Nucleon generalized form factors from lattice QCD near the physical quark mass

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Outline

An outline

1. GPD motivation
2. Lattice setup
3. Extracting GFFs
4. Excited state analysis
5. Results
6. Conclusion and outlook

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Generalized Parton Distributions (GPDs)

- introduced late ‘90s
- comprehensive description of the hadron structure

Contain information about

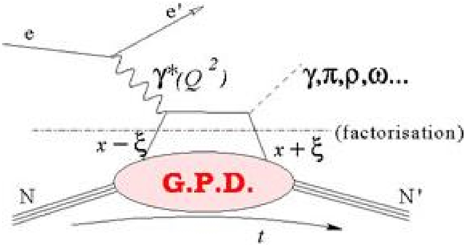
- traditional form factors and parton distributions (limiting cases)
- quark total angular momentum

In particular important for

- spin structure of the proton
- deeply virtual Compton scattering (DVCS)

Kinematic definitions

- $\Delta = P' - P$
- $\bar{P} = (P' + P)/2$
- $n \cdot n = 0$
- $n \cdot \bar{P} = 1$



Kinematic variables

- x: longitudinal momentum fraction
- $\xi = -n \cdot \Delta/2$: longitudinal momentum transfer
- $Q^2 = -t = \Delta^2 = P' - P$: momentum transfer

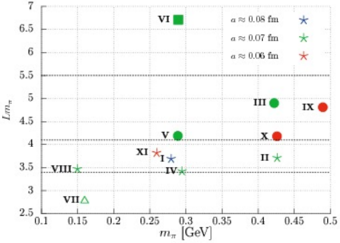
Outline

General setup

- $N_f = 2$ mass-degenerated NP improved Wilson-clover Fermions with Wilson Gauge action (RQCD, QCDSF)
- Apply Wuppertal smearing to suppress excited states
- Kinematic setup: $\vec{p}_f \cdot \vec{p}_f = 0$ and $\vec{p}_i \cdot \vec{p}_i \neq 0$

Todays setup

- Show results for ensemble VIII
- 150 MeV m_π
- $L \cdot m_\pi = 3.47$
- $t_f/a = 9, 12, 15$ with $t_{15} \sim 1\,\mathrm{fm}$



Outline

Consider Vector GFF

$$\langle N(P')|\, \mathcal{O}_V^{\mu\nu}\, |N(P)\rangle =$$

$$S(\mu,\nu)\bar{U}(P')\left[\mathrm{i}\gamma^{\mu}\bar{P}^{\nu}A_{2,0}(t) + \frac{\mathrm{i}\sigma^{\mu\alpha}\Delta^{\alpha}\bar{P}^{\nu}}{2m_N}B_{2,0}(t) + \frac{\Delta^{\mu}\Delta^{\nu}}{m_N}C_2(t)\right]U(P)$$

Remarks:

- Traces are already subtracted
- Euclidean signature (after Wick-Rotation)
- Compute the (kinematic) prefactors of the GFFs They define the coefficient matrix $\mathbb{M}$

Extract the GFF by solving an overdetermined system of equations:

$$\epsilon = \left[\mathrm{M} \cdot \begin{bmatrix} A_{20} \\ B_{20} \\ C_2 \end{bmatrix} - c_* \right]^T \cdot \mathrm{cov}^{-1}(c_*) \cdot \left[\mathrm{M} \cdot \begin{bmatrix} A_{20} \\ B_{20} \\ C_2 \end{bmatrix} - c_* \right]$$

Remarks:

- Find A_{20}, B_{20}, C_2 such that ϵ is minimized
- c_* can be fitted from lattice data

Outline

Imporant (first) step:

- Suppress excited states (Wuppertal smearing)

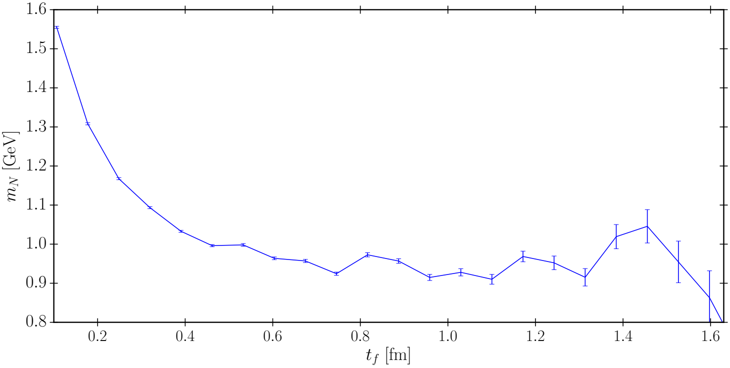


Figure : Effective mass plot of a nucleon (Ens. VIII).

