Magnetic properties of light nuclei from lattice QCD

I. Magnetic Moments & Polarisabilities

[NPLQCD PRL 113, 252001 (2014), 1506.05518]

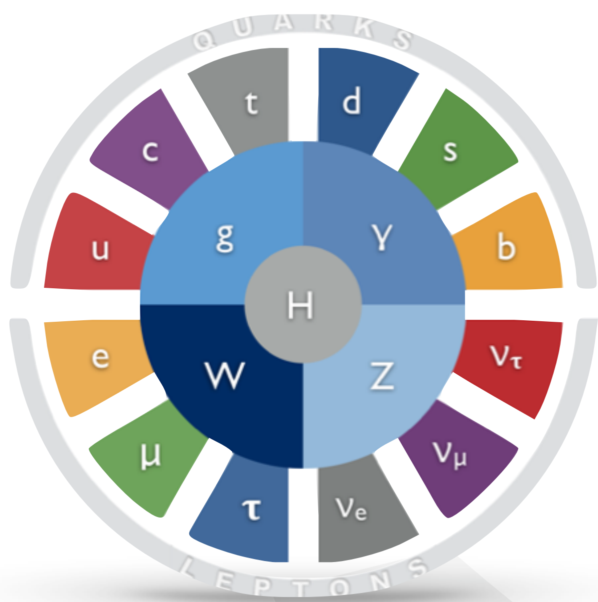
II. Thermal neutron capture cross-section: $np \rightarrow d\gamma$

[NPLQCD 1505.02422]

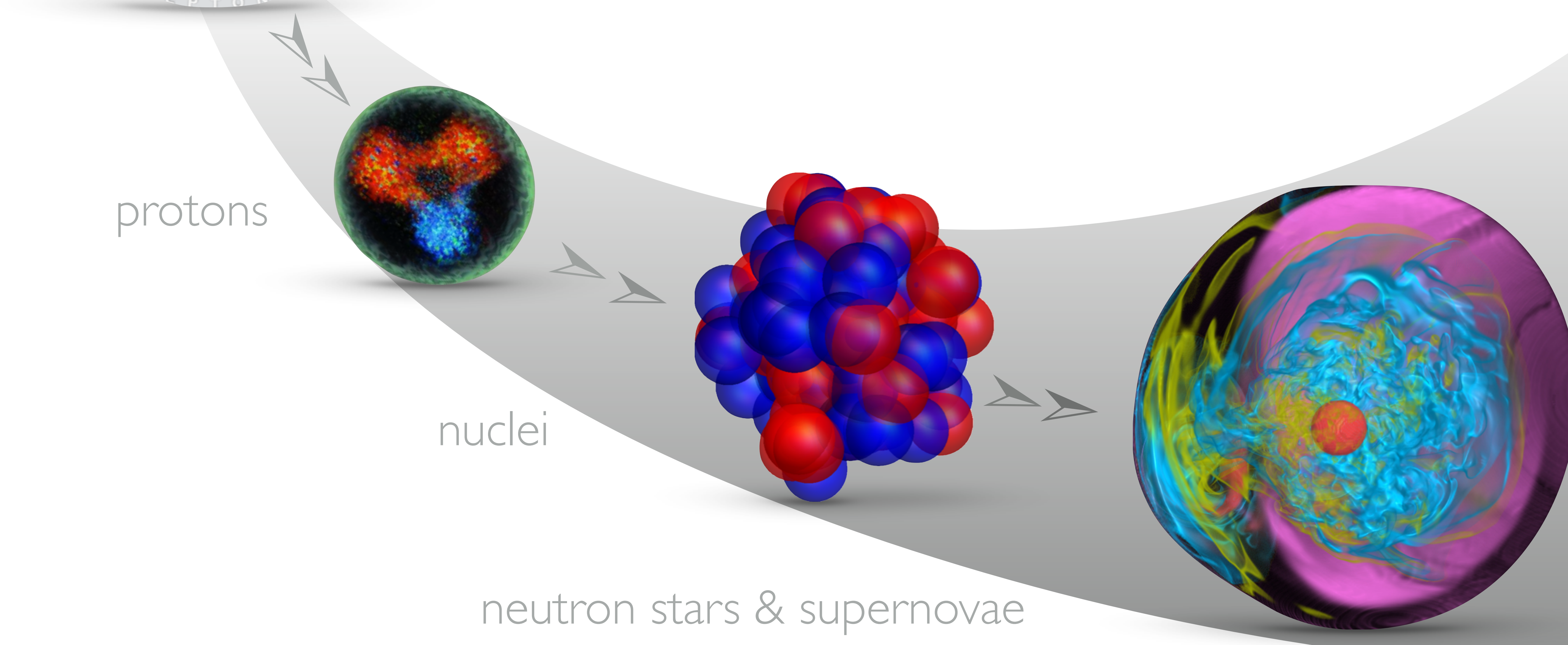
William Detmold, MIT

The 33rd International Symposium on Lattice Field Theory, Kobe, Japan, July 15th 2015

From Quarks to the Cosmos

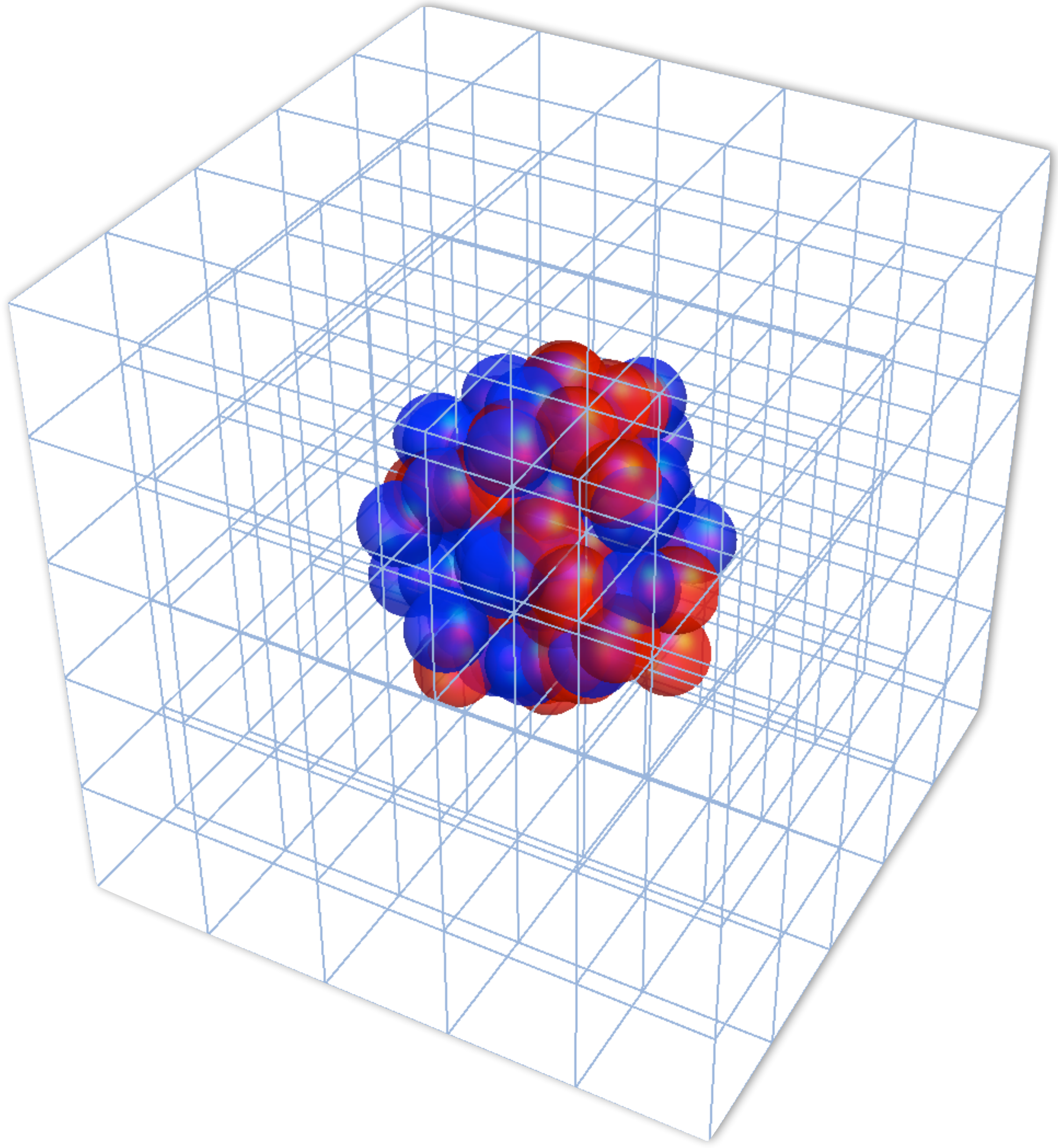


- Complexity of nuclear physics emerges from the Standard Model
  - LQCD + EFT: exciting prospect of quantitative connection



