

# Hadronic form factors for rare semi-leptonic *B* decays

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# Overview

Rare B decays and their importance

Phenomenological motivation

Theoretical Framework

B physics on the lattice

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Next steps

## Rare B decays and their importance

- ▶ What are they?

Decays that do not proceed through the  $b \rightarrow c$  transition.

- ▶ Why are they important?

Contributions from NP could be significant.

Can be used to measure CP violation.

- ▶ Interested in exclusive decays with one hadronic final state

- ▶  $b \rightarrow u$  with pseudoscalar final state (CKM suppressed)

$$B \rightarrow \pi l \nu \quad B_s \rightarrow K l \nu \quad \text{Previous talk by T. Kawanai}$$

$$B \rightarrow \pi l^+ l^- \quad B_s \rightarrow K l^+ l^-$$

- ▶  $b \rightarrow s(u)$  with vector final state (GIM(CKM) suppressed)

$$B_s \rightarrow \phi l^+ l^- \quad B \rightarrow K^* l^+ l^- \quad B_s \rightarrow K^* l^+ l^-$$

- ▶ We treat the final vector state as stable

- ▶ (Far) future simulate  $K^* \rightarrow K \pi$  Lellouch, Lüscher, 2000. Hansen, Sharpe, 2013

# Phenomenological motivation: Why $b \rightarrow u$

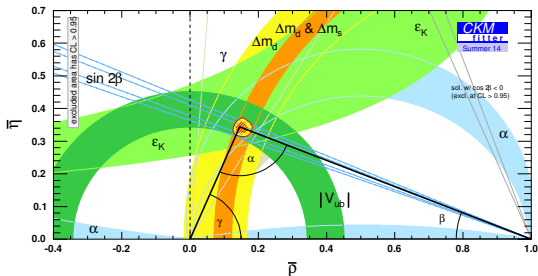
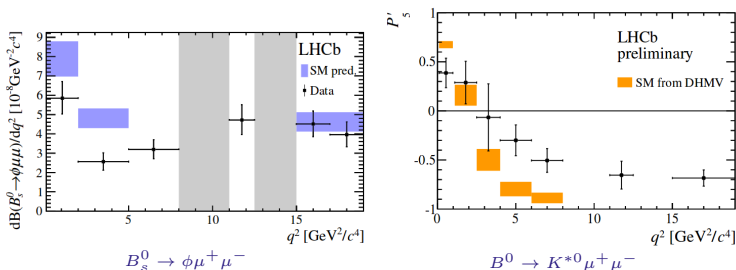


Figure from: CKMfitter Group (J. Charles et al.)

$$\begin{aligned}
 \frac{d\Gamma(B_s \rightarrow K^* \ell^+ \ell^-)}{dq^2} &= \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B_s}^3} \left( \left| \frac{1}{2m_{K^*} \sqrt{q^2}} \left\{ \frac{4m_{B_s}^2 |\vec{k}|^2}{m_{B_s} + m_{K^*}} A_2(q^2) \right. \right. \right. \\
 &\quad \left. \left. \left. - (m_{B_s}^2 - m_{K^*}^2 - q^2)(m_{B_s} + m_{K^*}) A_1(q^2) \right\} \right|^2 \right. \\
 &\quad \left. + \left| (m_{B_s} + m_{K^*}) A_1(q^2) + \frac{2m_{B_s} |\vec{k}|}{m_{B_s} + m_{K^*}} V(q^2) \right|^2 \right. \\
 &\quad \left. + \left| (m_{B_s} + m_{K^*}) A_1(q^2) - \frac{2m_{B_s} |\vec{k}|}{m_{B_s} + m_{K^*}} V(q^2) \right|^2 \right)
 \end{aligned}$$

# Phenomenological motivation: Why $b \rightarrow s(d)$



Figures from the LHCb Collaboration

- ▶  $d\Gamma(B_s \rightarrow \phi\ell^+\ell^-)/dq^2 = f(V, A_1, A_2)$
- ▶  $P'_5 = f(V, A_0, A_1, A_2, T_1, T_2, T_3)$
- ▶ Significant deviations from the Standard Model predictions
- ▶ Deviation in  $P'_5$  suggests a new physics contribution to  $C_9$