Excited state analysis

Strategy of the analysis

- Perform excited state analysis for GFFs with simultaneous fits to 2-pt and 3-pt functions (extract c_*)
- Carefully analyze introduced systematics (e.g. fit ranges)

This is challenging

- More noise due to derivative
- Even more noise if $Q^2 \neq 0$
- Fit ansatz more complicated if $Q^2 \neq 0$

Key ingredients

- Improve 3-pt signal by averaging data of equal rows of M
- Simultaneous fit of all c_* (stabilizing noisy channels)

Fit ansatz

Nucleon 2-pt function with one exponent:

$$C_2^{1e}(p,t) = z_N(\vec{p})\ \frac{E(\vec{p},m_N)+m_N}{E(\vec{p},m_N)}\ \mathrm{e}^{-E(\vec{p},m_N)t}$$

Nucleon 2-pt function with two exponents:

$$C_2^{2e}(p,t) = z_N(\vec{p})\ \frac{E(\vec{p},m_N)+m_N}{E(\vec{p},m_N)}\ \mathrm{e}^{-E(\vec{p},m_N)t} + z_{N'}(\vec{p})\ \frac{E'+m_{N'}}{E'}\ \mathrm{e}^{-E't}$$

Remarks

- Assume continuum dispersion relation for N but not for N'

Nucleon 3pt function:

$$\begin{aligned} C_3(p,t) = &\sqrt{z_N^0 \ z_N^{pi}} \ {\color{red}c} \ \mathcal{E}(t, m_N, E(\vec{p}, m_N)) + \\ &\sqrt{z_N^0 \ z_{N'}^{pi}} \ x \ \mathcal{E}(t, m_N, E') + \\ &\sqrt{z_{N'}^0 z_N^{pi}} \ x \ \mathcal{E}(t, m_N', E(\vec{p}, m_N)) + \end{aligned}$$

with

$$\mathcal{E}(t, M, E) = \mathrm{e}^{-M(t_{\text{sink}} - t)} \ \mathrm{e}^{-E(t - t_{\text{src}})}$$

Remarks

- Need to get c

Scan fit range dependency

- Fit range 2-pt 1 exp.: (9,25), (10,25), (11,25), (12,25), (13,25)
- Fit range 2-pt 2 exp.: (1,25), (2,25), (3,25), (4,25), (5,25)
- Fit range 3-pt: (2,14), (3,13), (4,12), (4,25), (5,25)

Do resampling

- Create bootstrap ensemble (ss = 150) for each fit range combination

Remarks

- Solve the $5 \cdot 5 \cdot 3 \cdot 151 = 11325$ overdetermined systems
- Create a histogram
- Weight the fits properly
- Mix statistical and systematic distributions

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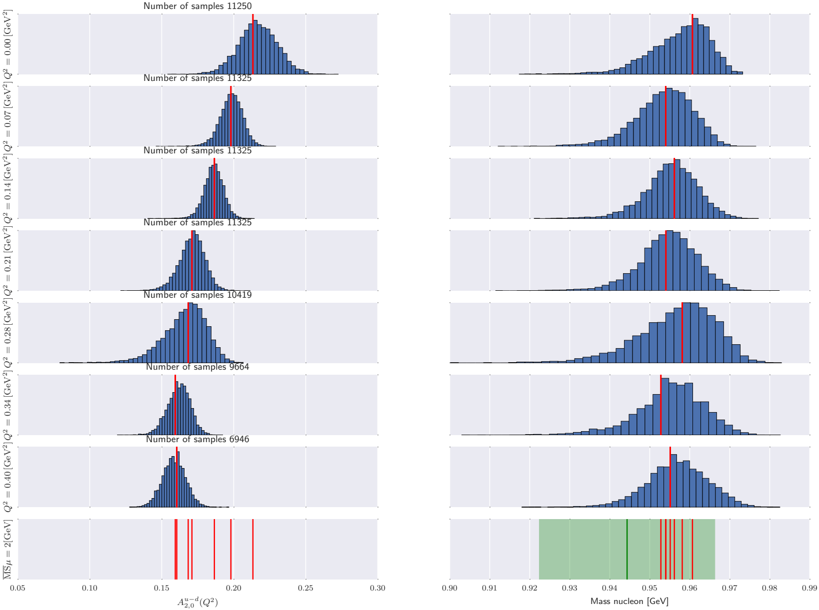
$A_{2,0}^{u-d}(Q^2)$ for Ens. VIII with $(t_f = 15)$