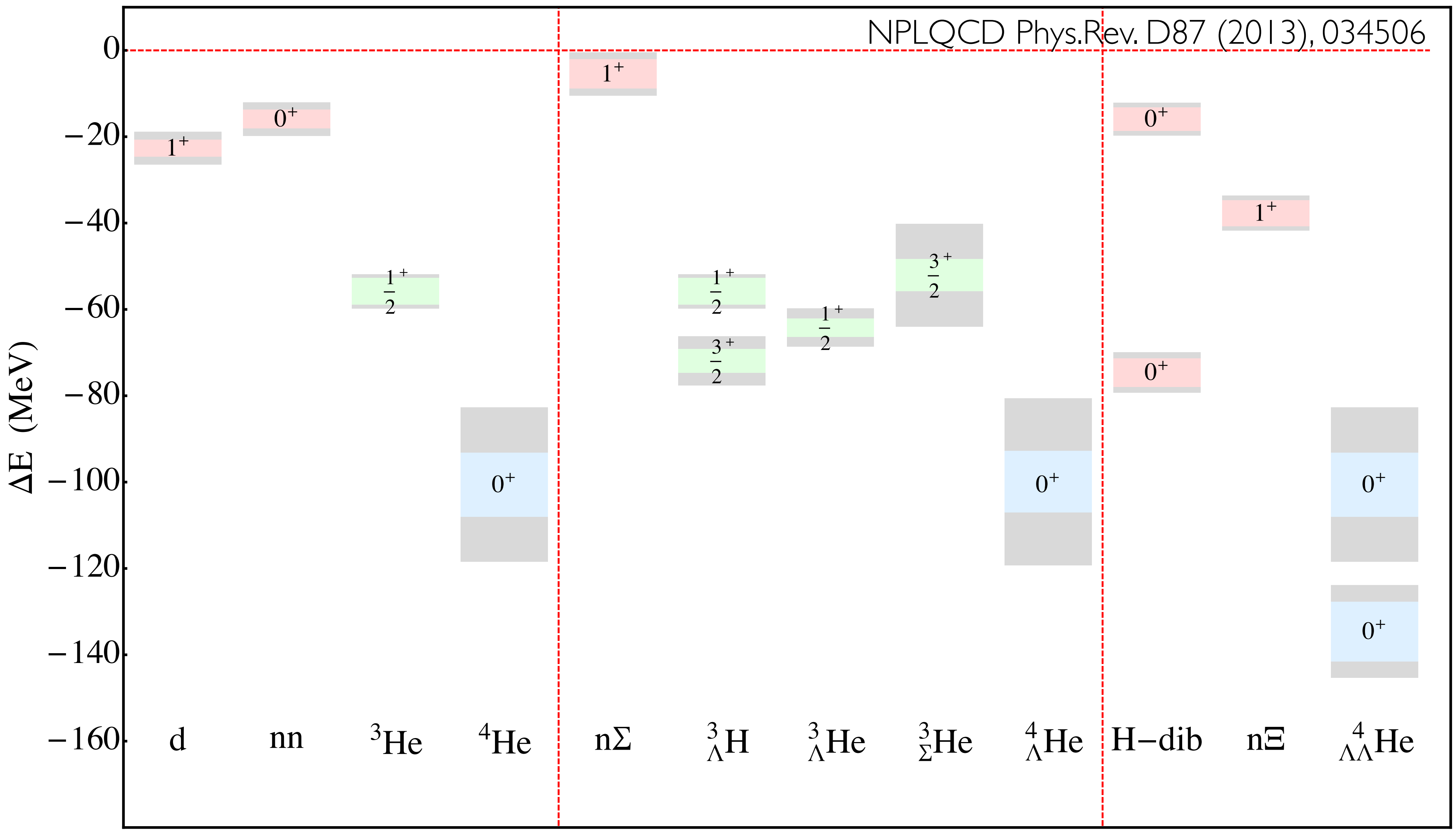
QCD for Nuclei

- In practice: a hard problem
- At least two exponentially difficult challenges
  - Noise statistical uncertainty grows exponentially with A
  - Contraction complexity grows fast



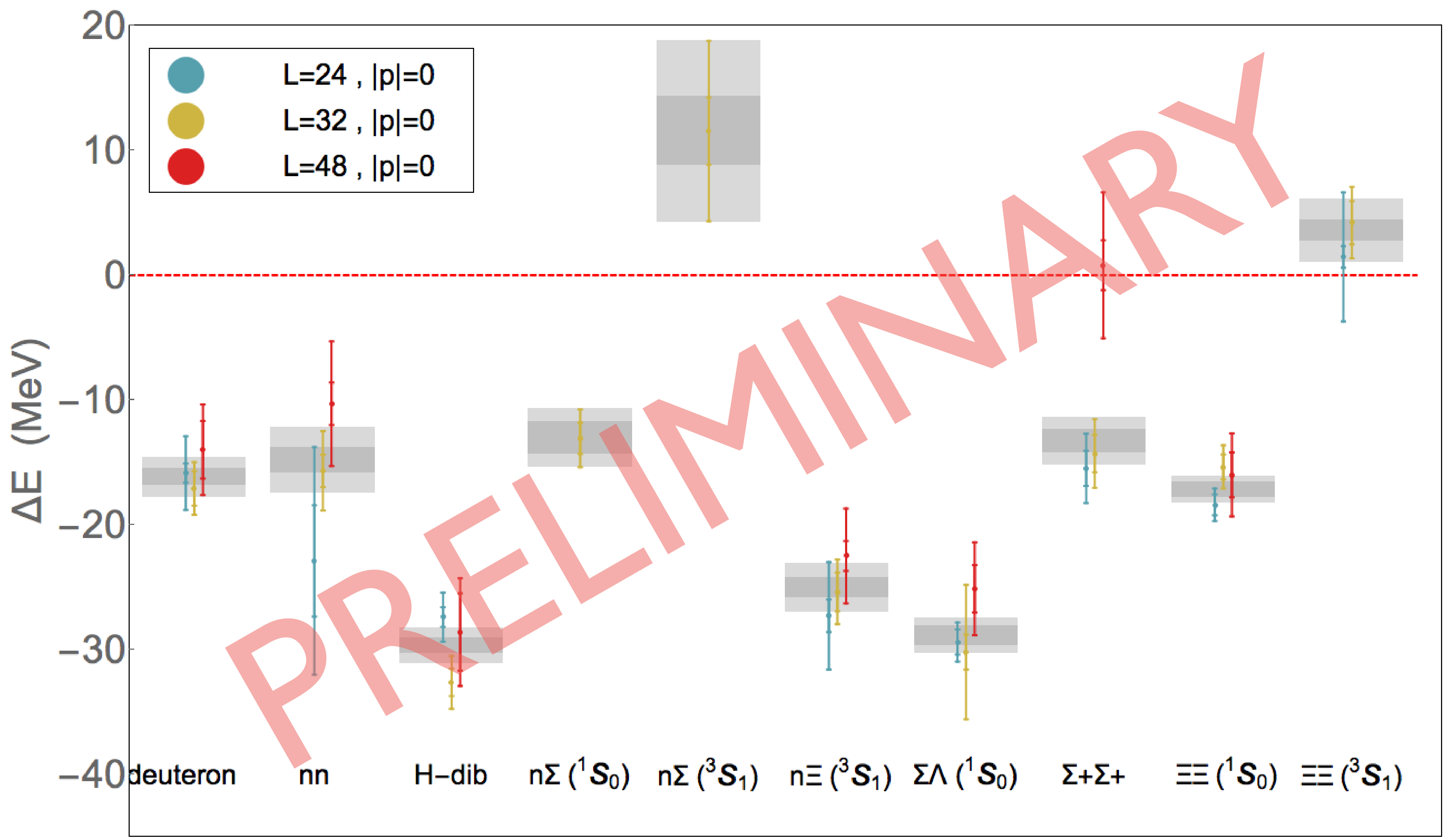
Light nuclei

- Light hypernuclear binding energies @ m_π=800 MeV



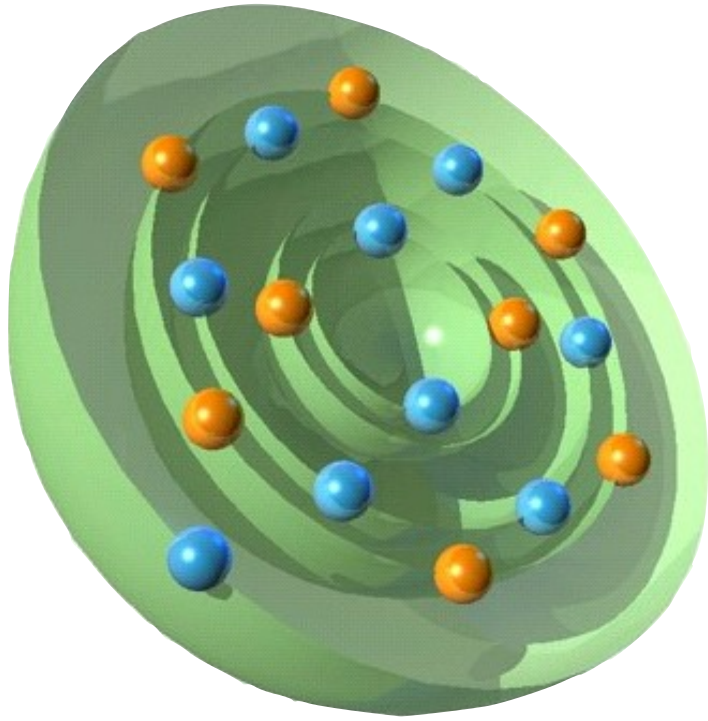
Light nuclei

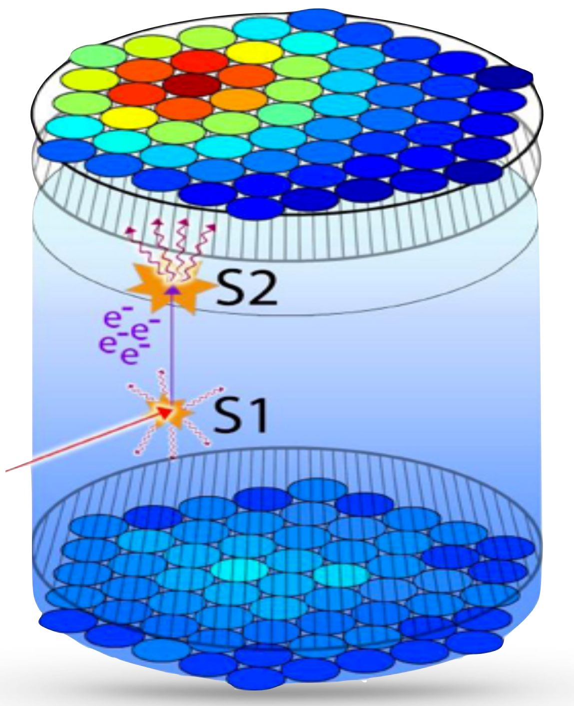
- Two baryon spectrum @ m_π=450 MeV



External currents and nuclei

- Nuclear matrix elements important in many contexts
  - Probes of nuclear structure
  - Neutrino-nucleus scattering
  - Dark matter direct detection experiments
  - …
- In many cases, no independent experimental information available (eg DM)
- Need SM calculation: nuclear physics is the new flavour physics!