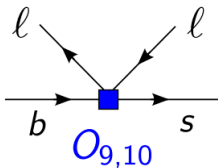
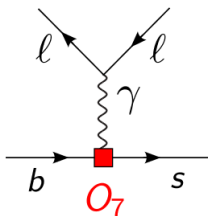
Descotes-Genon, et. al. 2013

- ▶ Most calculations performed in the high recoil region
- ▶ Only one unquenched lattice QCD calculation of

$$B \rightarrow K^*\ell^+\ell^- \text{ and } B_s \rightarrow \phi\ell^+\ell^- \quad \text{Horgan et. al. 2014}$$

# Theoretical framework: Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i O_i$$



$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s} \sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$

$$O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{\ell} \gamma_\mu \ell$$

$$O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{\ell} \gamma_\mu \gamma^5 \ell$$

# Theoretical framework: Form factors

$$\langle \phi(k, \varepsilon) | \bar{\psi} \gamma^\mu b | B_s(p) \rangle = V(q^2) \frac{2i \varepsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{m_{B_s} + m_\phi}$$

$$\begin{aligned} \langle \phi(k, \varepsilon) | \bar{\psi} \gamma^\mu \gamma_5 b | B_s(p) \rangle &= A_0(q^2) \frac{2m_\phi \varepsilon^* \cdot q}{q^2} q^\mu \\ &+ A_1(q^2) (m_{B_s} + m_\phi) \left[ \varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right] \\ &- A_2(q^2) \frac{\varepsilon^* \cdot q}{m_{B_s} + m_\phi} \left[ k^\mu + p^\mu - \frac{m_{B_s}^2 - m_\phi^2}{q^2} q^\mu \right] \end{aligned}$$

$$q_\nu \langle \phi(k, \varepsilon) | \bar{\psi} \sigma^{\nu\mu} b | B_s(p) \rangle = T_1(q^2) 4\varepsilon^{\mu\rho\tau\sigma} \varepsilon_\rho^* k_\tau p_\sigma$$

$$\begin{aligned} q_\nu \langle \phi(k, \varepsilon) | \bar{\psi} \sigma^{\nu\mu} \gamma_5 b | B_s(p) \rangle &= T_2(q^2) 2i \left[ \varepsilon^{*\mu} (m_{B_s}^2 - m_\phi^2) - (\varepsilon^* \cdot q)(p + k)^\mu \right] \\ &+ T_3(q^2) 2i (\varepsilon^* \cdot q) \left[ q^\mu - \frac{q^2}{m_{B_s}^2 - m_\phi^2} (p + k)^\mu \right] \end{aligned}$$



# Theoretical framework: Obtaining the V form factor for $B \rightarrow \phi l^+ l^-$

Taking the ratio of three to two point functions:

$$R_{V\mathcal{J}B}^{\mu\nu}(t, T, \vec{p}_V) = \frac{C_{VV B}^{\mu\nu}(t, T, \vec{p}_V)}{\sqrt{\frac{1}{3} \sum_i C_{VV}^{ii}(t, \vec{p}_V) \times C_{BB}(T-t)}} \sqrt{\frac{4E_V m_B}{e^{-E_V t} e^{-m_B(T-t)}}}$$
$$\xrightarrow{t, T \rightarrow \infty} \sum_{\lambda} \epsilon_{\mu}(p_V, \lambda) \langle V(p_V, \lambda) | \bar{\psi} \gamma^{\nu} b | B(p_B) \rangle$$

and using the relation

$$\sum_{\lambda} \epsilon^{\mu}(k, \lambda) \epsilon^{\nu*}(k, \lambda) = \frac{k^{\mu} k^{\nu}}{m_V^2} - g^{\mu\nu}$$

it can be shown that in the B-meson rest frame

$$V(q^2) = \frac{i\mathcal{R}_{VV B}^{ji}(\vec{k})(m_B + m_V)}{2m_B \epsilon^{0ijk} k_k} \quad (\text{no } i, j \text{ sum})$$

