Non-perturbative renormalization of Tensor currents in SF schemes

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in collaboration with

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ALPHA Collaboration

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Outline

- Motivation
- Tensor operator in SF formalism and O(a) improvement
- Perturbative 1-loop computation:
    1. Renormalization constants and cutoff effects
    2. Scheme matching and 2-loop SF Anomalous Dimension
- Non-Perturbative renormalization and running:
    3. Nf=0
    4. Nf=2
    5. Nf=3 (ongoing)

Motivation

1. The only bilinear missing in the ALPHA collaboration NP renormalization program

2. Tensor currents enter in some interesting physic processes like rare heavy meson decay processes.
   eg:

[Atoui et al. Eur.Phys.J. C74 (2014) 5, 2861]

$$B \to K^* l^+ l^-$$

[Horgan et al. Phys.Rev. D89 (2014) 9, 094501]

$$B \to \pi l^+ l^-$$

[Fermilab arXiv:1507.01618 [hep-ph]]
[Horgan et al. Phys.Rev.Lett. 112 (2014) 212003]

$$\cdots$$

[Talk by Straub at Moriond]

Tensor Operator O(a) improvement

O(a) operator improvement is achieved by considering

$$T^I_{\mu\nu} = T_{\mu\nu} + ac_T \left(\tilde{\partial}_\mu V_\nu - \tilde{\partial}_\nu V_\mu\right)$$

Where the improvement coefficient computed at 1-loop within the SF

[Sint et al. Nucl.Phys. B502 (1997) 251-268]

$$c_T = 0.00896(1) C_F g_0^2 + \mathcal{O}(g_0^4)$$

Focusing on the "electric" part of the operator the improvement reads as

$$T^I_{0k} = T_{0k} + ac_T \left(\tilde{\partial}_0 V_k - \tilde{\partial}_k V_0\right)$$

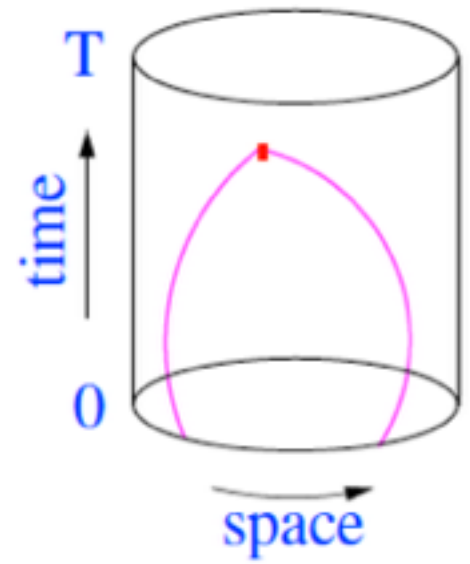
since we restrict ourselves to zero momentum the second term in the parenthesis vanishes.

SF correlation functions

In the SF scheme, contracting the tensor operator with a boundary source we define the tensor correlator

$$k_T(x_0) = -\frac{a^6}{2} \sum_{\mathbf{y},\mathbf{z}} \langle T_{0k}(x_0)(\bar{\zeta}(\mathbf{y})\gamma_k \zeta(\mathbf{z}))\rangle$$

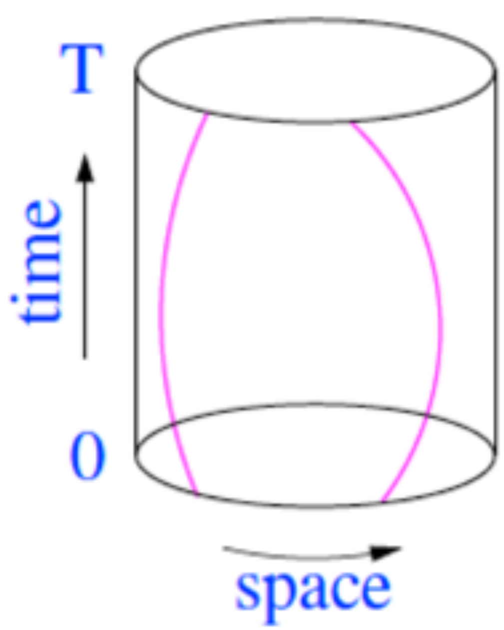
$$k_T^I(x_0) = k_T(x_0) + a c_T\, \tilde{\partial}_0 k_V(x)\Big|_{x_0}$$



and the "boundary-to-boundary" correlators

$$k_1 = -\frac{a^{12}}{6L^6} \sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \langle(\bar{\zeta}'(\mathbf{y}')\gamma_k \zeta'(\mathbf{z}')(\bar{\zeta}(\mathbf{y})\gamma_k \zeta(\mathbf{z}))\rangle$$

$$f_1 = -\frac{a^{12}}{2L^6} \sum_{\mathbf{y},\mathbf{z},\mathbf{y}',\mathbf{z}'} \langle(\bar{\zeta}'(\mathbf{y}')\gamma_5 \zeta'(\mathbf{z}')(\bar{\zeta}(\mathbf{y})\gamma_5 \zeta(\mathbf{z}))\rangle$$



SF Renormalization Scheme

We impose mass independent renormalization conditions of the form

$$Z_T^{(\alpha)}(g_0, L/a)\frac{k_T^I(L/2)}{f_1^{1/2-\alpha}k_1^\alpha} = \left.\frac{k_T(L/2)}{f_1^{1/2-\alpha}k_1^\alpha}\right|_{m_0=m_c, g_0=0}$$

defining a class of renormalization schemes that depend on the free parameter α and the angle θ entering spatial boundary conditions

Following the standard SF iterative renormalization procedure, we define the SSF

$$\Sigma_T^{(s)}(u, a/L) = \left.\frac{Z_T^{(s)}(g_0, a/2L)}{Z_T^{(s)}(g_0, a/L)}\right|_{\bar{g}^2(L)=u}$$

The continuum SSF is then defined as

$$\sigma_T(g^2) = \exp\left\{\int_g^{\sqrt{\sigma(g^2)}} dg' \frac{\gamma(g')}{\beta(g')}\right\} \longrightarrow \sigma_T^{(s)}(u) = \lim_{a\to 0}\Sigma_T^{(s)}(u, a/L)$$ composite op

[Capitani et al. Nucl.Phys. B544 (1999) 669-698]

$$-\log(2) = \int_g^{\sqrt{\sigma(g^2)}} dg' \frac{1}{\beta(g')} \longrightarrow \sigma(u) = \bar{g}^2(2L),\ u = \bar{g}^2(L)$$ coupling

[Della Morte et al. Nucl.Phys. B713 (2005) 378-406]

SFPT Renormalization

We impose mass independent renormalization conditions of the form

$$Z_T^{(\alpha)}(g_0, L/a)\frac{k_T^I(L/2)}{f_1^{1/2-\alpha}k_1^\alpha} = \left.\frac{k_T(L/2)}{f_1^{1/2-\alpha}k_1^\alpha}\right|_{m_0=m_c,g_0=0}$$

Expanding in powers of g_0^2

$$Z_T(g_0, L/a) = 1 + \sum_{n=1}^{\infty} g_0^{2n} Z_T^{(n)}(L/a) \quad \text{and} \quad k_T(x_0) = \sum_{n=0}^{\infty} g_0^{2n} k_T^{(n)}(x_0)$$

at 1-loop the improvement for the tensor correlator reads

$$k_T^I(x_0) = k_T^{(0)}(x_0) + g_0^2 k_T^{(1)}(x_0) + a g_0^2 c_T^{(1)} \tilde{\partial}_0 k_V^{(0)}(x)\Big|_{x_0} + \mathcal{O}(a g_0^4)$$

and the 1-loop renormalization constant for $\alpha = 0$

$$Z_T^{(1)}(x_0, L/a) = -\left\{\frac{\bar{k}_T^{(1)}(x_0)}{k_T^{(0)}(x_0)} - \frac{\bar{f}_1^{(1)}}{2f_1^{(0)}} + a c_T^{(1)}\frac{\tilde{\partial}_0 k_V^{(0)}(x)|_{x_0}}{k_T^{(0)}(x_0)}\right\}, \quad \bar{F}^{(1)} = F^{(1)} + F_{bi}^{(1)} + m_c^{(1)}\frac{\partial}{\partial m_0}F^{(0)}$$

[Sint and Weisz Nucl.Phys. B545 (1999) 529-542]

1-loop SSF cutoff effects

In order to study the approach to the continuum limit we define the relative deviation

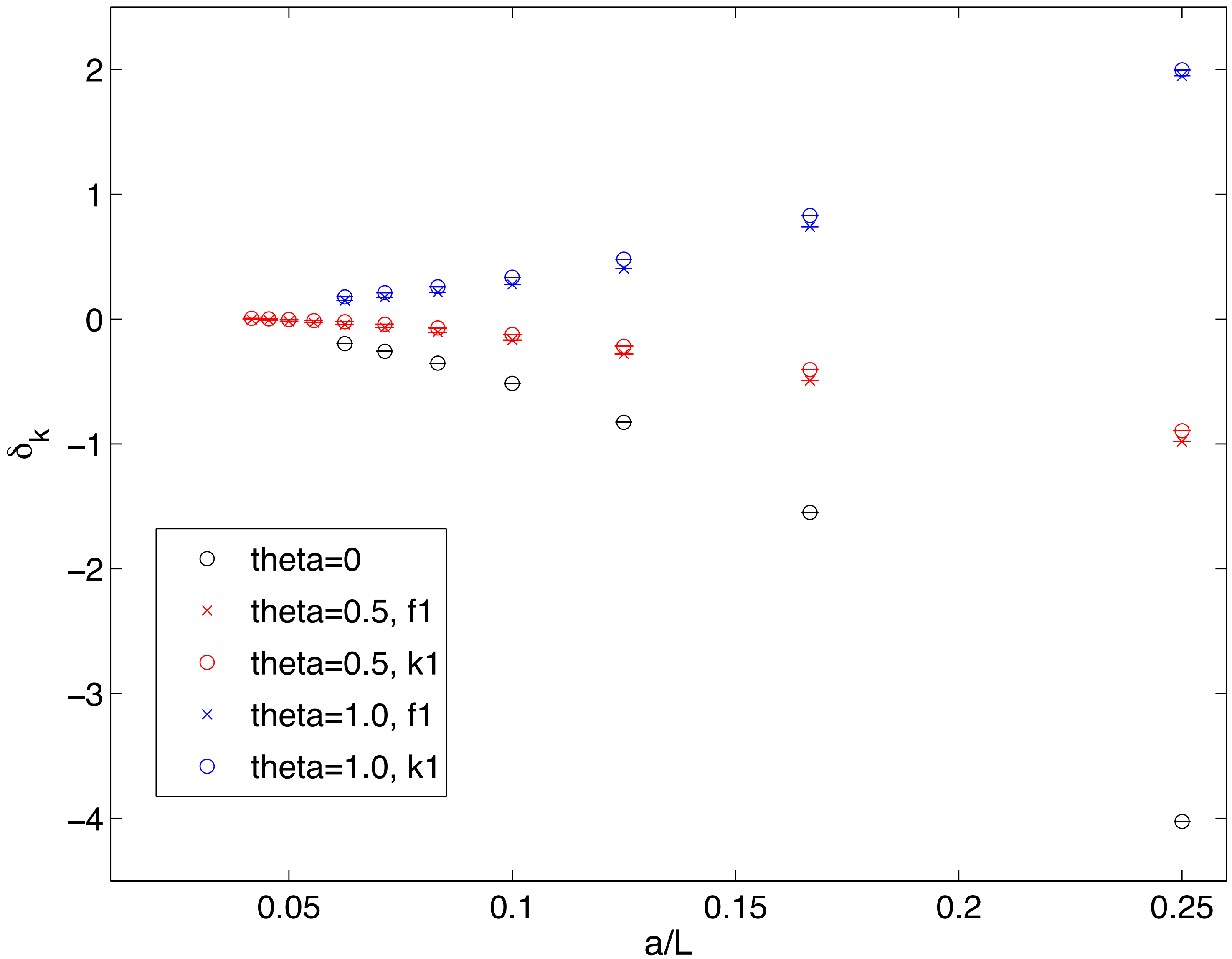
$$\Delta_k(g_0^2, L/a) = \frac{\Sigma_T(g_0^2, L/a)\big|_{u=g_{SF}^2(L)} - \sigma_T(u)}{\sigma_T(u)} = \delta_k g_0^2 + \mathcal{O}(g_0^4)$$