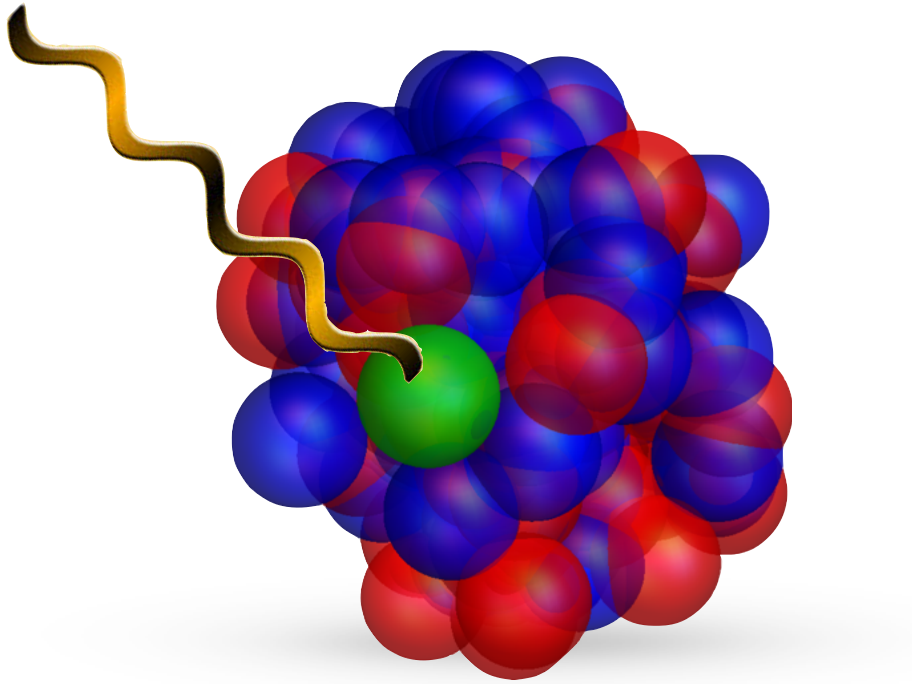


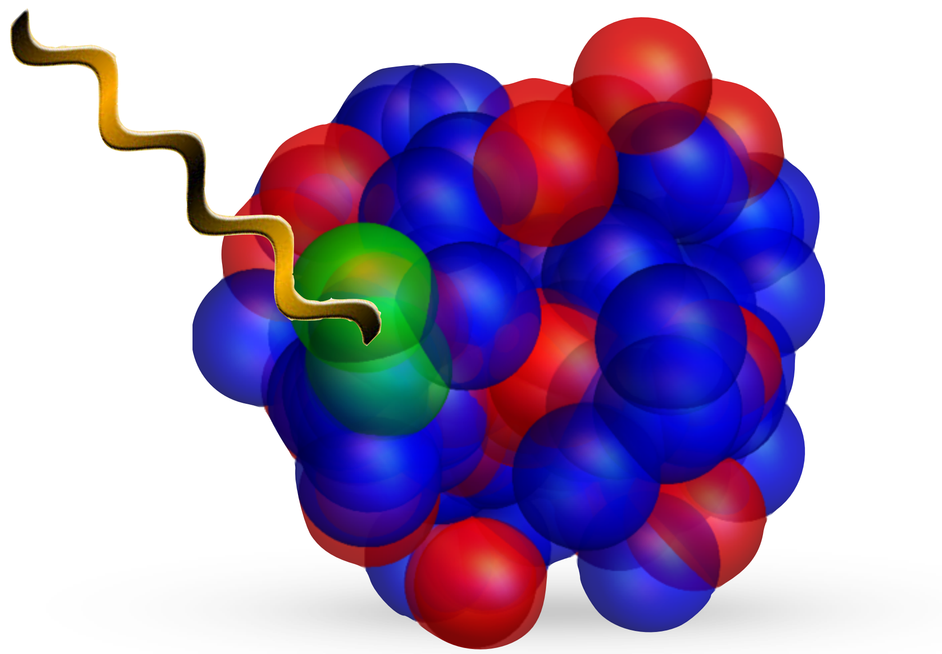


http://www.hep.ucl.ac.uk/darkMatter/

External currents and nuclei

- Xe in LQCD not likely any time soon
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading
    but non-negligible
- LQCD: determine one body current from single nucleon
- LQCD: determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei



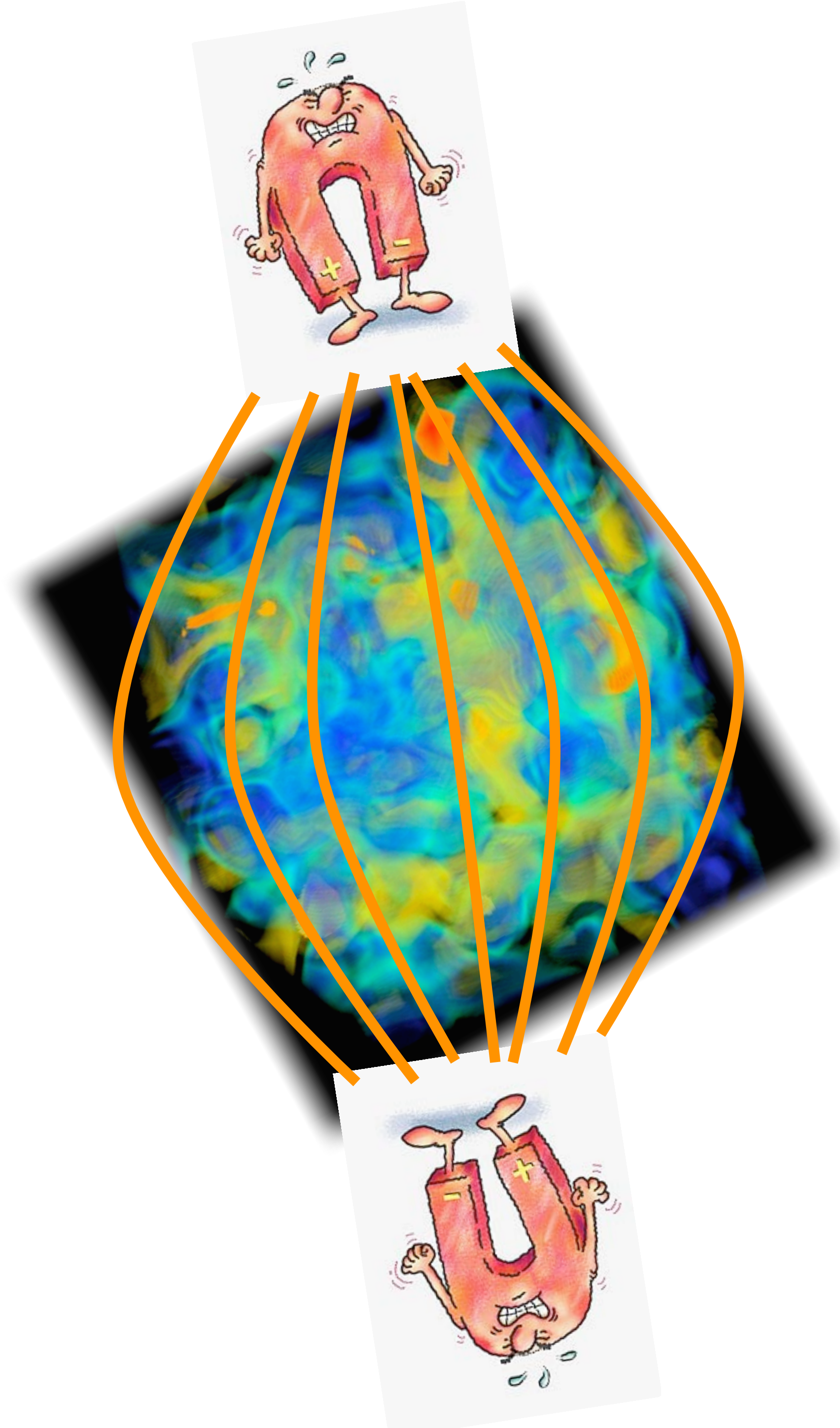


Background field methods

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: constant B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Magnetic moments, polarisabilities, …
  - Not restricted to simple EM fields (axial, twist-2,…)



- Magnetic field in *z*-direction (quantised n)

$$U_\mu^{\rm QCD} \longrightarrow U_\mu^{\rm QCD} \cdot U_\mu^{(Q)}$$

$$U_\mu^{(Q)}(x) \;=\; e^{i\frac{6\pi Q_q \tilde{n}}{L^2} x_1 \delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L} x_2 \delta_{\mu,1} \delta_{x_1,L-1}}$$