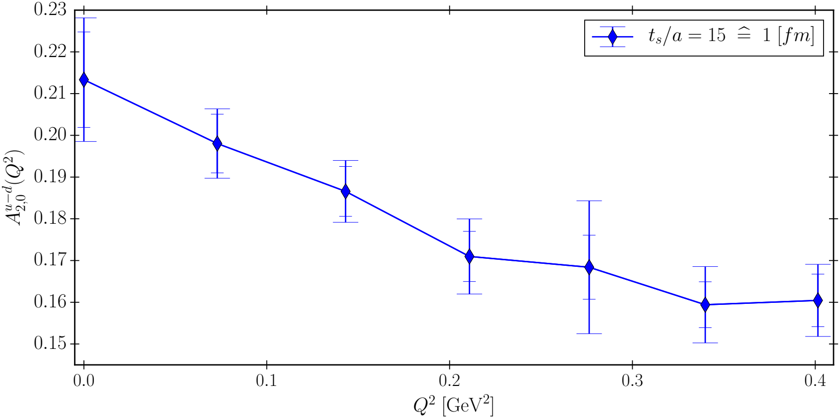


Figure : Inner error is statistical, outer error is total

Remarks:

- Linear behavior for small Q^2 as expected by LO χ-PT
- Finite volume effects of $\langle x \rangle_{u-d} = A_{2,0}^{u-d}(Q^2 = 0)$ are small (χ-PT result)

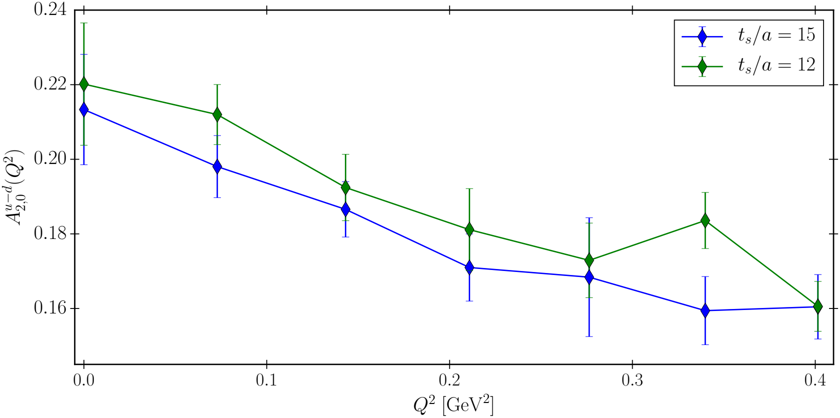


Figure : Comparison of two different sink locations.

Remarks:

- $A^{u-d}_{2,0}(Q^2 = 0) = \langle x \rangle_{u-d} \neq 0.155$ is no x-state effect

Ji's sumrule

$$J^{u-d} = \frac{A_{2,0}^{u-d}(Q^2=0) + B_{2,0}^{u-d}(Q^2=0)}{2}$$

Remarks

- $B_{2,0}^{u-d}(Q^2=0)$ not directly accessible.
- Calculate $(\, A_{2,0}^{u-d}(Q^2 \neq 0) + B_{2,0}^{u-d}(\neq 0)\,)/2$.
- Perform linear fit and draw histogram (preliminary)

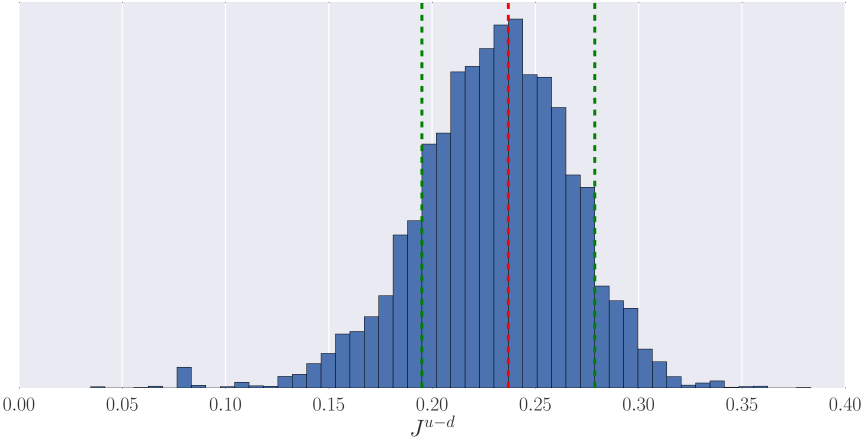


Figure : $J^{u-d} = 0.237 \pm 0.042$ for Ens. VIII with $(t_s/a = 15)$.

Outline

Conclusion

We have demonstrated our excited states analysis for some GFFs on our lightest ensemble. At least in this case excited states are under control at a source sink separation of $1\ [\mathrm{fm}]$.

To do

- Apply analysis to all ensembles.
- Apply different weights to the histograms.
- Perform combined χ-PT fits to all ensembles simultaneously.