[Sint et al. JHEP 0603 (2006) 089]

$$\delta_k = \frac{Z_T^{(1)}(2L/a) - Z_T^{(1)}(L/a)}{\gamma_0 \log(2)} - 1$$

[Sint et al. Nucl.Phys. B545 (1999) 529-542]

for O(a) improved fermions



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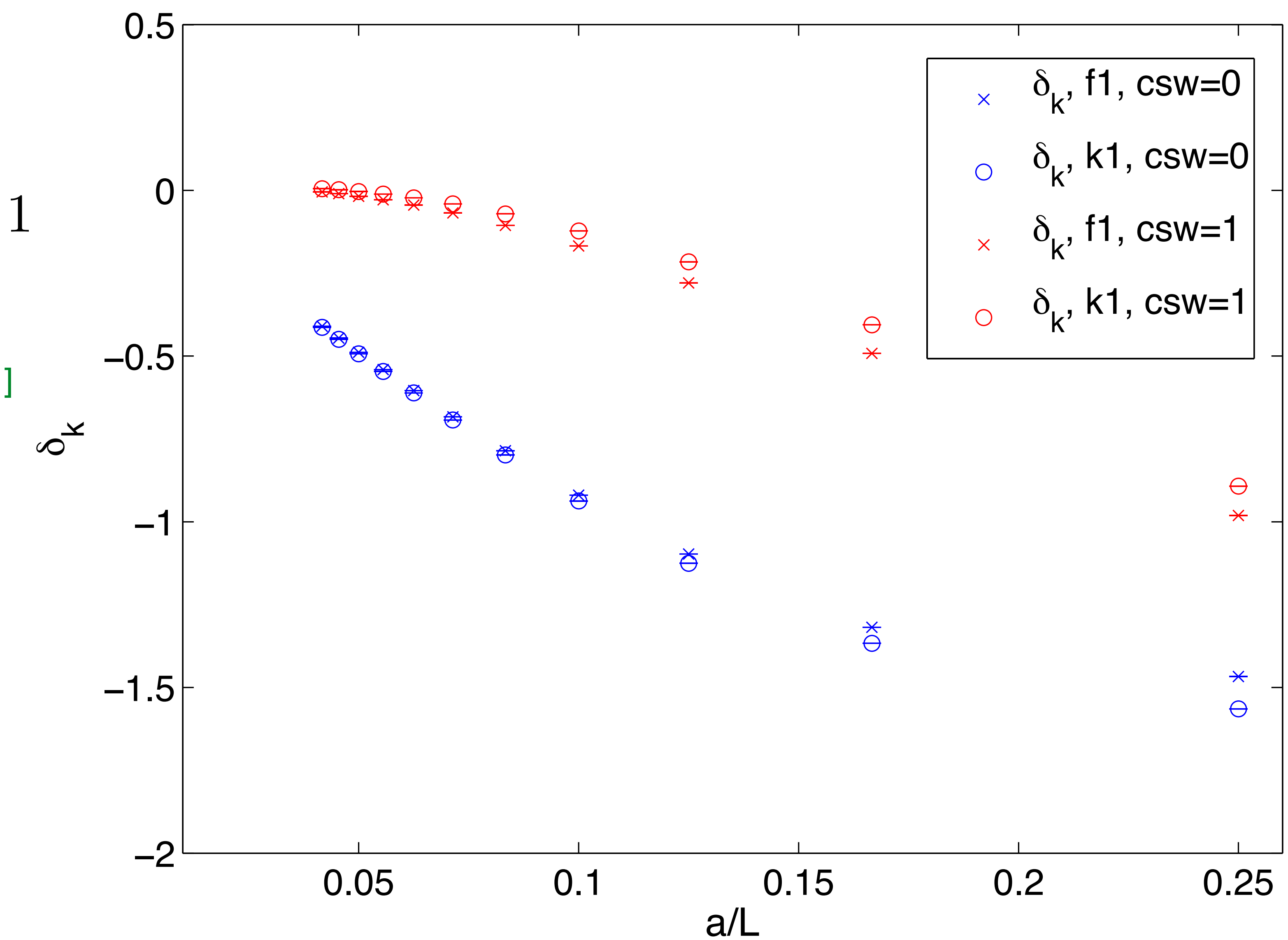
at $\theta = 0.5$



SFPT Renormalization

Considering the asymptotic expansion in powers of a/L

$$Z_T^{(1)}(L/a) \simeq \sum_{\nu=0}^{\infty} \left(\frac{a}{L}\right)^{\nu} \left\{ r_\nu + s_\nu \log\left(\frac{L}{a}\right) \right\}$$

[Bode et al. Nucl.Phys. B608 (2001) 481]

and focusing on $\nu = 0$ coefficients:

For the two schemes and for the three values of theta we checked $s_0 = \gamma_0$

$$\gamma(g) = -g^2(\gamma^{(0)} + g^2\gamma^{(1)} + g^4\gamma^{(2)} + \mathcal{O}(g^6))$$
$$\beta(g) = -g^3(b^{(0)} + g^2 b^{(1)} + g^4 b^{(2)} + \mathcal{O}(g^6))$$

$$\gamma_T^{(0)} = \frac{2C_F}{(4\pi)^2}$$

θ	$\gamma_0/s_0^{f_1}$	$\gamma_0/s_0^{k_1}$
0	$1.01(1)$	$1.01(1)$
0.5	$1.00(7)$	$1.00(7)$
1	$0.97(3)$	$0.97(5)$

SFPT Renormalization

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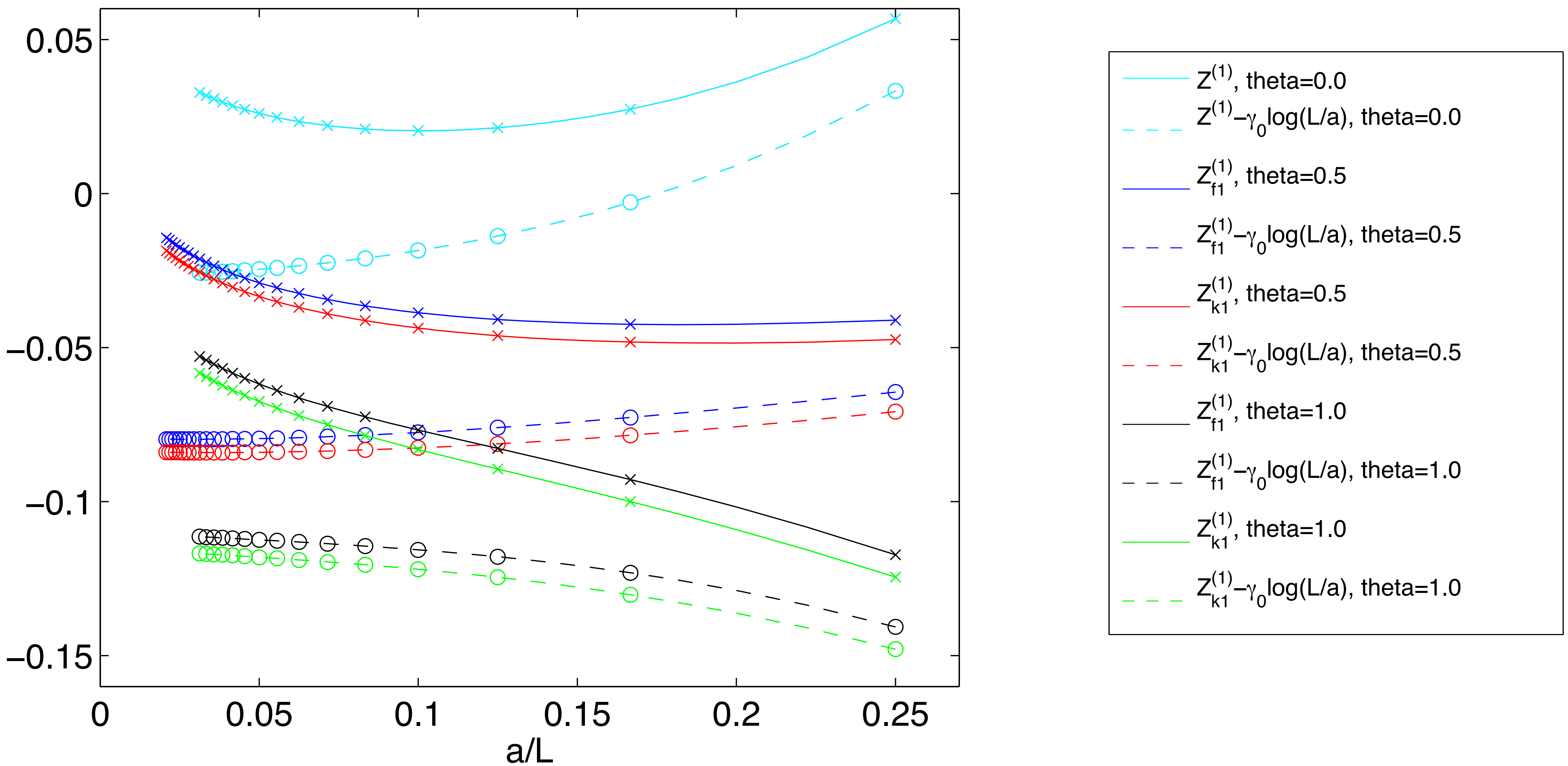