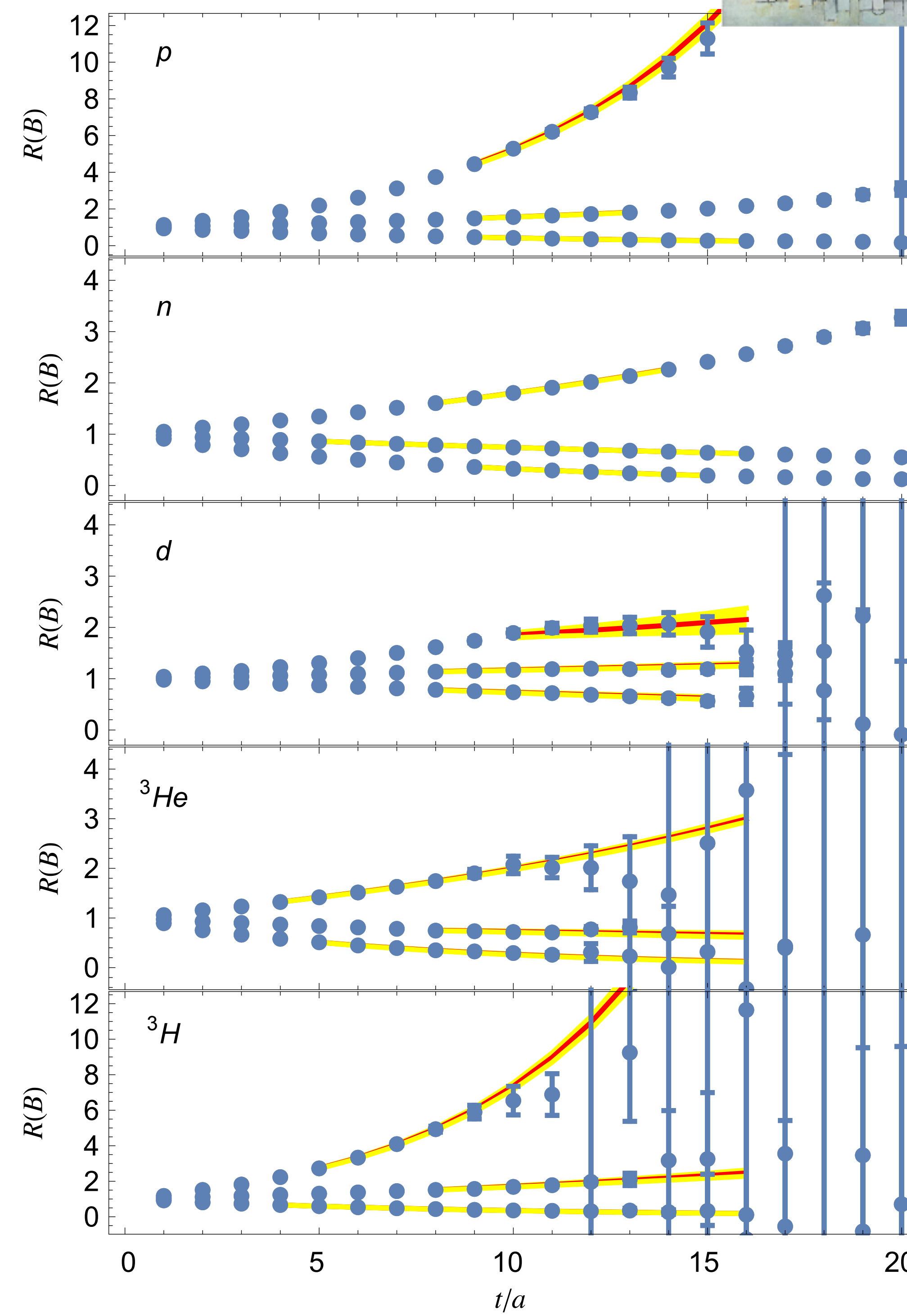
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E_{+j}^{(B)} - E_{-j}^{(B)} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

$$R(B) = \frac{C_j^{(B)}(t)\; C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\; C_j^{(0)}(t)} \;\stackrel{t\to\infty}{\longrightarrow}\; Z e^{-\delta E^{(B)} t}$$

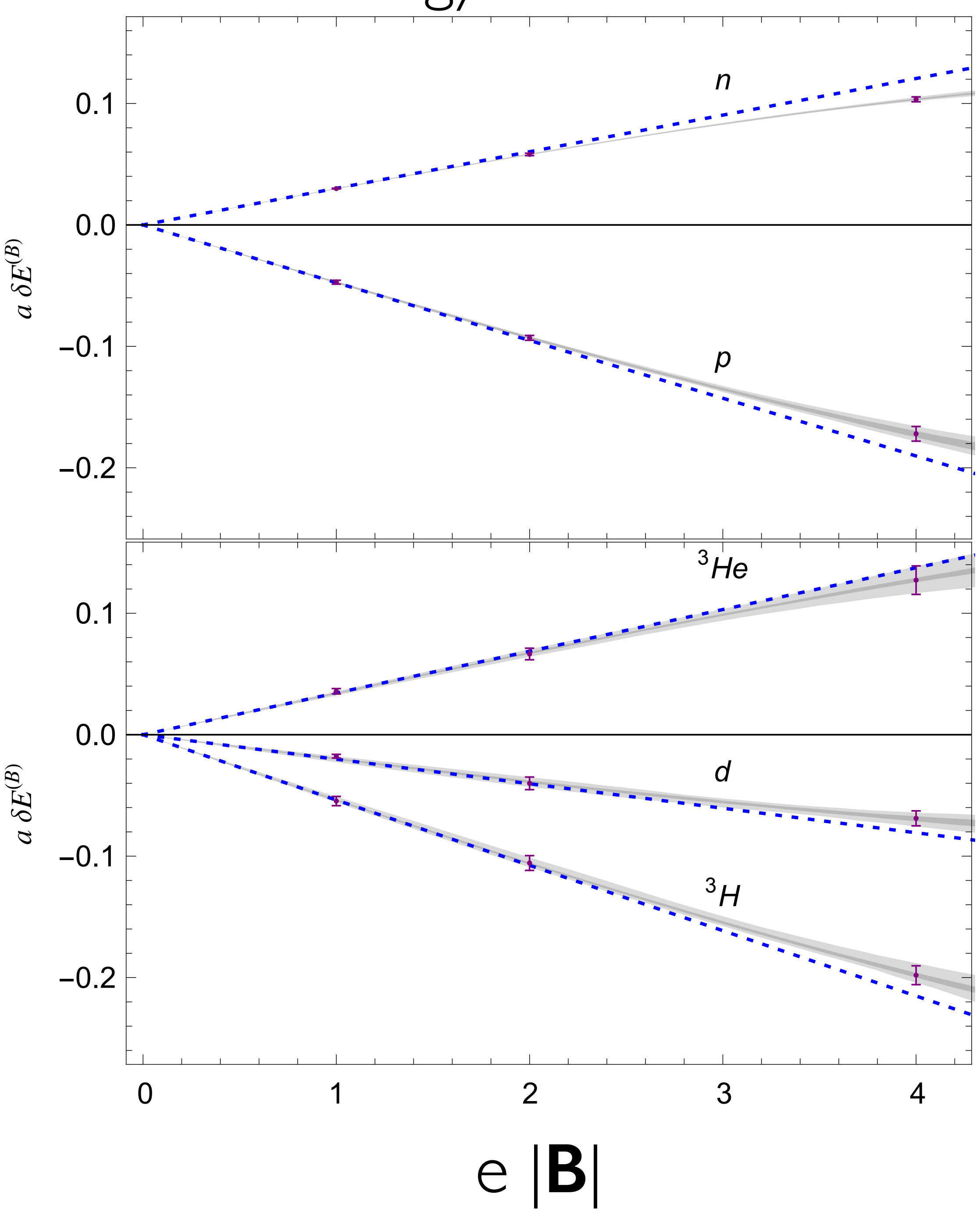
- Careful to be in single exponential region of each correlator



[NPLQCD 1409.3556]

Magnetic moments of nuclei

Energy shift vs B



[NPLQCD PRL 2014]

