# $K\pi$ and $\pi\pi$ scattering

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# The first slide

- Talk is about resonances appearing in $\pi\pi$ and $K\pi$ scattering.
- In lattice QCD, we like to deal with stable hadrons.
- Particles such as $\rho$ or $K^*$ decay strongly and cannot be treated a stable.
- Strong decay widths are about 150 or 50 MeV, depending on which one it is.
- The two processes are analoguous, both conceptually and in some calculation steps.

# What is new?

- Various studies have been done before on $I = 1 \pi\pi$ and $I = 1/2 K\pi$ scattering.
- Resonance behaviour is observed.
- We are close to the physical pion mass.
- Here, $2m_\pi < m_\rho$ and $m_\pi + m_K < m_{K^*}$ $(2\sqrt{m_\pi^2 + (2\pi/L)^2} < m_\rho)$.
- We use two lattice sizes – $V = 48^3 \times 64$ and $64^4$ – with $N_f = 2$, $a \simeq 0.071$ fm, $m_\pi \simeq 150$ MeV.
- We work on 'medium sized' lattices (3.5, 4.5 fm), with slightly smaller than ideal $m_\pi L$.

# What do we actually want?

- Channels of interest are I=1 $\pi\pi$ and I=1/2 $K\pi$.
- We expect there is a resonance in these channels and want to study it.
- Use 2-particle operators.

$$\pi(p_1)\pi(p_2) = \pi^+(p_1)\pi^-(p_2) - \pi^-(p_1)\pi^+(p_2)$$

$$\pi(p_1)K(p_2) = \sqrt{\frac{2}{3}}\pi^+(p_1)K^0(p_2) - \sqrt{\frac{1}{3}}\pi^0(p_1)K^+(p_2)$$

- Also corresponding 1-particle operators $\rho(P)$ and $K^*(P)$ made with $\gamma_\mu$, $\gamma_\mu\gamma_t$, $\nabla_\mu$.
- Aim to determine resonance parameters.

# Our methods

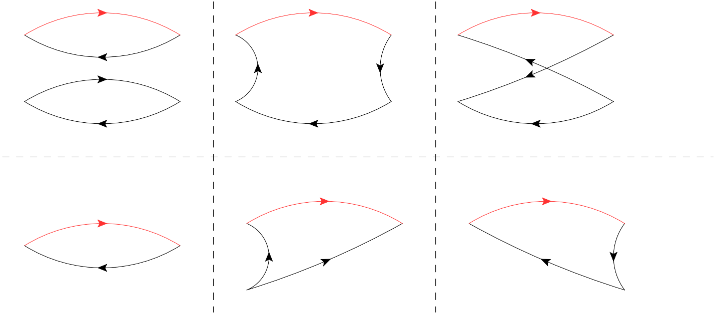
- Besides having two volumes, we also use moving frames to increase the number of points in $W_{CM}$ we can access.
- To extract the resonance parameters, we need enough data in the region of the resonance.
- Stochastic sources: to generate propagators required for all the $2 \to 2$ and $2 \to 1$ correlations, this needs sequential inversions.
- This means we have to project onto momentum for the sequential sources.

# States we look at

| $\pi\pi$ | | | | |
|---|---|---|---|---|
| $L$ | $P$ | Irrep | $\mathcal{O}_{\pi\pi}$ | $\mathcal{O}_{\rho}$ |
| 48 | $(0,0,0)$ | $T_1$ | $\pi(1,0,0)\pi(-1,0,0)$ | $\rho_\mu$ |
| 48 | $(0,0,1)$ | $E$ | $\pi(1,0,0)\pi(-1,0,1)-\pi(-1,0,0)\pi(1,0,1)$ | $\rho_x,\rho_y$ |
| 48 | $(0,1,1)$ | $A_1$ | $\pi(1,0,0)\pi(-1,1,1)+\pi(-1,0,0)\pi(1,1,1)$ | $\rho_y+\rho_z$ |
| 48 | $(0,1,1)$ | $B_1$ | $\pi(1,0,0)\pi(-1,1,1)-\pi(-1,0,0)\pi(1,1,1)$ | $\rho_x$ |
| 64 | $(0,0,1)$ | $E$ | $\pi(1,0,0)\pi(-1,0,1)-\pi(-1,0,0)\pi(1,0,1)$ | $\rho_x,\rho_y$ |
| 64 | $(0,1,1)$ | $A_1$ | $\pi(1,0,0)\pi(-1,1,1)+\pi(-1,0,0)\pi(1,1,1)$ | $\rho_y+\rho_z$ |
| 64 | $(0,1,1)$ | $B_1$ | $\pi(1,0,0)\pi(-1,1,1)-\pi(-1,0,0)\pi(1,1,1)$ | $\rho_x$ |
| $K\pi$ | | | | |
| $L$ | $P$ | Irrep | $\mathcal{O}_{K\pi}$ | $\mathcal{O}_{K^*}$ |
| 48 | $(1,1,0)$ | $B_2$ | $\pi(1,0,0)K(0,1,0)$ | $K^*_x-K^*_y$ |
| 64 | $(0,0,0)$ | $T_1$ | $\pi(1,0,0)K(-1,0,0)$ | $K^*\mu$ |
| 64 | $(0,0,1)$ | $E$ | $\pi(1,0,0)K(-1,0,1)-\pi(-1,0,0)K(1,0,1)$ | $K^*_x,K^*_y$ |
| 64 | $(0,1,1)$ | $A_1$ | $\pi(1,0,0)K(-1,1,1)+\pi(-1,0,0)K(1,1,1)$ | $K^*_y+K^*_z$ |
| 64 | $(0,1,1)$ | $B_1$ | $\pi(1,0,0)K(-1,1,1)-\pi(-1,0,0)K(1,1,1)$ | $K^*_x$ |

- 2 criteria: expected to be close to resonance (by non-interacting energy) and not too many diagrams (cross terms).