Scheme Matching

SF anomalous dimensions are given by

$$\gamma_{SF}^{(1)} = \gamma_{ref}^{(1)} + 2b_0\chi^{(1)} - \gamma^{(0)}\chi_g^{(1)}$$

where

$$\chi^{(1)} = \chi_{SF,ref}^{(1)} = \chi_{SF,lat}^{(1)} - \chi_{ref,lat}^{(1)} \quad \text{and} \quad \chi_g^{(1)} = 2b_0\log(\mu L) - \frac{1}{4\pi}(c_{1,0} + c_{1,1}N_f)$$

[Sint and Sommer Nucl.Phys. B465 (1996) 71-98]

and the matching coefficient SF-LAT are the finite parts of the 1-loop renormalization coefficients

$$\chi_{SF,lat}^{(1)} = r_0$$

considering two “ref” schemes $ref = \begin{cases} \overline{\mathrm{MS}} \\ \mathrm{RI}' \end{cases}$

[Skouroupathis et al. Phys.Rev. D79 (2009) 094508]
[Capitani et al. Nucl.Phys. B593 (2001) 183-228]
[Gracey Nucl.Phys. B667 (2003) 242-260]

θ	$\gamma_{SF}^{(1),f_1}$	$\gamma_{SF}^{(1),k_1}$
0	0.014330(13)-0.00067164(42)N_f	0.014330(13)-0.00067164(42)N_f
0.5	0.006945(55)-0.0002241(16)N_f	0.006361(65)-0.0001886(16)N_f
1	0.002693(28)-0.00003364(84)N_f	0.001956(16)-0.00007830(50)N_f

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[Skouroupathis et al (2009)]
0811.4264 [hep-lat]
[Capitani et al. (2000)]
hep-lat/0007004
[Gracey (2003)]

θ	$\gamma_{SF}^{(1),f_1}/\gamma_0$	$\gamma_{SF}^{(1),k_1}/\gamma_0$
0	0.8486(8)-0.039772(25)N_f	0.8486(8)-0.039772(25)N_f
0.5	0.4113(33)-0.0132(1)N_f	0.4376(33)-0.0111(1)N_f
1	0.1594(16)-0.0019(5)N_f	0.1158(10)-0.0046(3)N_f

$$\gamma_{\bar{M}S}^{(1)}/\gamma_0$$

$$0.1910 - 0.091N_f$$

NP computation of SSF

$$Z_T^{(\alpha)}(g_0, L/a)\frac{k_T^I(L/2)}{f_1^{1/2-\alpha}k_1^\alpha} = \left.\frac{k_T(L/2)}{f_1^{1/2-\alpha}k_1^\alpha}\right|_{g_0=0}$$

Reminding the definition of the SSF

$$\Sigma_T^{(s)}(u, a/L) = \left.\frac{Z_T^{(s)}(g_0, a/2L)}{Z_T^{(s)}(g_0, a/L)}\right|_{\bar{g}^2(L)=u}$$

We have performed NP calculation for several values of Nf

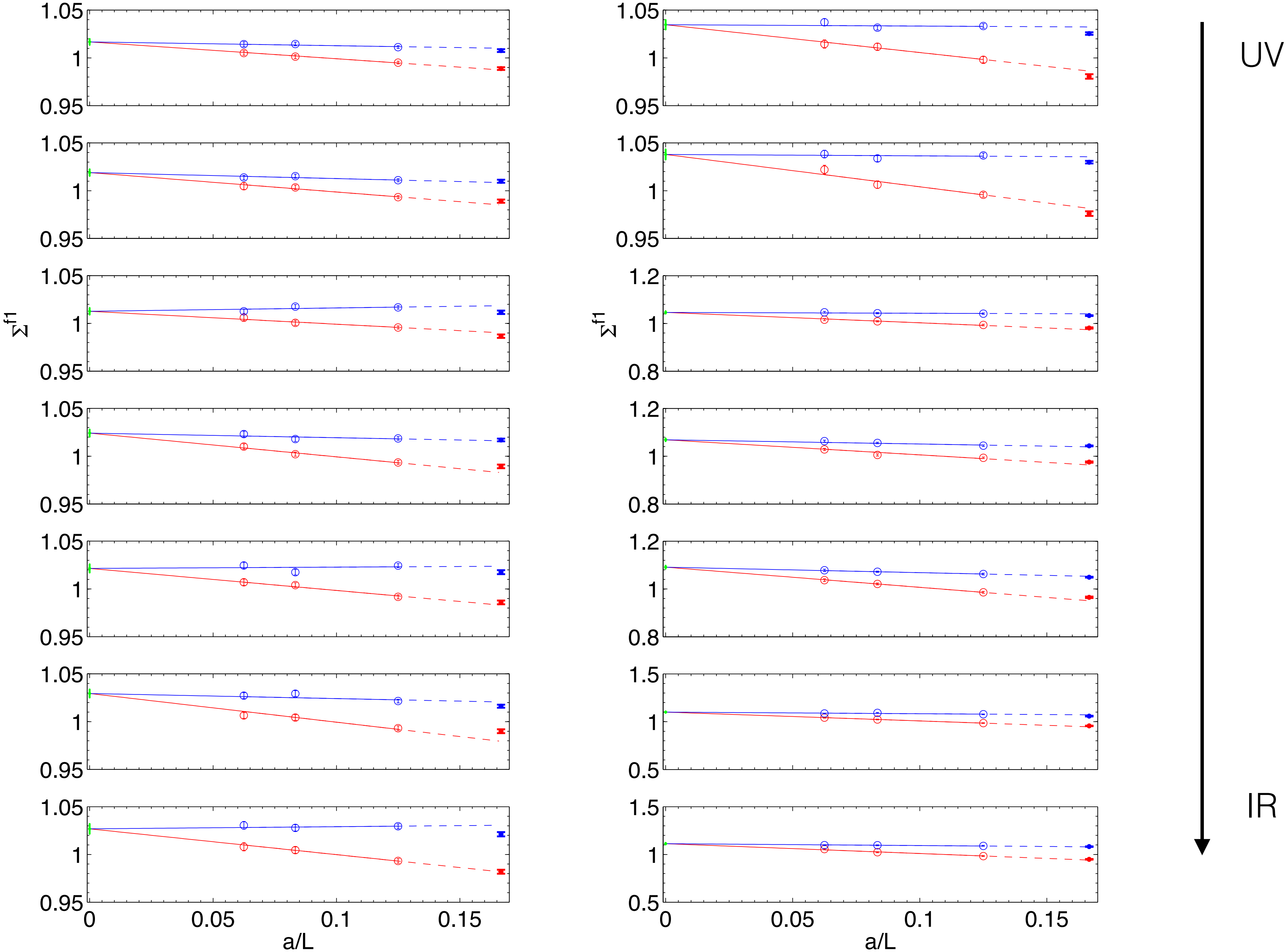
N_f	# coup	θ	status
0	14	0.5	preliminary
2	6	0.5	preliminary
3	8(SF)*	0.5	ongoing-preliminary