Magnetic moments of nuclei

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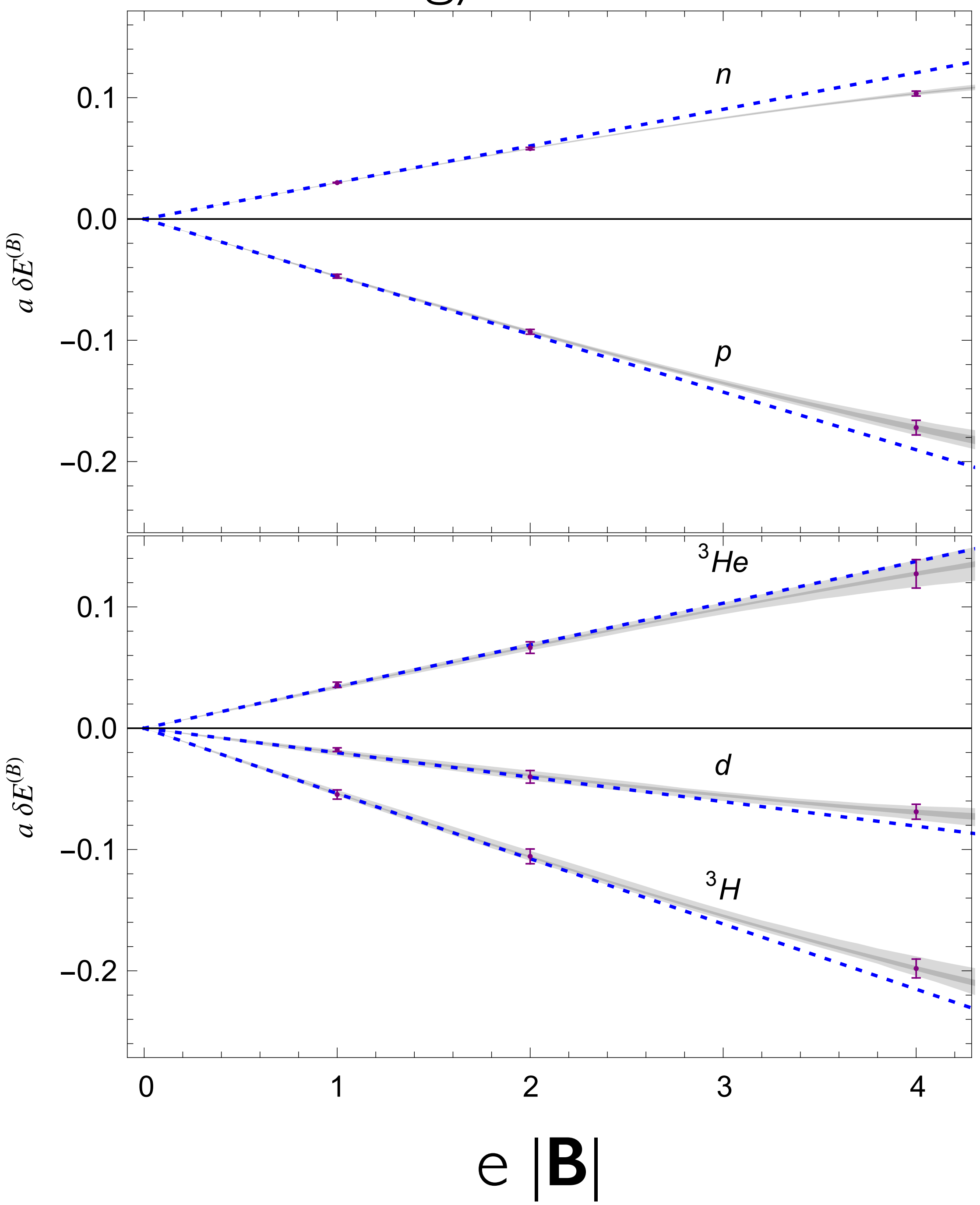
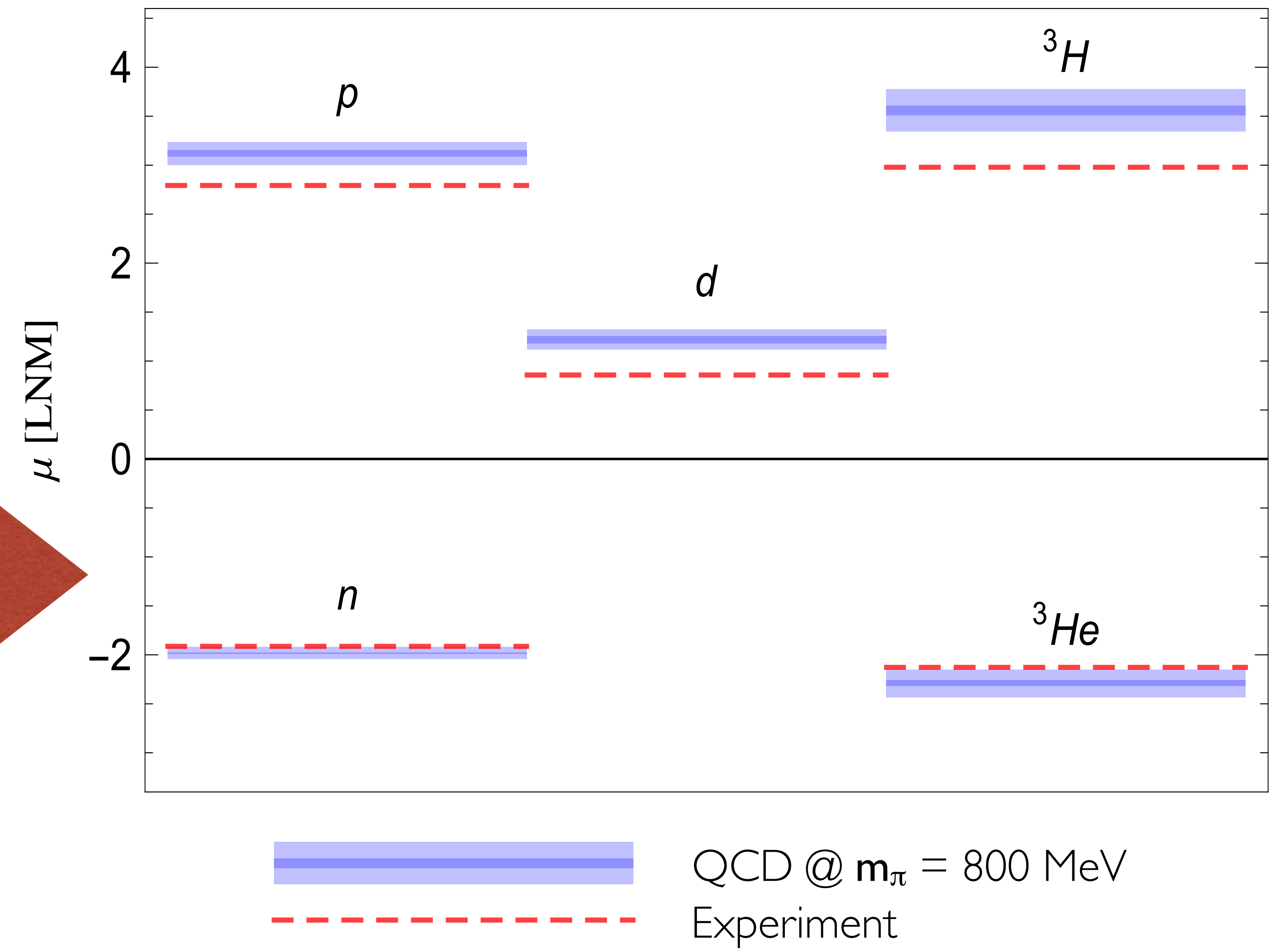




	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL 2014]

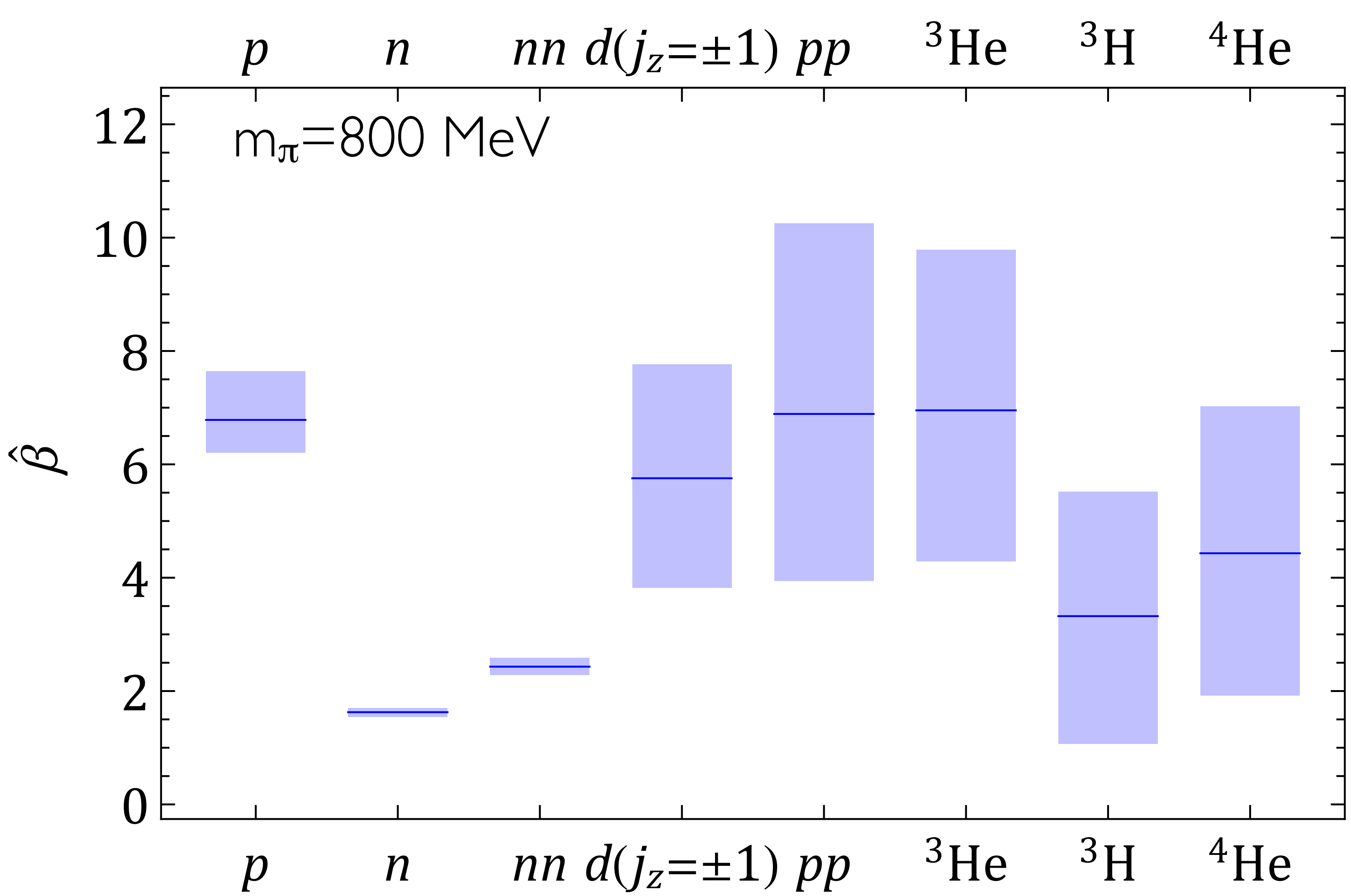
[NPLQCD 1506.05518]

