Wilson Fermions with Four Fermion Interactions

Jarno Rantaharju, Vincent Drach, Martin Hansen, Ari Hietanen, Claudio Pica, Francesco Sannino,
CP3 -Origins, IFK & IMADA, University of Southern Denmark

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Four fermion interactions

In composite Higgs

- A standard way of explaining SM fermion masses
- Potentially significant effects on dynamics
- Deform a conformal model into a walking one
- Significant effects on the spectrum (top coupling)[1]

Preserve chiral symmetry: NJL-models

- Ungauged
- Wilson fermions, chiral symmetry

[1] R. Foadi, M. T. Frandsen and F. Sannino, Phys. Rev. D **87** (2013) 9, 095001
S. Di Chiara, R. Foadi and K. Tuominen, Phys. Rev. D **90** (2014) 11, 115016

Four fermion interactions

- Composite Higgs models need to explain fermion masses
- Generated at a high energy scale Λ_{UV}

Generates all allowed terms:

$$\frac{a}{\Lambda_{UV}^2}(\bar{\Psi}_{SM}\Psi_{SM})^2 + \frac{b}{\Lambda_{UV}^2}\bar{\Psi}_{SM}\Psi_{SM}\bar{\Psi}_{TC}\Psi_{TC} \quad + \frac{c}{\Lambda_{UV}^2}(\bar{\Psi}_{TC}\Psi_{TC})^2$$

- Mixing term provides masses
- First term includes flavour changing neutral currents

$$\frac{c}{\Lambda_{ETC}^2}(\bar{\Psi}_{TC}\Psi_{TC})^2$$

- The third term appears in the technicolor Lagrangian
- Can influence dynamics at technicolor scale
- Increased anomalous dimension, spontaneous chiral symmetry breaking
- Tunable parameter γ

[1] K. Yamawaki, hep-ph/9603293
H. S. Fukano and F. Sannino, Phys. Rev. D **82** (2010) 035021

The Nambu-Jona-Lasinio Model

$$L = \bar{\Psi}\partial\!\!\!/\Psi + \gamma\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$

- A chirally symmetric four fermion interaction $(N_F = 2)$
- A model for spontaneous symmetry breaking

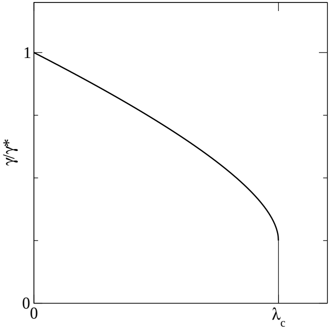
$$\gamma > \gamma^* = \frac{2\pi^2}{N\Lambda^2}$$

Gauged NJL

$$L = F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\not{D}\Psi + \gamma\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$

Now

$$\gamma_c = \frac{1}{4}\left(1 + \sqrt{1 - \frac{\lambda}{\lambda_c}}\right)^2 \gamma^*$$



The Lattice Model [2]

- For lattice, need to rewrite with pseudo fermions
- To make quadratic, rewrite with auxiliary fields

$$L = L_G + \bar{\Psi}\not{D}\Psi + \gamma\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$

$$\to L = L_G + \bar{\Psi}\not{D}\Psi + \sigma\bar{\Psi}\Psi + \pi^a\bar{\Psi}i\gamma_5\tau_a\Psi + \frac{\sigma^2 + \pi^a\pi^a}{4\gamma}$$

$$\langle\sigma\rangle = 2\gamma\,\langle\bar{\Psi}\Psi,\rangle \quad \langle\pi\rangle = 2\gamma\langle\bar{\Psi}i\gamma_5\tau_3\Psi\rangle$$

[2] K. M. Bitar and P. M. Vranas, Phys. Rev. D **50** (1994) 3406
S. Aoki, S. Boettcher and A. Gocksch, Phys. Lett. B **331** (1994) 157
A. Ali Khan, M. Gockeler, R. Horsley, P. E. L. Rakow, G. Schierholz and H. Stuben, Nucl. Phys. Proc. Suppl. **34** (1994) 655
J. B. Kogut, J. F. Lagae and D. K. Sinclair, Phys. Rev. D **58** (1998) 034504
S. Catterall, R. Galvez, J. Hubisz, D. Mehta and A. Veernala, Phys. Rev. D **86** (2012) 034502