## B physics in the lattice

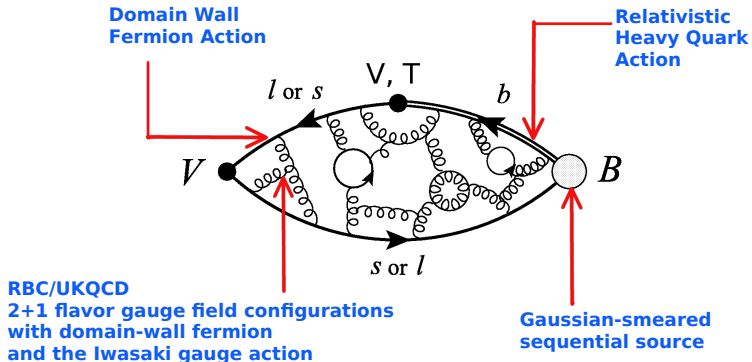
- ▶ Problem:  $ma > 1$
- ▶ Solution: Adapt the lattice to describe heavy quark physics in a carefully circumscribed kinematic range [El-Khadra, et. al., 1997](#)
- ▶ How?:

The relativistic heavy quark action

$$S = \sum_n \bar{\psi}_n \left( m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - \frac{a}{2} (D_0)^2 - \frac{a}{2} \zeta (\vec{D})^2 + \sum_{\mu, \nu} \frac{ia}{4} c_p \sigma_{\mu\nu} F_{\mu\nu} \right) \psi_n$$

- ▶ By tuning  $m_0$ ,  $\zeta$  and  $c_p$  all discretization errors of order  $O(|p|a)$  and  $O(ma)^n$  can be removed [El-Khadra, et al., 1997](#);  
[S. Aoki, et al., 2001](#); [Christ, et. al., 2006](#); [Lin and Christ, 2006](#)
- ▶ Nonperturbative tuning performed following [Y. Aoki et al. 2012](#)

# Details of the lattice calculation

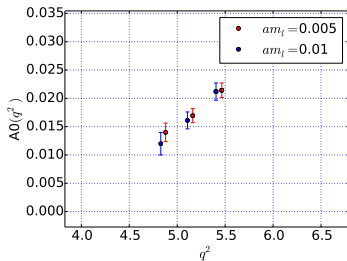
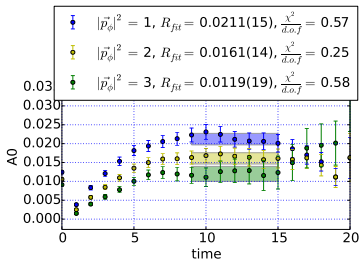
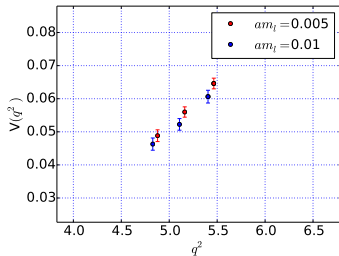
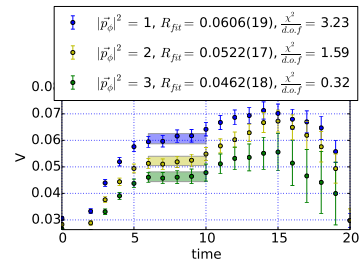