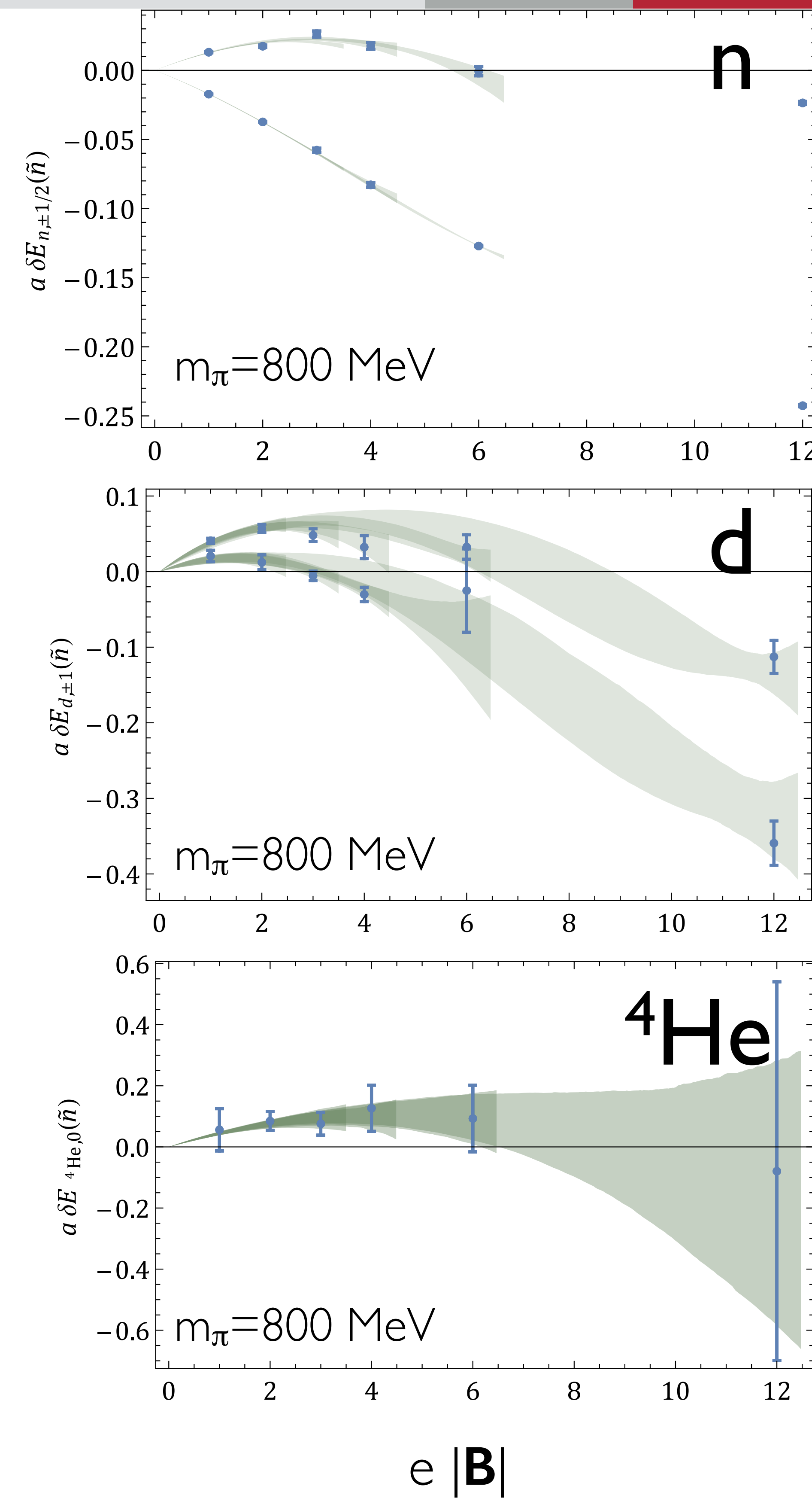
- Second order shifts

$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- Care required with Landau levels

- Polarisabilities (dimensionless units)





[NPLQCD 1506.05518]

- Second order shifts

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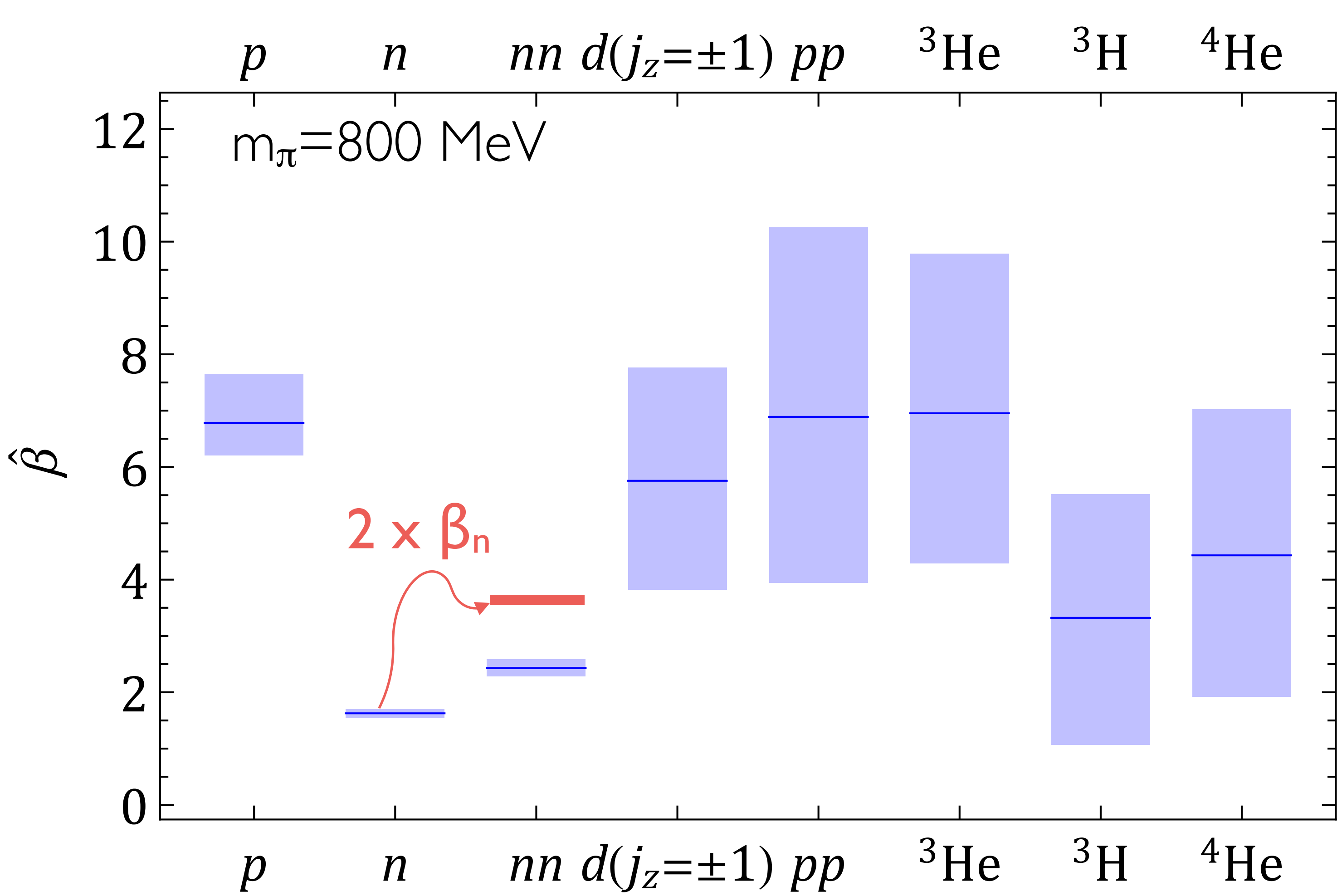
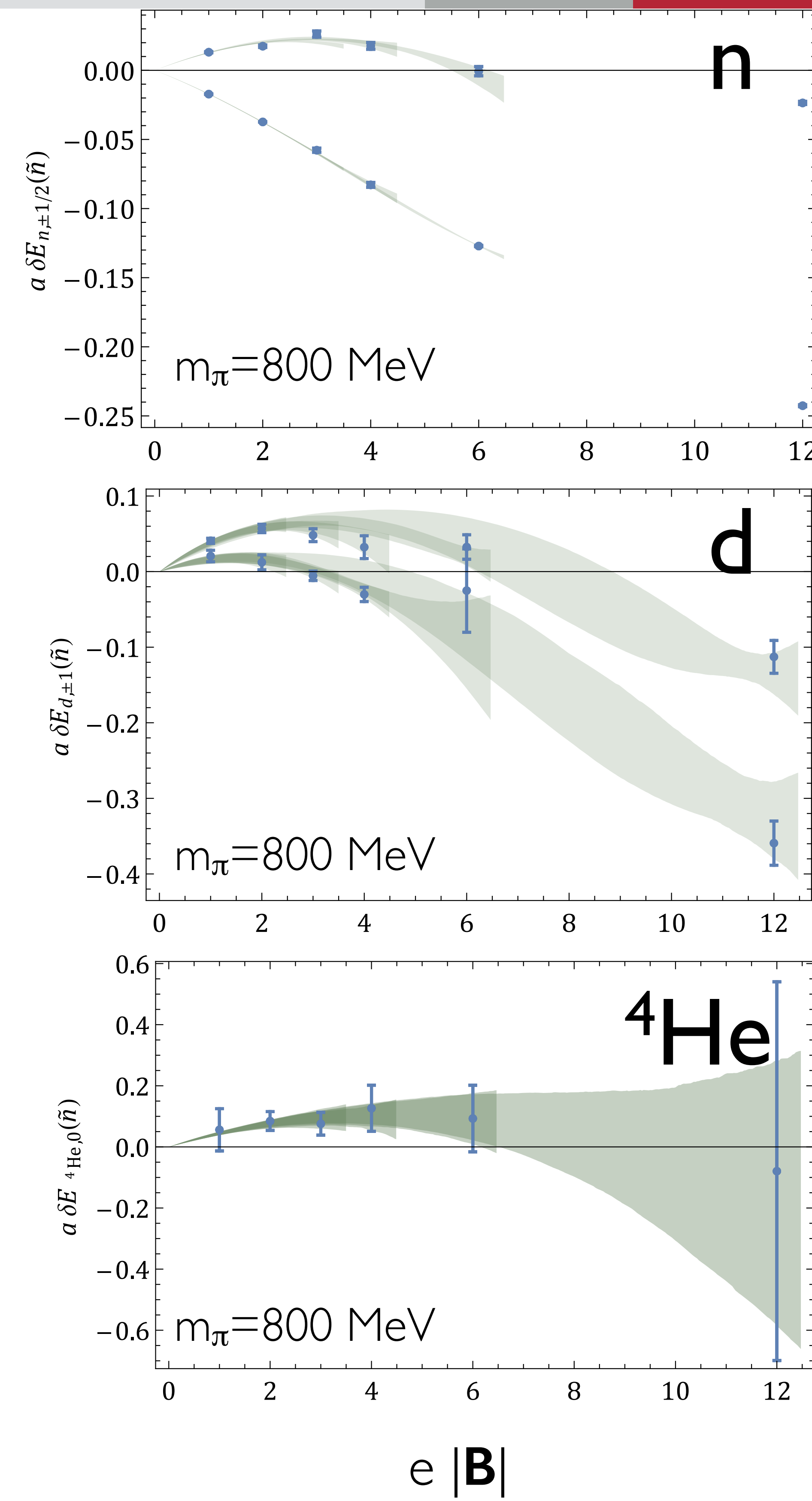
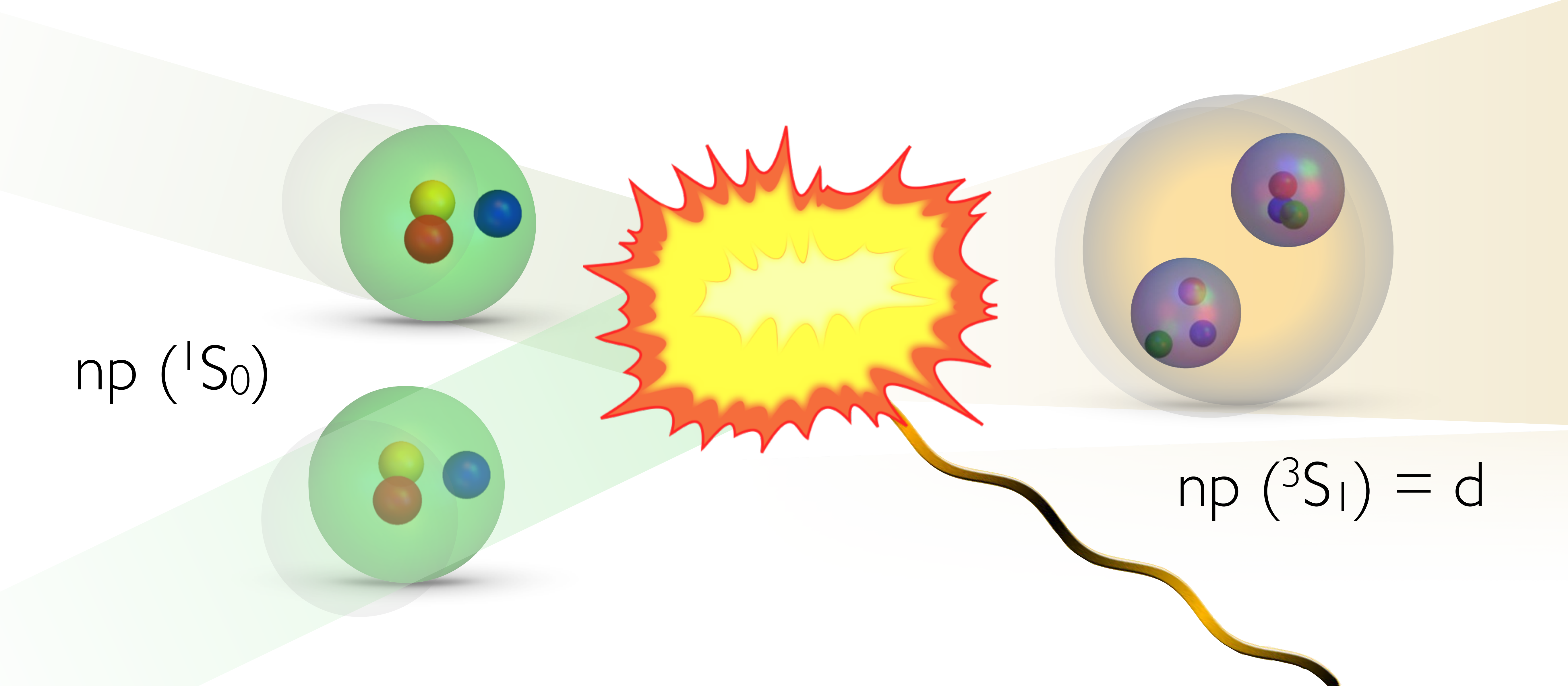