The Lattice Model

- For lattice, need to rewrite with pseudo fermions
- Integrate out fermions

$$\int d\bar{\Psi} d\Psi e^{\sum_{x,y} \bar{\Psi}_x \left[\not{D}_{x,y} + \delta_{x,y}(\sigma_x + \pi_x^a i\gamma_5 \tau_a) \right] \Psi_y}$$

$$= \mathrm{Det} \begin{bmatrix} \not{D}_{x,y} + \delta_{x,y}(\sigma + i\pi_3\gamma_5) & \delta_{x,y}(i\pi_1 + \pi_2)\gamma_5 \\ \delta_{x,y}(i\pi_1 - \pi_2)\gamma_5 & \not{D}_{x,y} + \delta_{x,y}(\sigma - i\pi_3\gamma_5) \end{bmatrix}$$

- This is complex, sign problem
  $\rightarrow$ Restrict to $\pi_1 = \pi_2 = 0$
- Only one axial symmetry

The Lattice Model

Wilson fermion action:

$$\begin{aligned} L = {} & F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(\not{D}_W + m_0)\Psi \\ & + \gamma\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_3\Psi\bar{\Psi}i\gamma_5\tau_3\Psi\right) + \delta_\gamma\bar{\Psi}\Psi\bar{\Psi}\Psi \end{aligned}$$

- Chiral symmetry broken
  $\rightarrow$ Possible corrections m_0, δ_γ
- Restored when $\langle\partial_\mu\langle A_\mu^3(x)\rangle O\rangle = 0$

The Lattice Model

Ward identities:

$$\partial_\mu \langle A^3_\mu(x) O\rangle = 2m_0 \langle P^3(x) O\rangle + 4\delta_\gamma \langle S^0(x) P^3(x) O\rangle + \langle a X^3(x) O\rangle$$

Variation of Wilson term X^3 renormalises as

$$\begin{aligned} aX^3(x) &= a\bar{X}^3(x) + \frac{c_m}{a} P^3(x) + (Z_A - 1)\,\partial_\mu A^3_\mu(x) \\ &\quad + a c_A \partial_\mu \partial_\mu P^3(x) + a^2 c_m^{(\mu\nu\nu)} \partial_{(\mu}\partial_\nu\partial_{\nu)} A^3_\mu(x) \\ &\quad + a^2 c_\gamma S^0(x) P^3(x) + O(a^3) \\ c &= c\left(\beta, \frac{\gamma}{a^2}, \frac{\delta_\gamma}{a^2}, am_0\right) \end{aligned}$$

When $\gamma/a^2 \ll 1$, no correction to δ_γ

$$am_0 + c_m = 0 \text{ and } \delta_\gamma = 0$$

First step: ungauged NJL

Check δ_G

- Tree-level mass $m = 2(\gamma + \delta_\gamma)\langle\bar{\Psi}\Psi\rangle + m_0$
- Jump in $\langle\bar{\Psi}\Psi\rangle \to m = 0$
- Symmetric if axial current conserved:

$$\tilde{m} = \frac{\partial_\mu \langle A^3_\mu(t) A^3_\mu(0)\rangle}{2\langle P^3(t) A^3_\mu(0)\rangle}$$

$$\text{if } \delta^R_m = 0,\ m = \tilde{m}$$

- Spontaneous symmetry breaking implies $m_\pi = 0,\ m_\rho \neq 0$

The Lattice Model

Configurations not flavor symmetric

$\rightarrow$ Disconnected diagrams

$$M_{u,d} = \sum_{\mu} \partial_{\mu,y,x}\gamma_{\mu} + \delta_{x,y}\left(\sigma(x) \pm i\pi(x)\gamma_5\right)$$

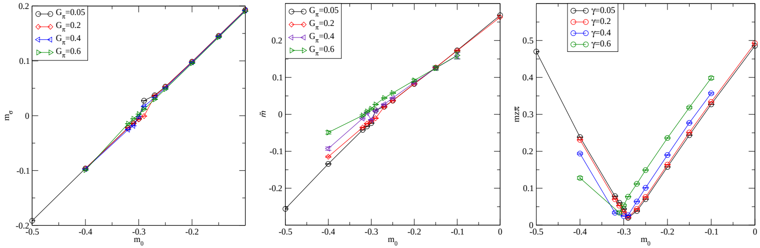
The disconnected loop is

$$\mathrm{Tr}\,(S_u - S_d)\,(x;y)\Gamma = -\mathrm{Tr}\,\frac{\delta_{x,y}2i\pi(x)\gamma_5}{M_u M_d}\Gamma$$