# Things we calculate



# Extraction of states

- Energy levels extracted by diagonalising correlator matrix.

$$C_{ij} = \sum_{\alpha} \frac{Z_{\alpha}^{i} Z_{\alpha}^{*j}}{2m_{\alpha}} e^{-m_{\alpha} t}.$$

- In cases we look at, expectation is to see 2 close-by states.
- When we have $E_L$, we use it to find $k_{cm}^2$:

$$k_{cm}^2 = \frac{(E_L^2 - (m_1^2 + m_2^2))^2 - 4m_1^2 m_2^2}{4E_L^2}.$$

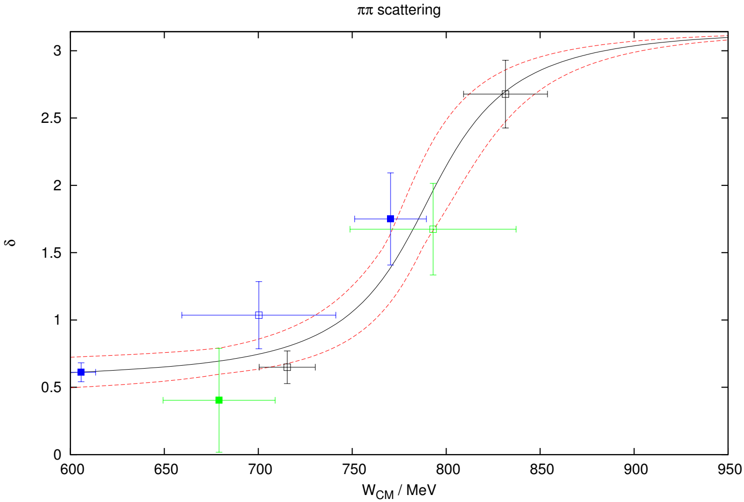
# Extraction of phase shift

- For $P = 0$, momenta is quantised as $2\pi/L$.
- Resonance parameterised by phase shift $\delta$, where we compare measured $k_{cm}$ to those allowed in finite box (with periodic boundary conditions).

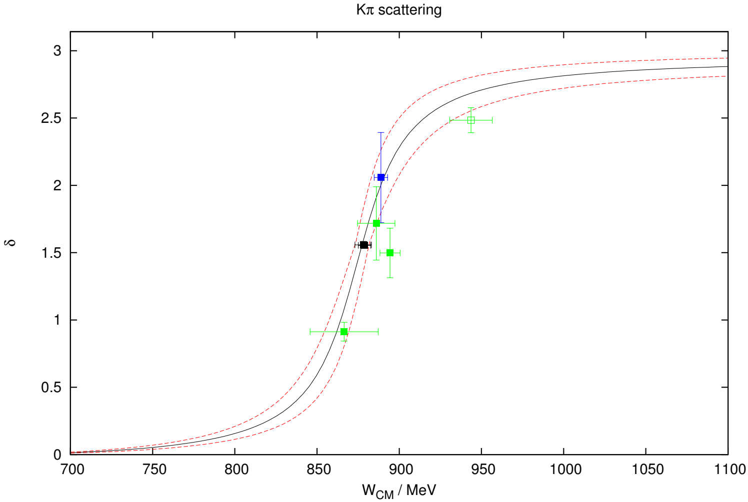
$$Z_{lm}(q^2) = \sum_{\vec{z}} \frac{\mathcal{Y}_{lm}(\vec{z})}{\vec{z}^2 - q^2}.$$

- $q^2$ is the centre of mass momentum equivalent to the 2-particle energy level and $\vec{z}$ the allowed momentum vector.
- And then $\cot\delta =$ some straightforward-ish combination of $Z_{lm}(q^2)$ depending on $P$, irrep and meson masses.

# Preliminary results - $\pi\pi$



# Preliminary results - $K\pi$



# Extract the resonance parameters

- We use a Breit-Wigner parameterisation.

$$\tan\delta = \frac{g^2}{6\pi}\frac{p_{cm}^3}{E_{cm}(m_R^2 - E_{cm}^2)}$$

- Can also use use a damping factor for $p_{cm}^3$ above resonance – useful for $\pi\pi$.
- Find $m_R = 778(11)$, $g = 4.46(44)$ for $\pi\pi$ and $m_R = 877(4)$, $g = 5.65(72)$ for $K\pi$.
- PDG: $m_\rho = 775$ MeV, $\Gamma_\rho = 148$ MeV $\rightarrow g = 5.97$; $m_{K^*} = 892$ MeV, $\Gamma_{K^*} = 51$ MeV $\rightarrow g = 5.69$.

# Summary

- We are investigating hadronic resonances in simulations of $\pi\pi$ and $K\pi$ in which the pion mass is 150 MeV.
- So far, things looks promising.

- Currently continuing to gain statistics for both lattice volumes.
- Related project looking at $DK$ scattering on the same lattice configurations.
- Interested in decays involving unstable hadrons, although it is not clear how or if this will manifest itself.