Thermal Neutron Capture Cross-Section

[NPLQCD 1505.02422]

- Thermal neutron capture cross-section: np→dγ
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: 2-body contributions ~10%



np→dγ in pionless EFT

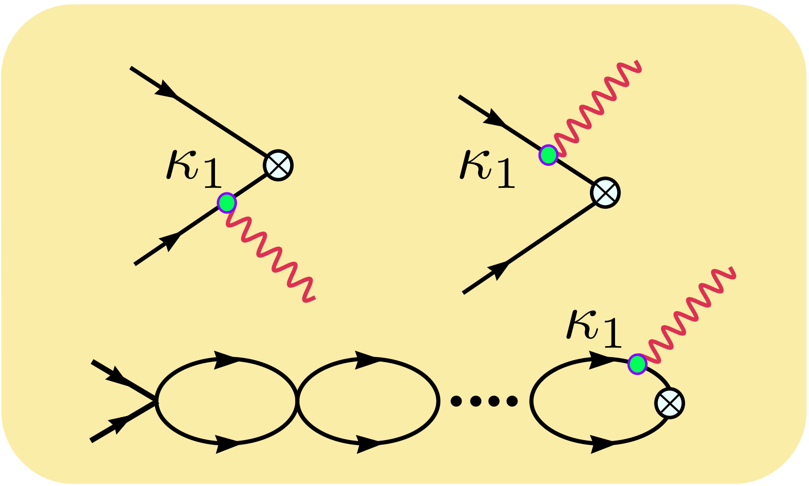
$$Z_d = 1/\sqrt{1-\gamma_0 r_3}$$

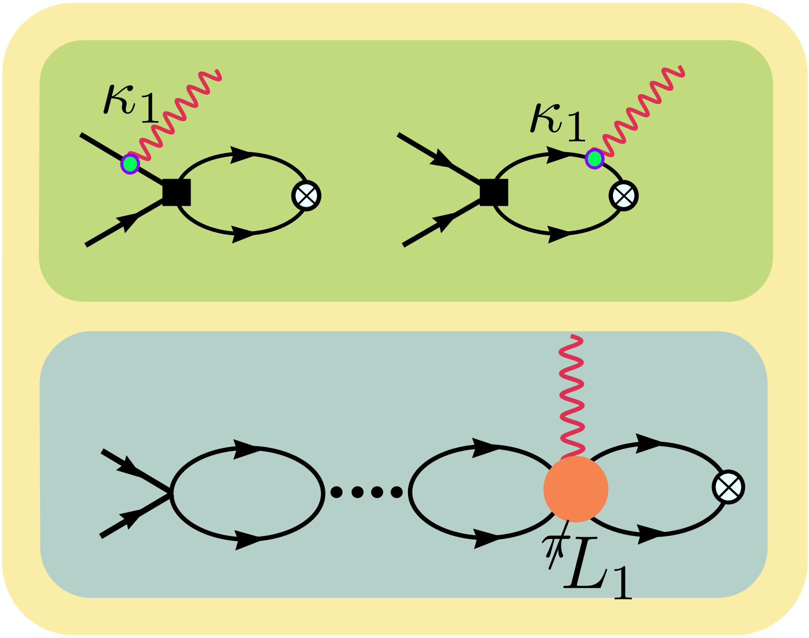
- Cross-section at threshold calculated in pionless EFT

$$\sigma(np\to d\gamma) = \frac{e^2(\gamma_0^2+|\mathbf{p}|^2)^3}{M^4\gamma_0^3|\mathbf{p}|}|\tilde{X}_{M1}|^2 + \ldots$$

- EFT expansion at LO given by mag. moments NLO contributions from short-distance two nucleon operators

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2 - i|\mathbf{p}|} \times \left[\frac{\kappa_1\gamma_0^2}{\gamma_0^2+|\mathbf{p}|^2}\left(\gamma_0-\frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2\right)+\frac{\gamma_0^2}{2}l_1\right]$$





- Phenomenological description with 1% accuracy for $E < 1$ MeV
  - Short distance (2-body) contributes ~10%