* talks by S. Sint & P. Fritzsch

Continuum Extrapolation

$N_f = 0$

$$\sigma_T^{(s)}(u) = \lim_{a \to 0} \Sigma_T^{(s)}(u, a/L)$$



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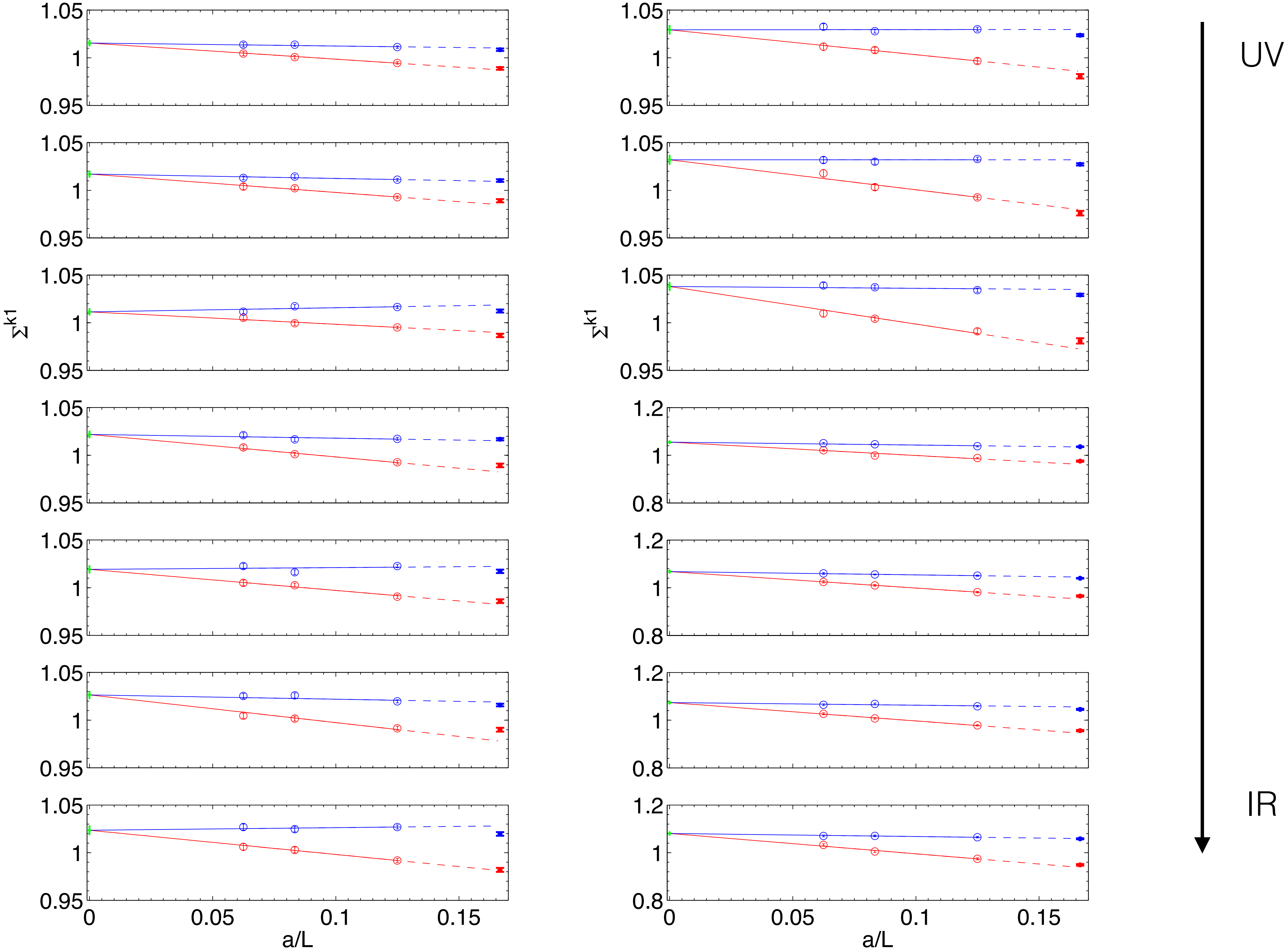
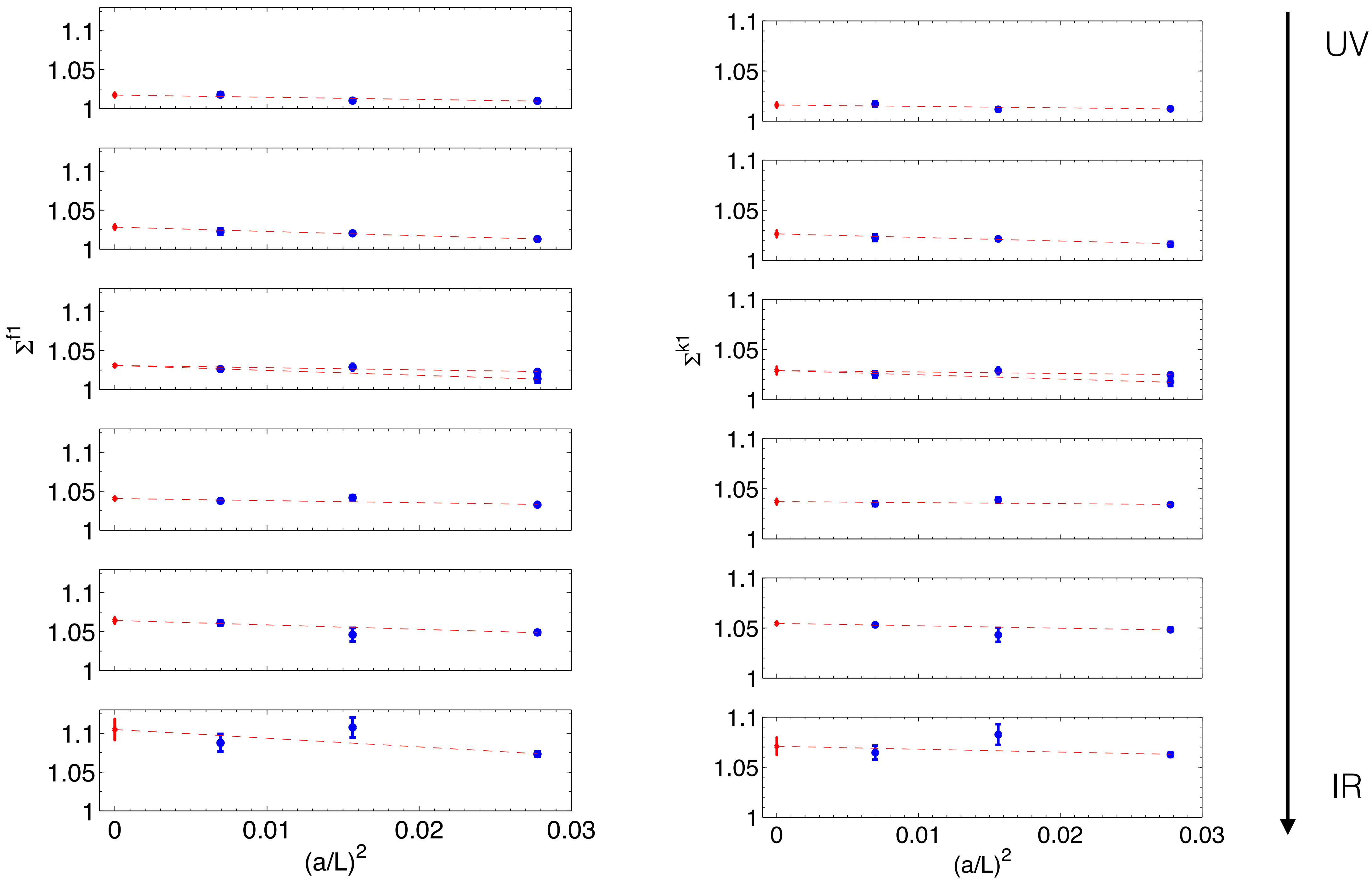


Continuum Extrapolation

$N_f = 2$

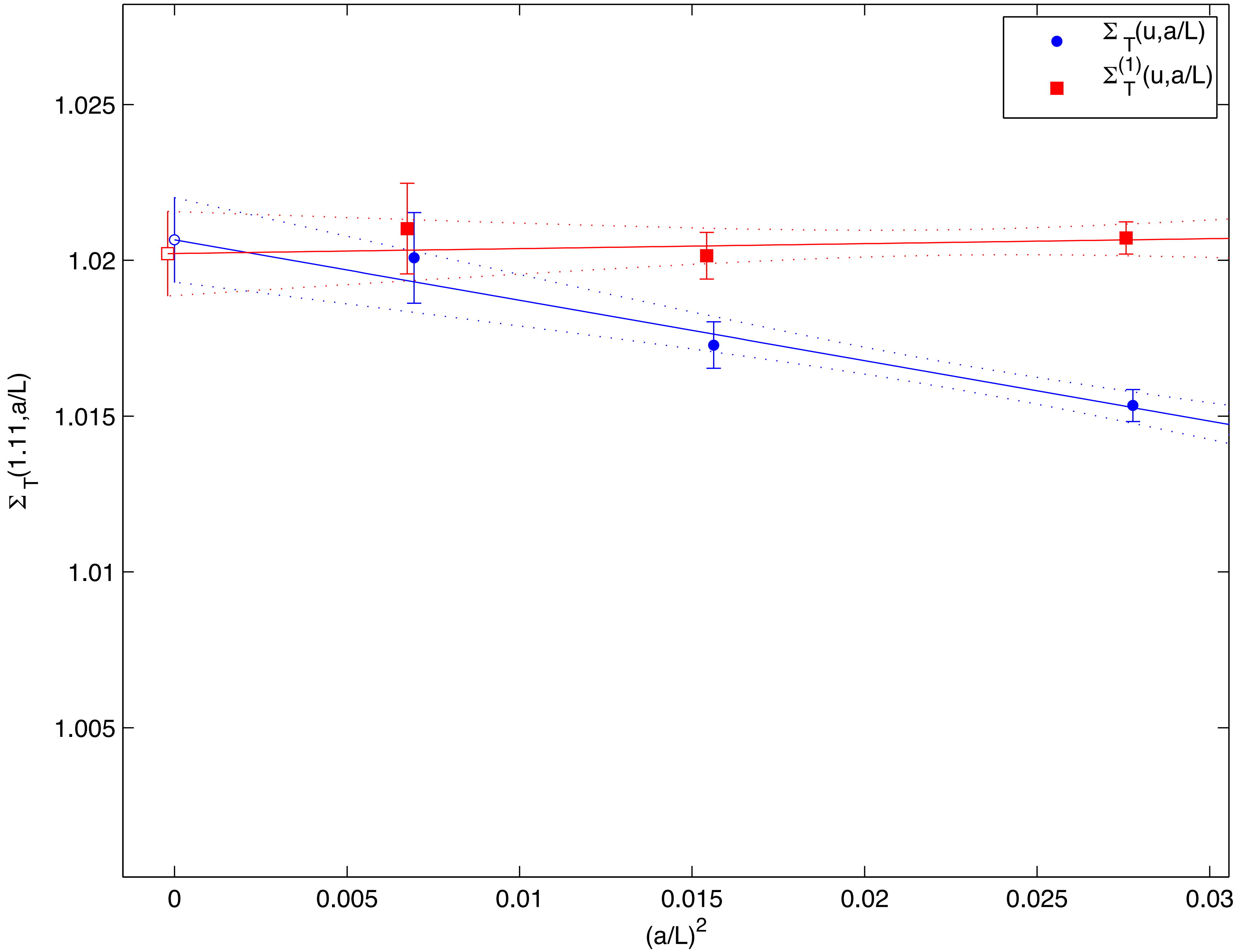
$$\sigma_T^{(s)}(u) = \lim_{a \to 0} \Sigma_T^{(s)}(u, a/L)$$



Continuum Extrapolation

$N_f = 3$

$$\sigma_T^{(s)}(u) = \lim_{a\to 0} \Sigma_T^{(s)}(u, a/L) \qquad \Sigma_T^{(1)}(u, a/L) = \frac{\Sigma_T(u, a/L)}{1 + \delta_k(a/L)u}$$