Noisy, need many configurations of π, σ

Small coupling

$$\gamma_\sigma = \gamma + \delta_\gamma = 0.04a^2, \quad L = 8^3 \times 16$$



Small coupling

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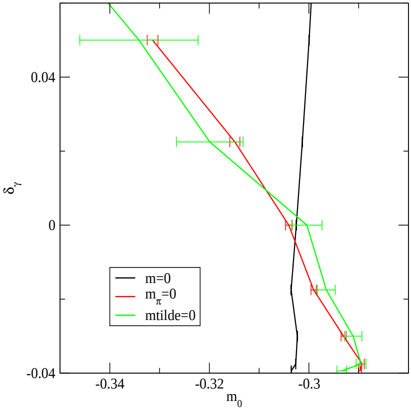
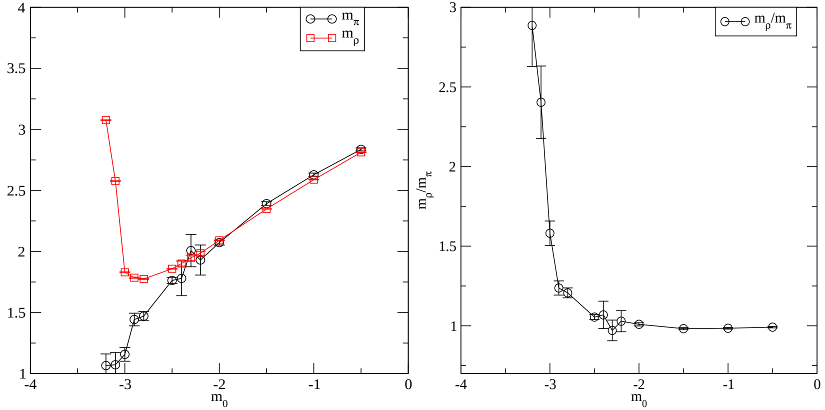


Large coupling

$$\gamma = 0.4225a^2, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16$$



Four fermion interactions

- Needed for fermion masses
- Can affect dynamics

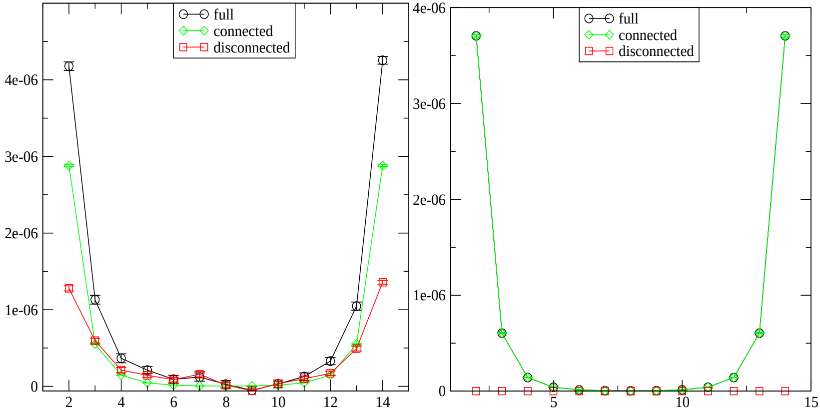
Ungauged NJL with Wilson fermions

- Signs of spontaneous symmetry breaking
- Large σ and π statistics needed for disconnected diagrams

To do

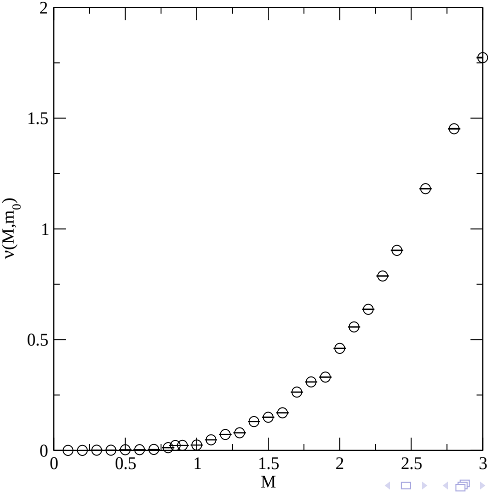
- Full phase space
- Scaling check
- Improve disconnected diagrams
- Renormalisation on mode number
- Full gauged model (SU(2) adjoint)

$$\gamma = 0.4225a^2, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16, \quad m_0 = 2.9$$



Small coupling

$$\gamma = 0.04a^2, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16, \quad m = 0.25$$



Large coupling

$$\gamma = 2.25a^2, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16, \quad m_0 = -1.5$$