon = 0.01$
10^{-8}	85%	86%
10^{-3}	51%	66%
5×10^{-3}	28%	45%

Calculation of connected EDM measurement on each configuration is about 1.5 times the cost of V/A form factors measurements.

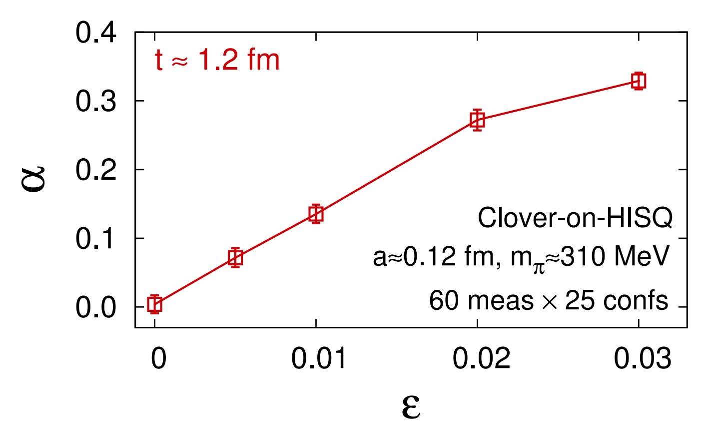
Numerical Tests
Propagator



Preliminary; Connected Diagrams Only

Numerical Tests

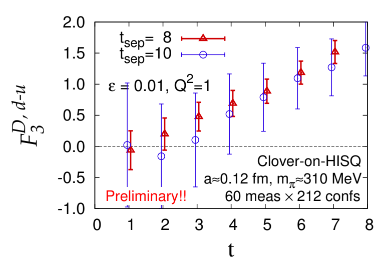
Linearity

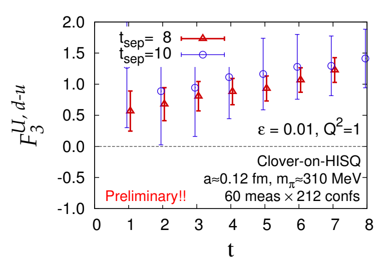


Preliminary; Connected Diagrams Only

Numerical Tests

F_3 Form factor





Preliminary; Connected Diagrams Only

Conclusions

Future

- Disconnected diagrams.
- Continuum limit.

  Most divergent mixing with $\frac{\alpha_s}{a^2}\bar{\psi}\gamma_5\psi$.

  nEDM due to this same as due to $\frac{\alpha_s}{ma^2}G\cdot\tilde{G}$.

  Current estimates of nEDM due to
  - $\mathrm{CEDM}^{\overline{\mathrm{MS}}} \Rightarrow O(1)$
  - $\frac{\alpha_s}{ma^2}\Theta G\cdot\tilde{G} \Rightarrow \frac{O(0.1)}{5\mathrm{MeV}\,a^2}O(10^{-3})$e-fm $= O(1)$

    at $a \approx 0.1\mathrm{fm}$.

  Expect O(1–10) cancellation.
- Fermions with better chiral symmetry.