Continuum Extrapolation

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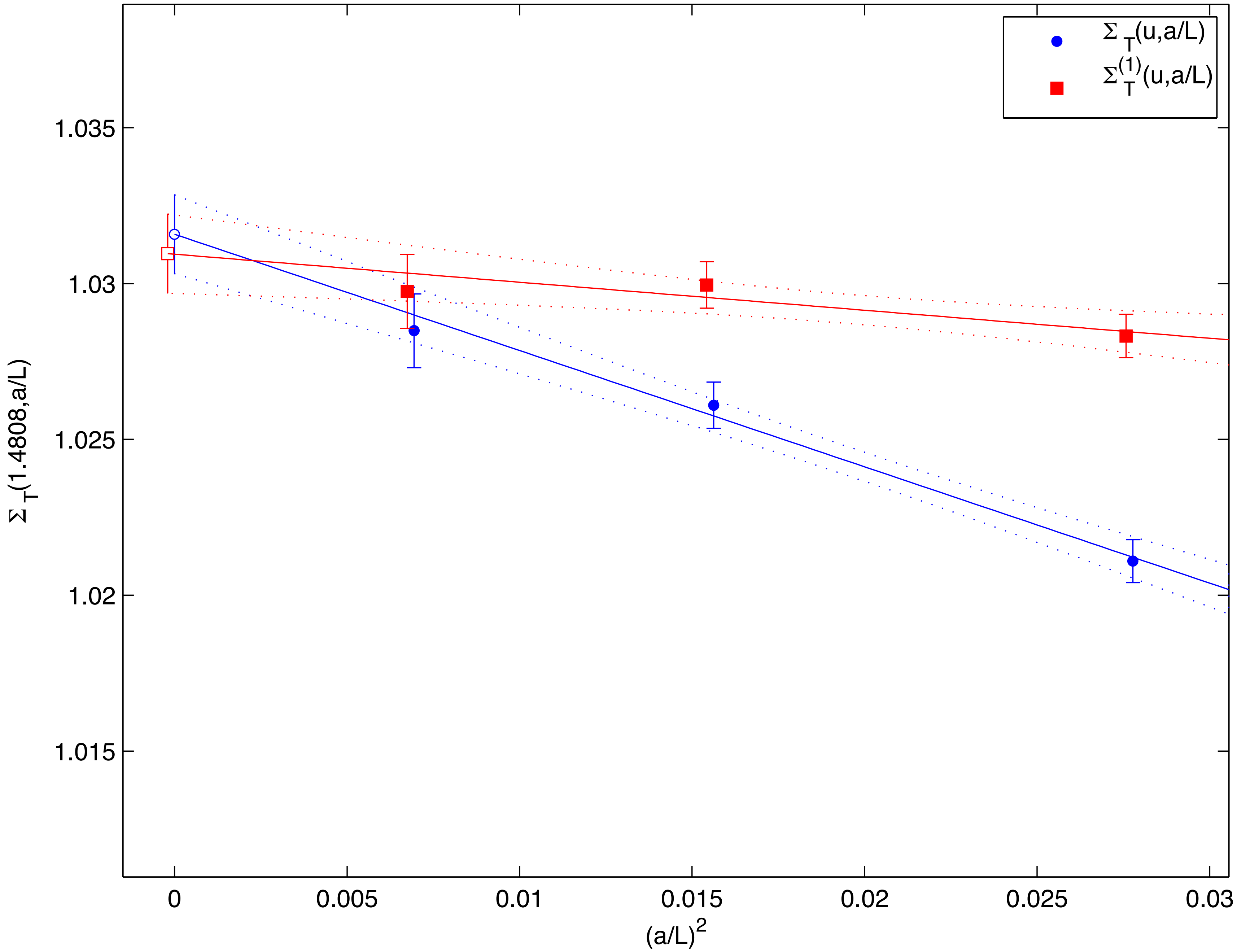

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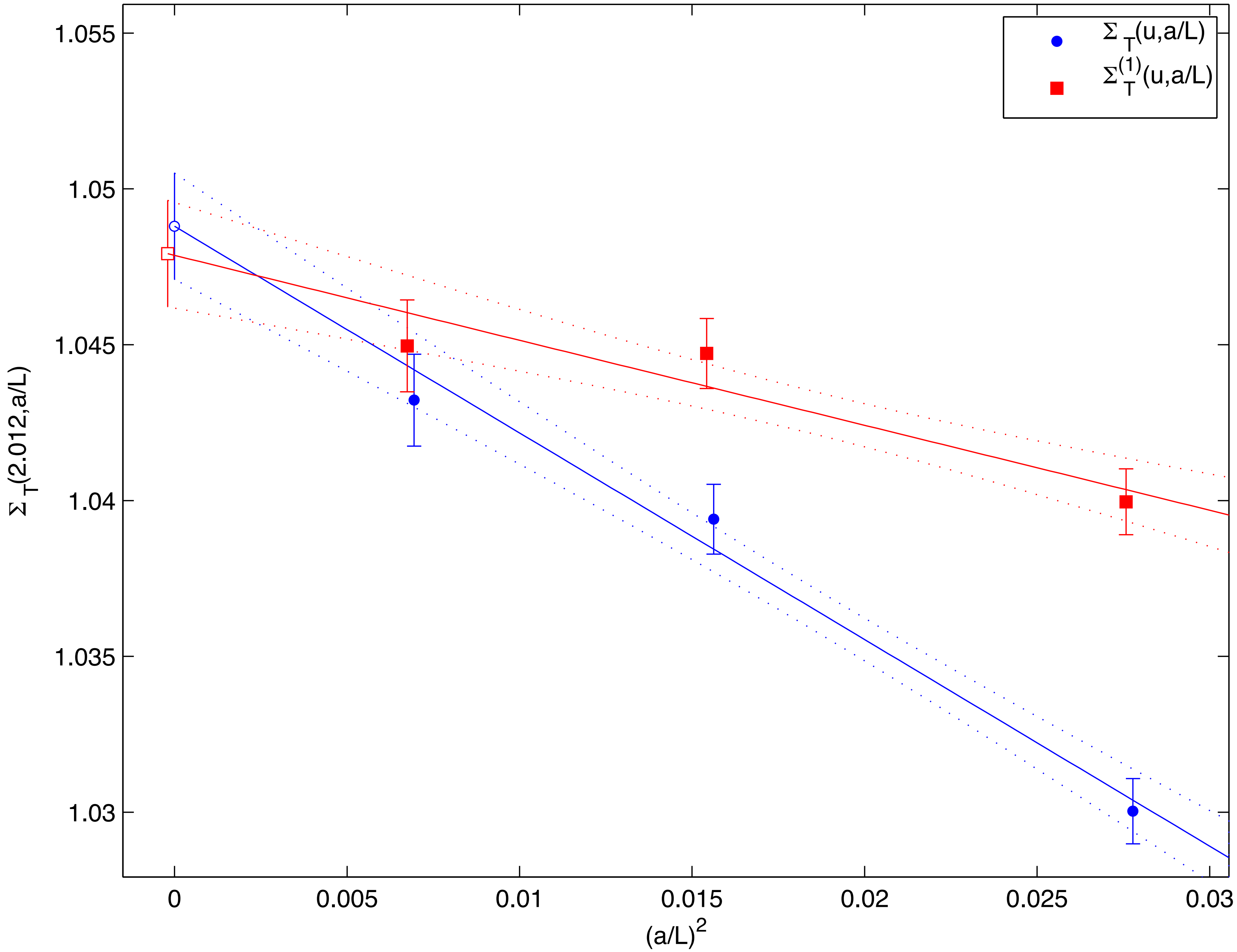


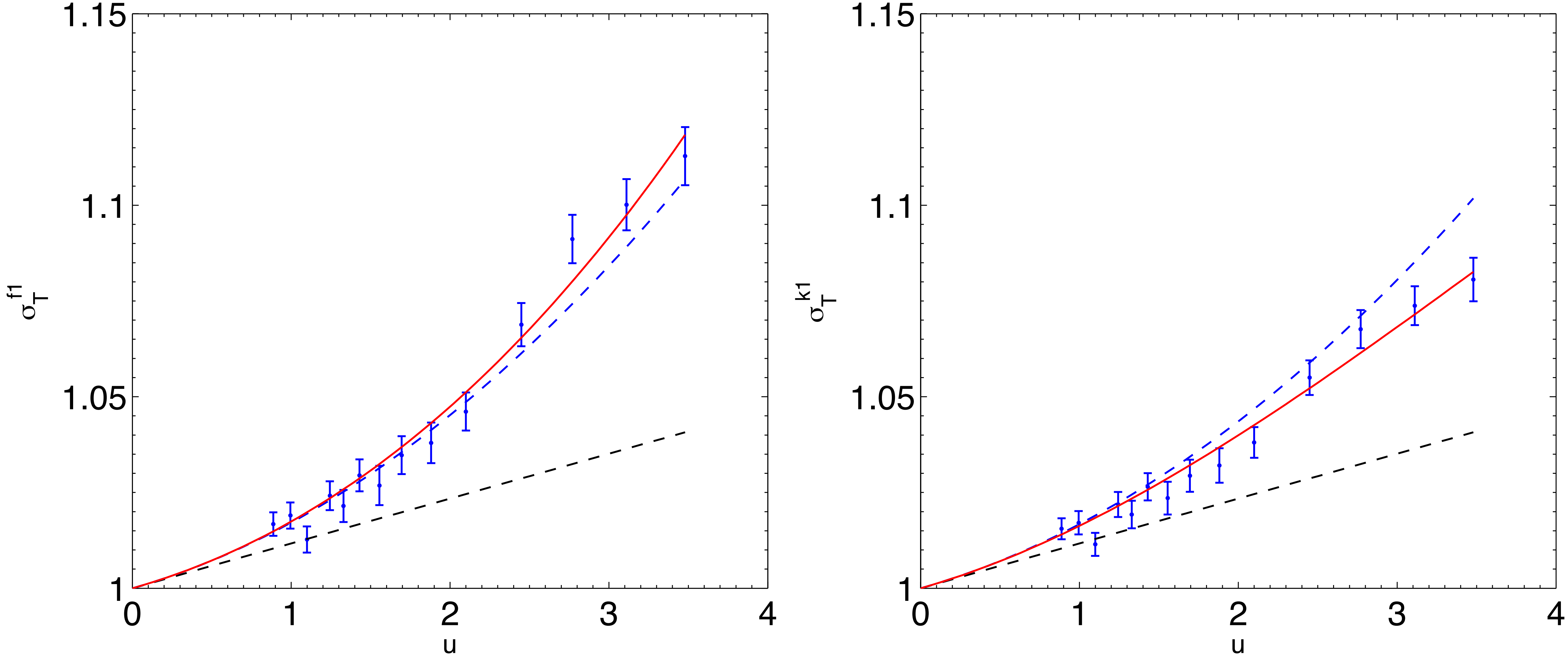
SSF

$N_f = 0$

$$\sigma_T^{(s)}(u) = 1 + \sigma_s^{(1)} u + \sigma_s^{(2)} u^2 + \sigma_s^{(3)} u^3 + \mathcal{O}(u^4)$$

$$\sigma_2 = \gamma^{(1)} \log(2) + \left[\frac{1}{2} (\gamma^{(0)})^2 + b_0 \gamma^{(0)} \right] (\log(2))^2$$