Parameters of the calculation

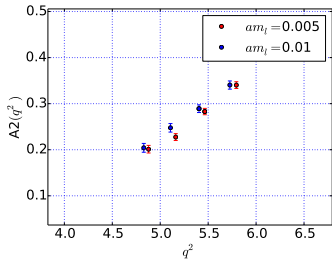
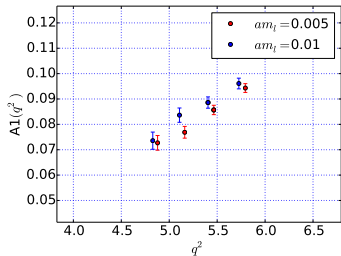
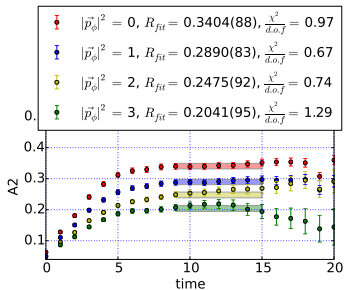
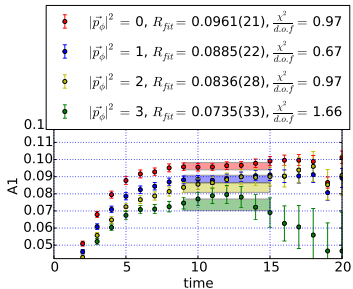
$L^3 \times T$	$a^{-1}$ [GeV]	$am_l$	$am'_s$	$M_\pi$ [MeV]	total # of configs
$24^3 \times 64$	1.785(5)	0.005	0.0343	338	1636
$24^3 \times 64$	1.785(5)	0.01	0.0343	434	1419

$a^{-1}$  taken from T. Blum et. al. 2014  $am'_s$  close to the physical strange quark mass

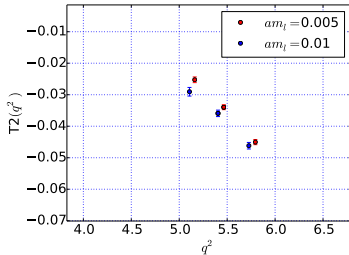
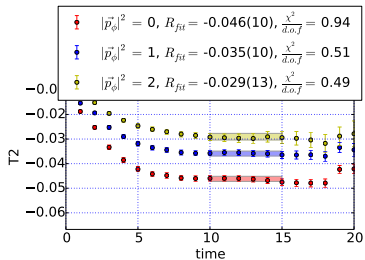
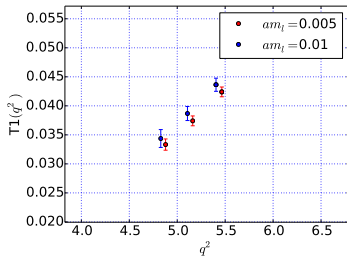
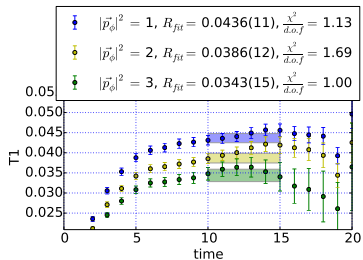
# First results: $B_s \rightarrow \phi \ell^+ \ell^-$



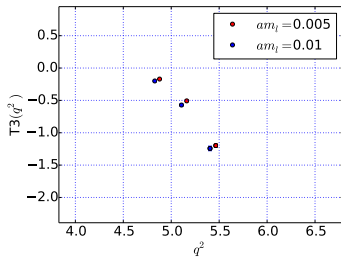
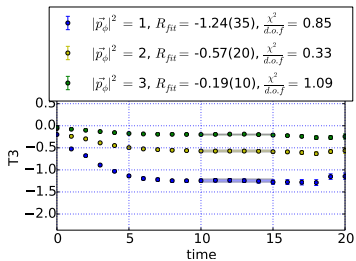
# First results: $B_s \rightarrow \phi \ell^+ \ell^-$



# First results: $B_s \rightarrow \phi \ell^+ \ell^-$



# First results: $B_s \rightarrow \phi \ell^+ \ell^-$



## Next steps

- ▶ Implement  $O(a)$  improvement.
- ▶ Obtain results for all ensembles (finer lattice spacing, physical pions).
- ▶ Perturbative computation of heavy-light renormalization factors and coefficients for  $O(a)$  improvement.
- ▶ Combined chiral-continuum extrapolation.
- ▶ Kinematic extrapolation to low  $q^2$  using the z-expansion.



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