$$\sigma_1 = \gamma^{(0)} \log(2)$$



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SSF

$N_f = 2$

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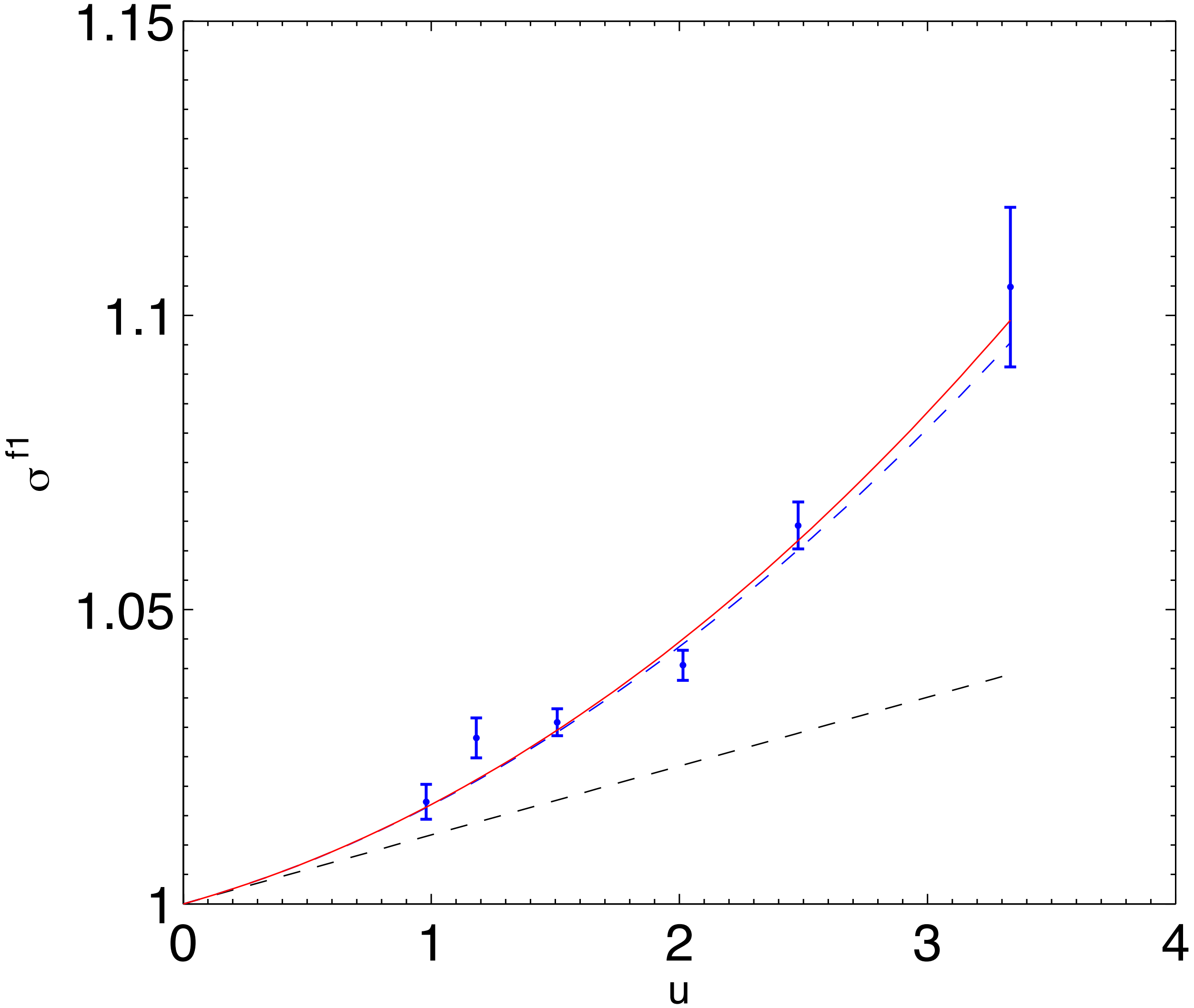
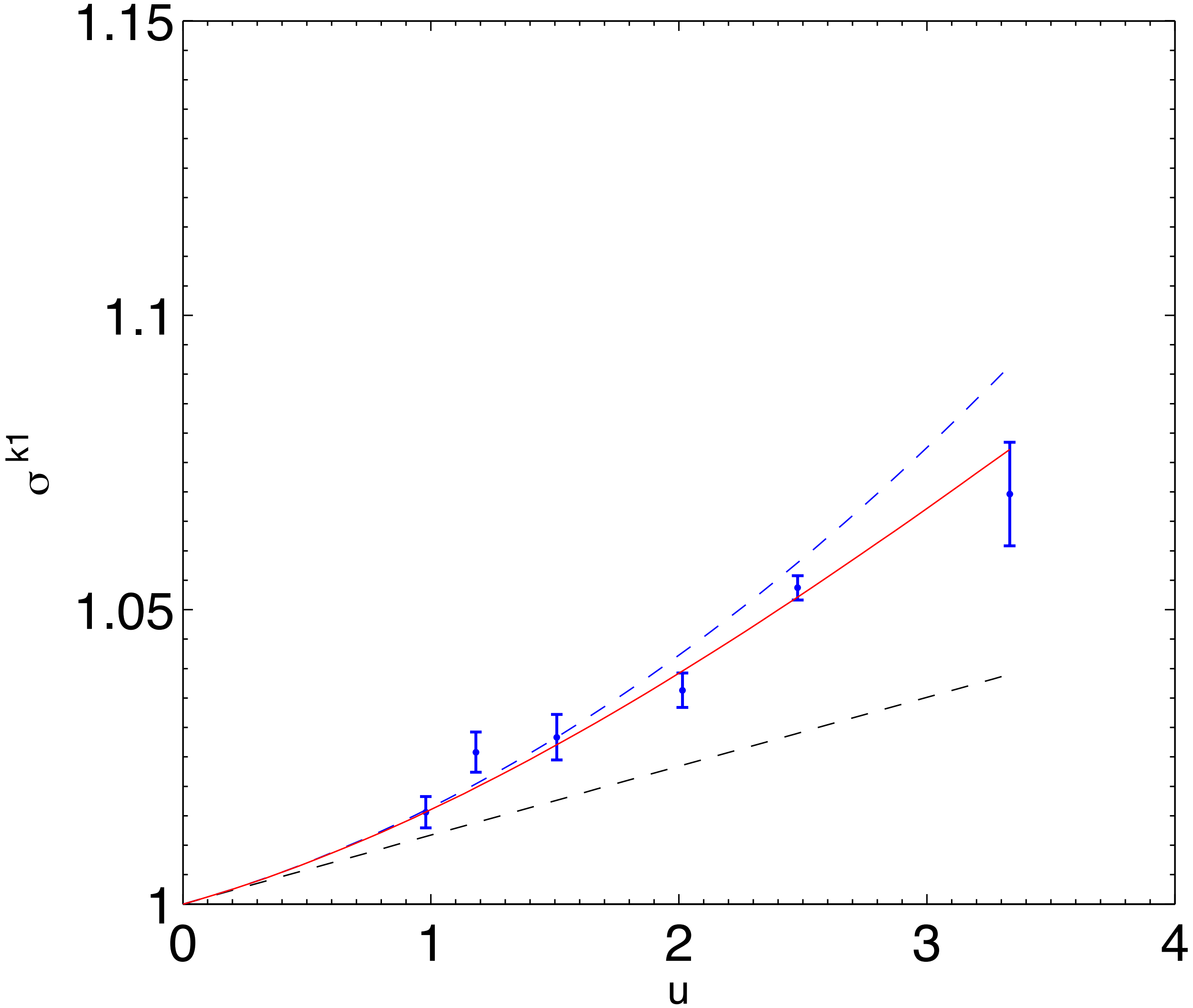
$$\sigma_1 = \gamma^{(0)} \log(2)$$





Kobe, Lattice 2015

SSF

$N_f = 3$

$$\sigma_T^{(s)}(u) = 1 + \sigma_s^{(1)} u + \sigma_s^{(2)} u^2 + \sigma_s^{(3)} u^3 + \mathcal{O}(u^4)$$

$$\sigma_2 = \gamma^{(1)} \log(2) + \left[\frac{1}{2} (\gamma^{(0)})^2 + b_0 \gamma^{(0)} \right] (\log(2))^2$$

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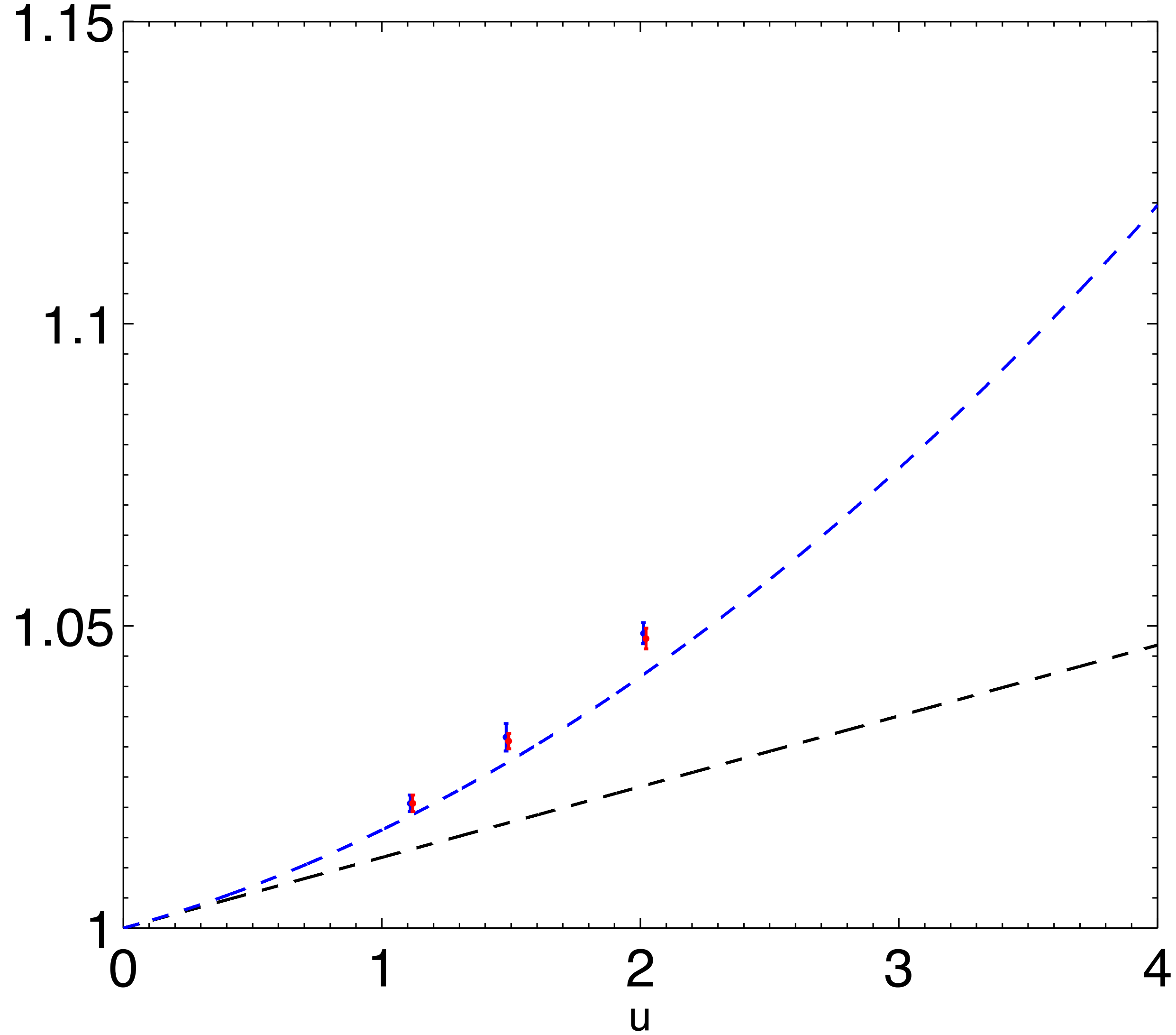


Running

The RG evolution between two scales is defined as

$$U(\mu_2, \mu_1) = \exp\left\{\int_{\bar{g}(\mu_1)}^{\bar{g}(\mu_2)} dg \frac{\gamma(g)}{\beta(g)}\right\} = \lim_{a\to 0} \frac{Z(g_0, a\mu_2)}{Z(g_0, a\mu_1)}$$

Once defined the perturbative evolution coefficient as

$$\hat{c}(\mu) = \frac{O_{RGI}}{O(\mu)} = \left[\frac{\bar{g}^2(\mu)}{4\pi}\right]^{-\frac{\gamma_0}{2b_0}} \exp\left\{-\int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right)\right\}$$

We can match it to the NP computations to an high energy $\mu_{PT} = 2^n \mu_{HAD}$

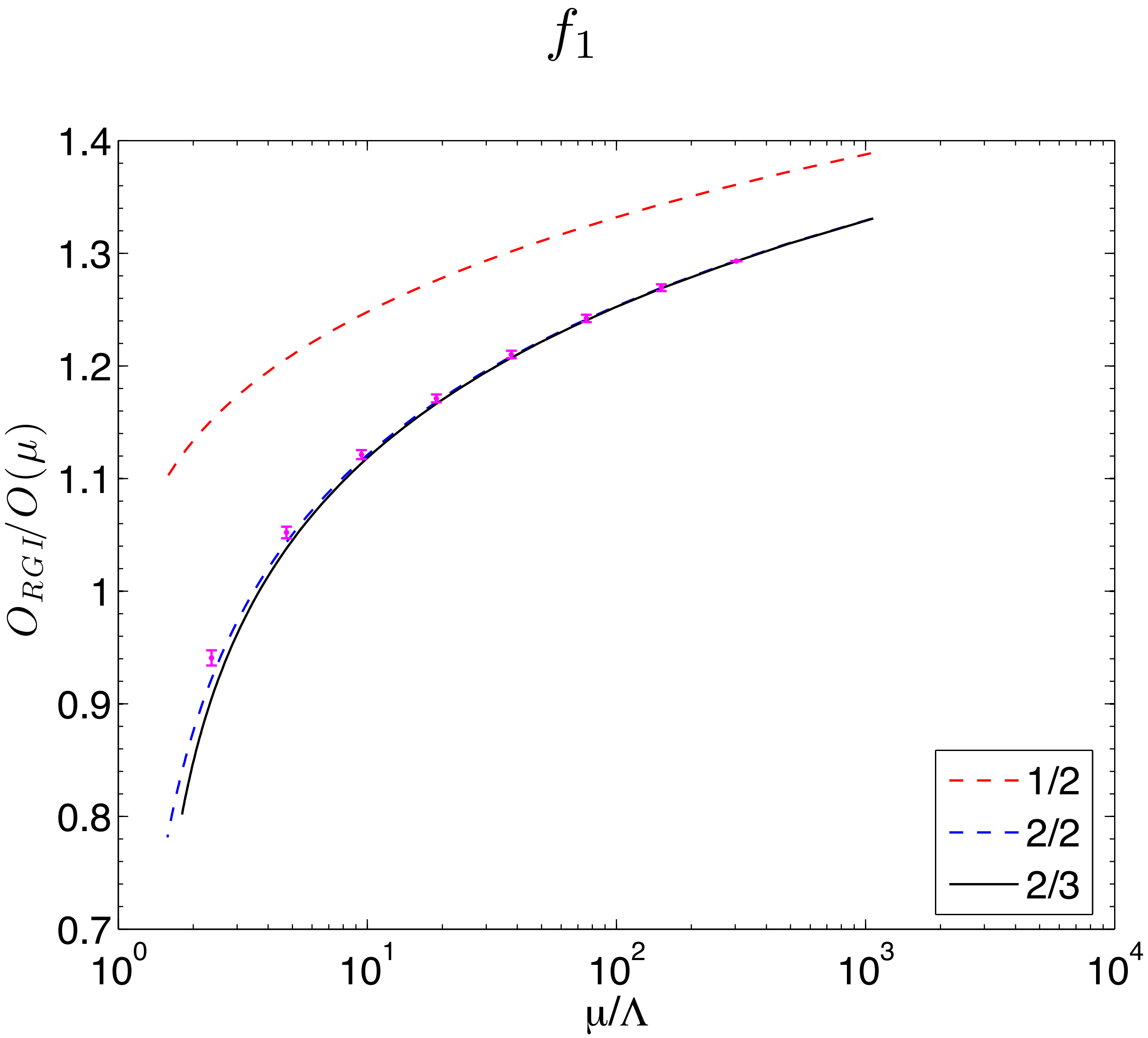
$$\hat{c}(\mu_{HAD}) = \hat{c}(\mu_{PT}) U(\mu_{PT}, \mu_{HAD})$$

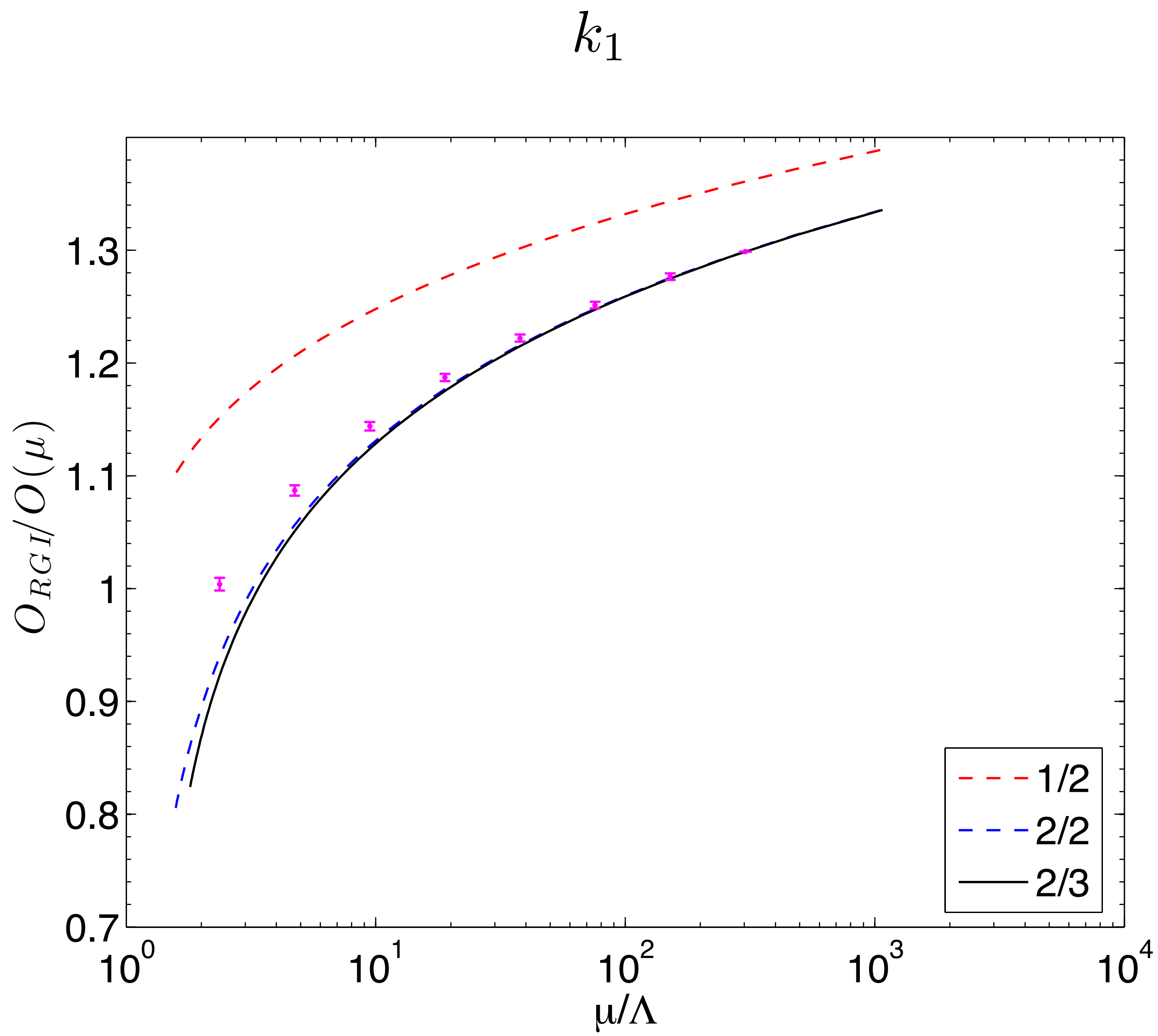
Where U is given by products of SSFs

$$\prod_{i=1}^{n} [\sigma_T(u_i)]^{-1}$$

Running

$N_f = 0$





	f_1	k_1
$\frac{O_{RGI}}{O(\mu_{had})}$	0.940(7)	1.004(6)

$\mu_{PT}/\Lambda = 302.9751$

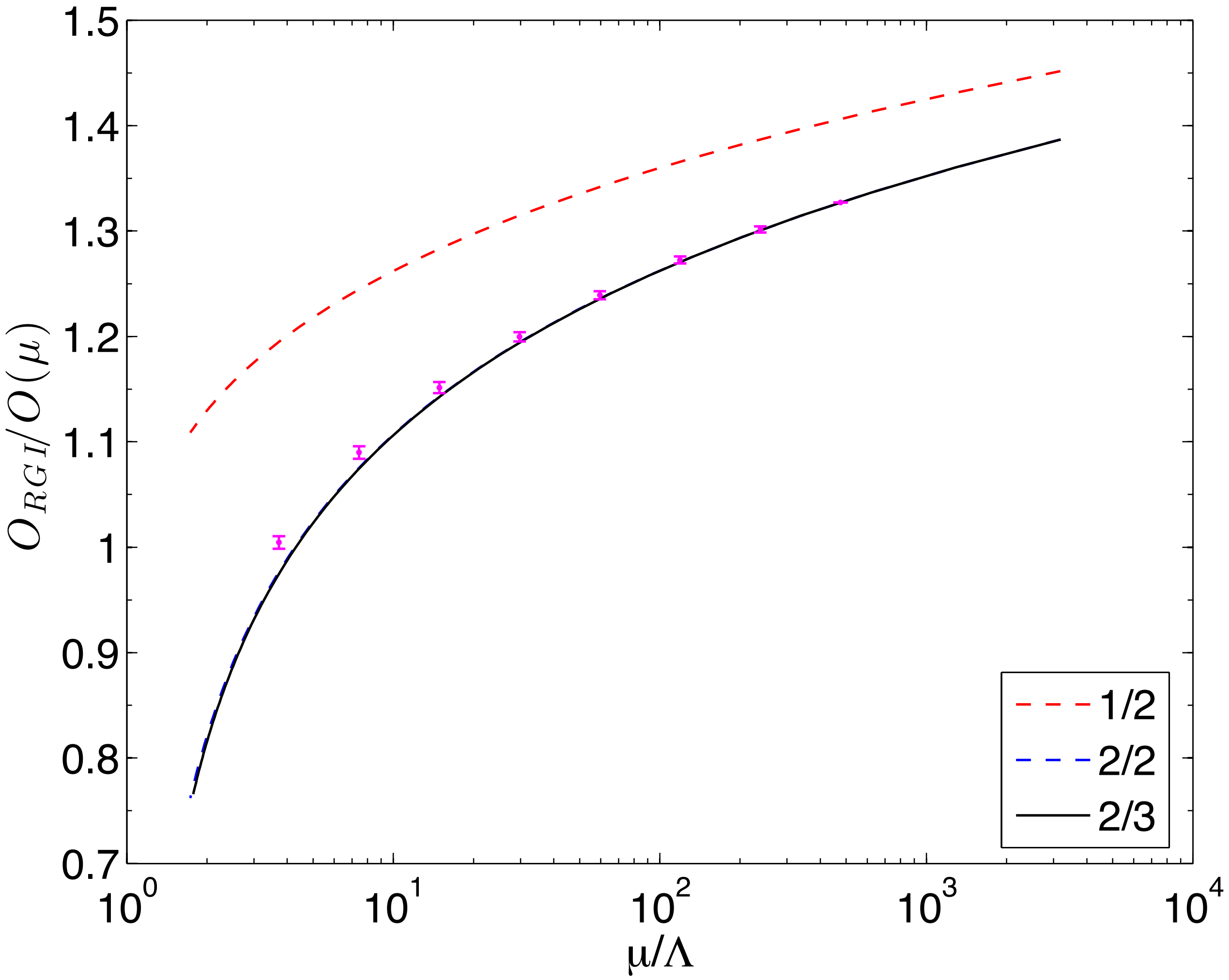
[S. Capitani et al., Nucl. Phys. B544 (1999) 669]

$225\,\mathrm{MeV} \longrightarrow 28.84\,\mathrm{GeV}$

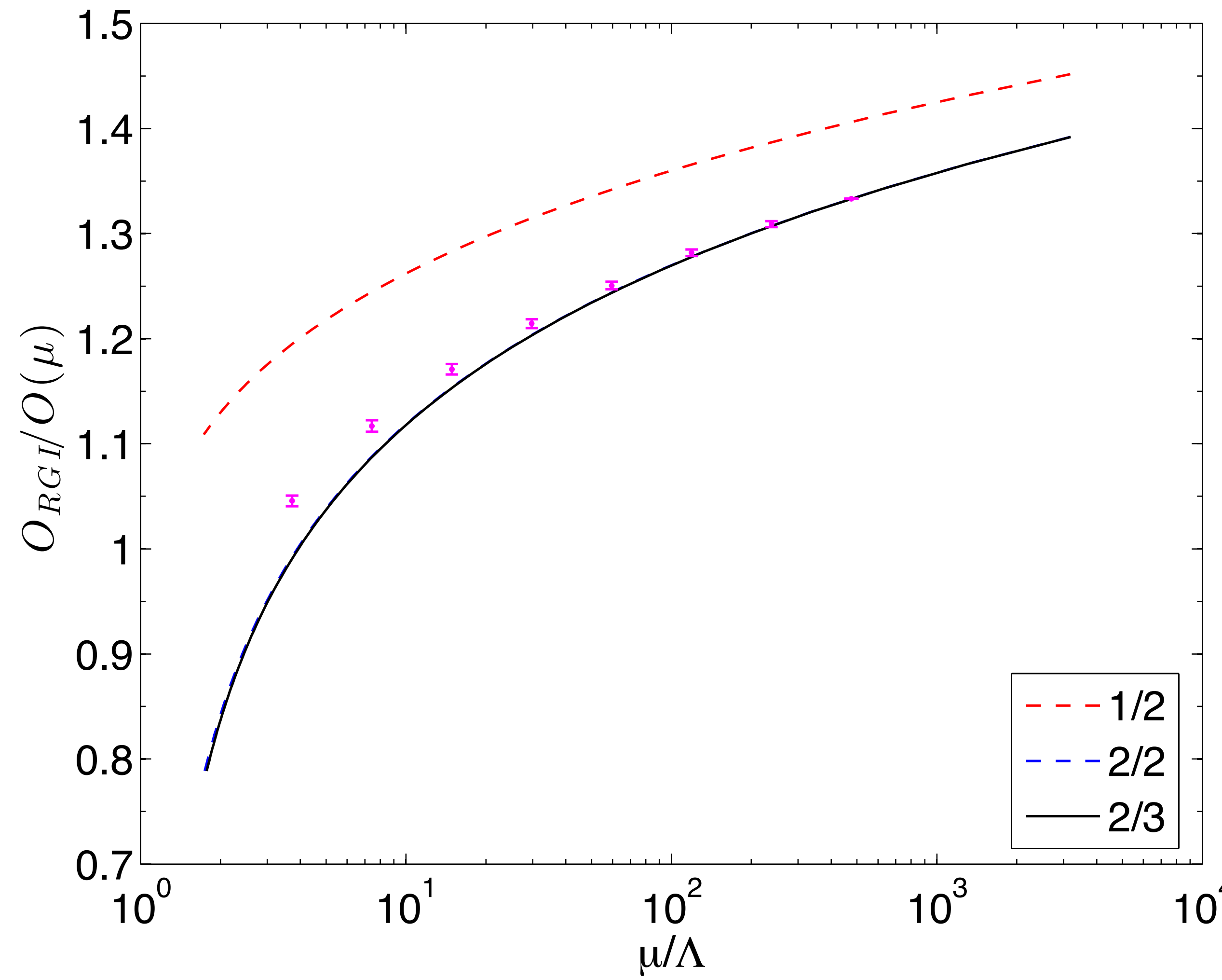
Running

$N_f = 2$

f_1



k_1



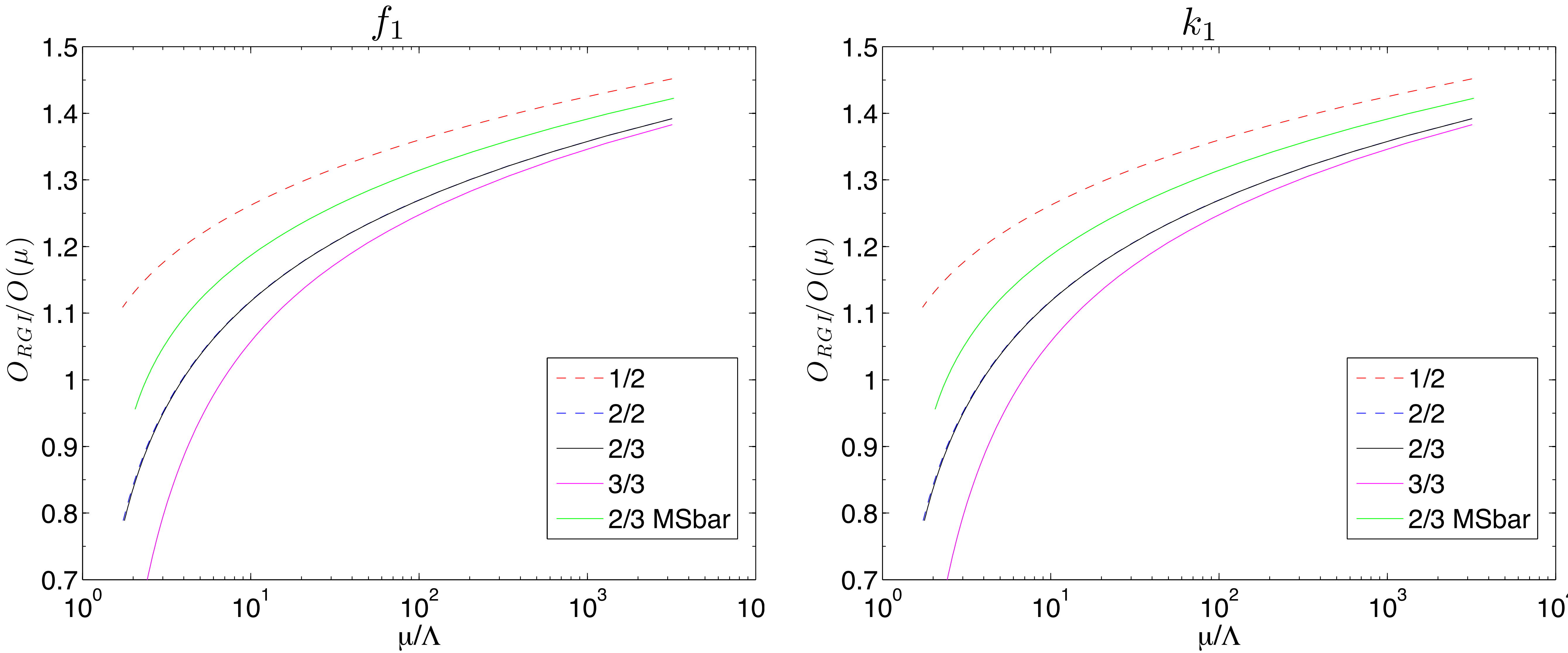
	f_1	k_1
$\frac{O_{RGI}}{O(\mu_{had})}$	1.004(9)	1.045(7)

$$\frac{\mu_{PT}}{\Lambda} = 477.8539$$

383 MeV → 49.1 GeV

PT running systematics

$N_f = 2$



If $\frac{\gamma^{(1)}}{\gamma_0} = \frac{\gamma^{(2)}}{\gamma^{(1)}}$

$$\frac{O^{f_1}_{RGI}}{O(\mu_{PT})} = 1.327(16)$$

$$\frac{O^{k_1}_{RGI}}{O(\mu_{PT})} = 1.333(14)$$

PT systematics is not negligible!

Conclusions

Perturbative Side:

- We have computed 1-loop renormalization coefficients and studied cutoff effects as a function of the angle theta
- From a matching with perturbative schemes we extracted the NLO anomalous dimensions in SF schemes

Non-Perturbative Side:

- We have computed renormalization constants for Nf=0,2,3 (ongoing)
- NP running over various orders of magnitude via recursive procedure through SSF
- RGI matching factors have been computed.
- Nf=3 in progress in parallel with the mass (Patrick Fritzsch's Talk)

Backup