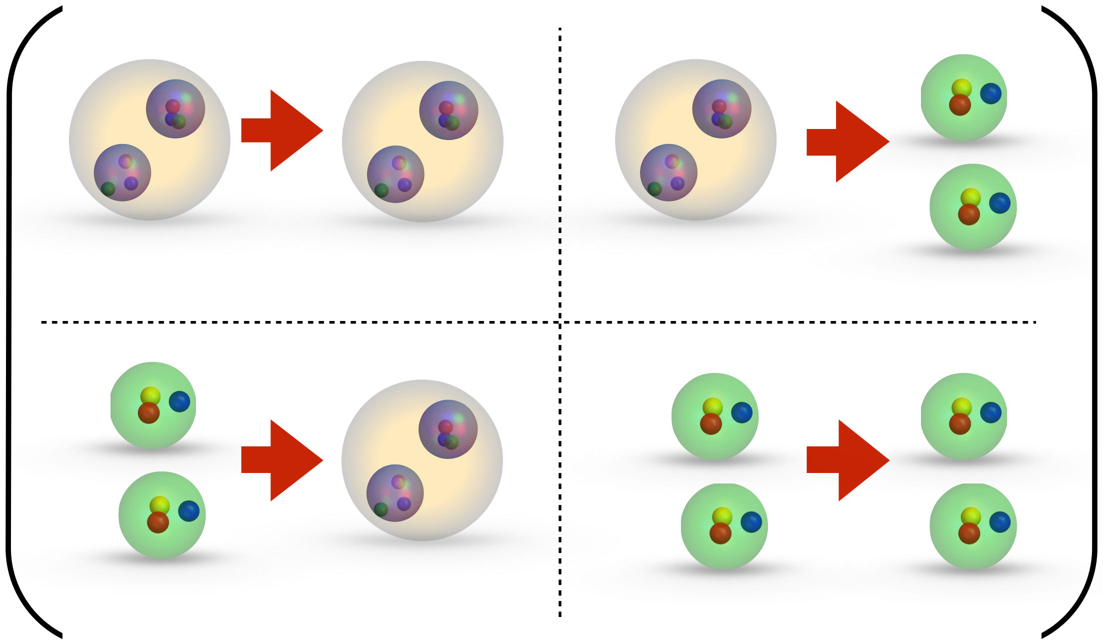
MECs: Riska, Phys.Lett. B38 (1972) 193
Hokert et al, Nucl.Phys. A217 (1973) 14

EFT: Chen et al.,Nucl.Phys. A653 (1999) 386
Chen et al, Phys.Lett. B464 (1999) 1
Rupak Nucl.Phys. A678 (2000) 405

$np \to d\gamma$

- Presence of magnetic field mixes $I_z=J_z=0$ $^3S_1$ and $^1S_0$ *np* systems

H =



- Wigner SU(4) super-multiplet (spin-flavour) symmetry relates $^3S_1$ and $^1S_0$ states (diagonal elements approximately equal)
  - Shift of eigenvalues determined by transition amplitude

$$\Delta E_{^3S_1,^1S_0} = \mp \left(\kappa_1 + \overline{L}_1\right) \frac{eB}{M} + \ldots$$

- More generally eigenvalues depend on transition amplitude
  [WD, & M Savage 2004, H Meyer 2012]

[NPLQCD 1505.02422]

np→dγ

Lattice 2-nucleon correlator with $^3S_1$ source and $^1S_0$ sink

- $I_z = J_z = 0$ correlation matrix

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

- Generalised eigenvalue problem

$$[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}\mathbf{C}(t;\mathbf{B})[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}v = \lambda(t;\mathbf{B})v$$

- Ratio of correlator ratios to extract 2-body

$$R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{\lambda_+(t;\mathbf{B})}{\lambda_-(t;\mathbf{B})} \xrightarrow{t\to\infty} \hat{Z}\exp\left[2\ \Delta E_{^3S_1,^1S_0} t\right]$$

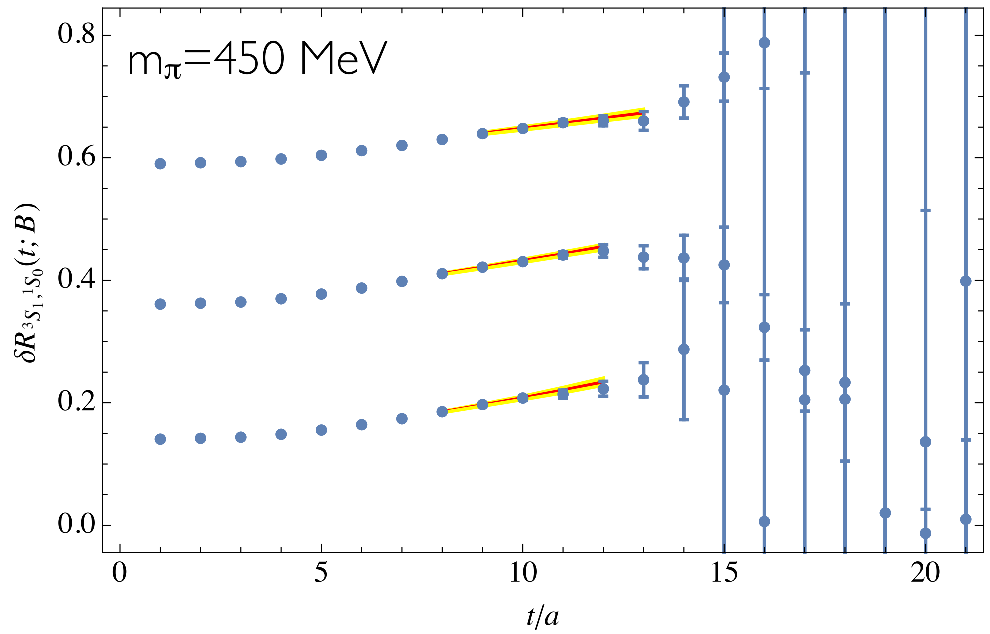
$$\delta R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{R_{^3S_1,^1S_0}(t;\mathbf{B})}{\Delta R_p(t;\mathbf{B})/\Delta R_n(t;\mathbf{B})} \to A\ e^{-\delta E_{^3S_1,^1S_0}(\mathbf{B})t}$$

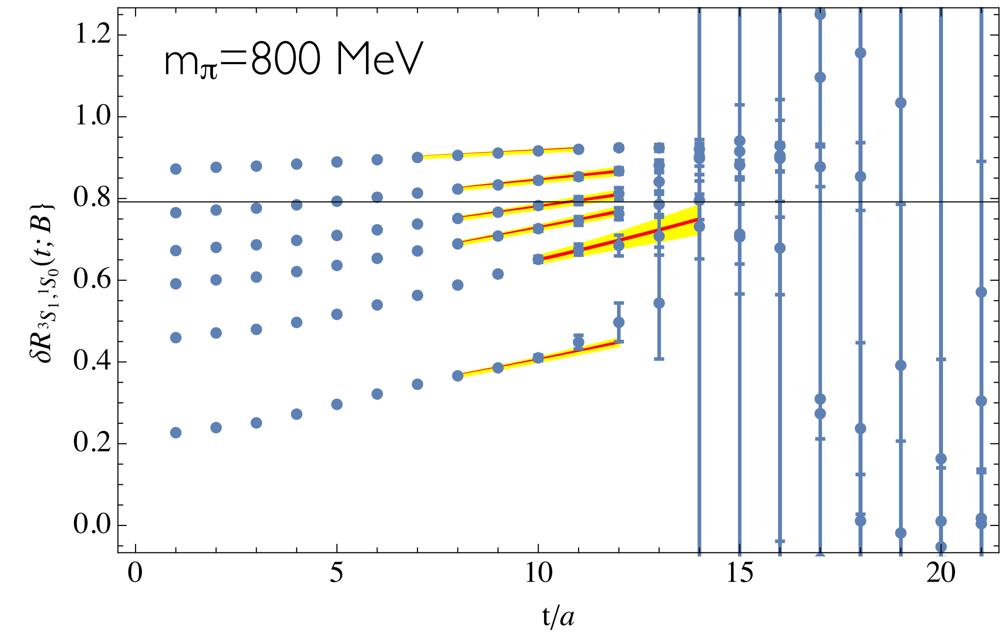
$$\begin{aligned}\delta E_{^3S_1,^1S_0} &\equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \\ &\to 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)\end{aligned}$$

[NPLQCD 1505.02422]

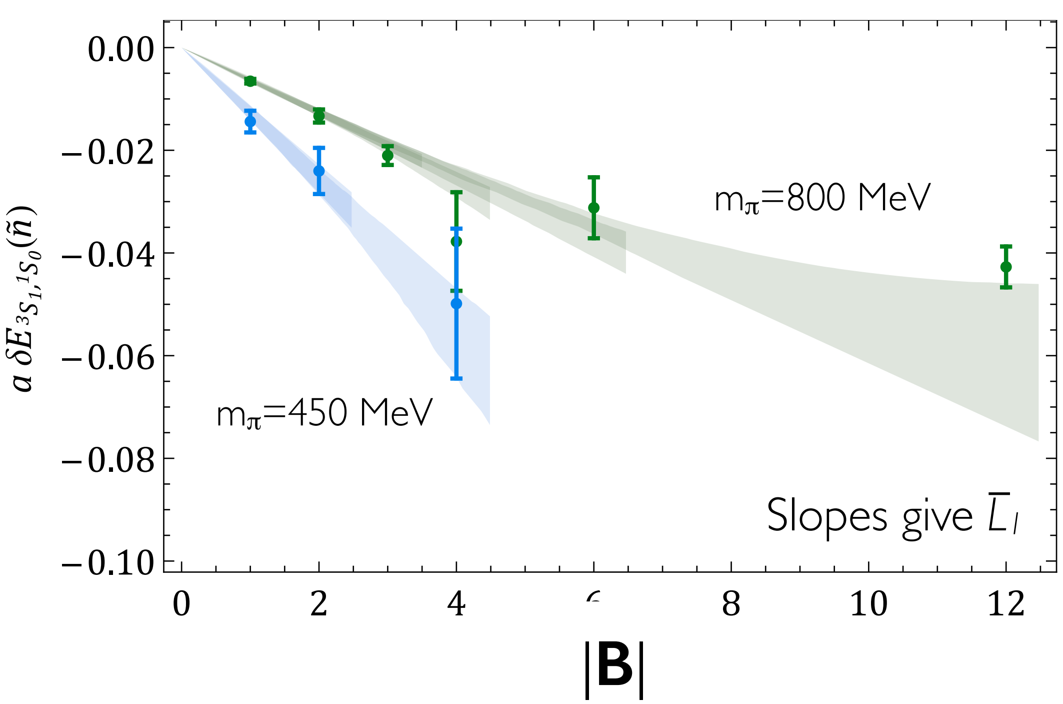
np→dγ

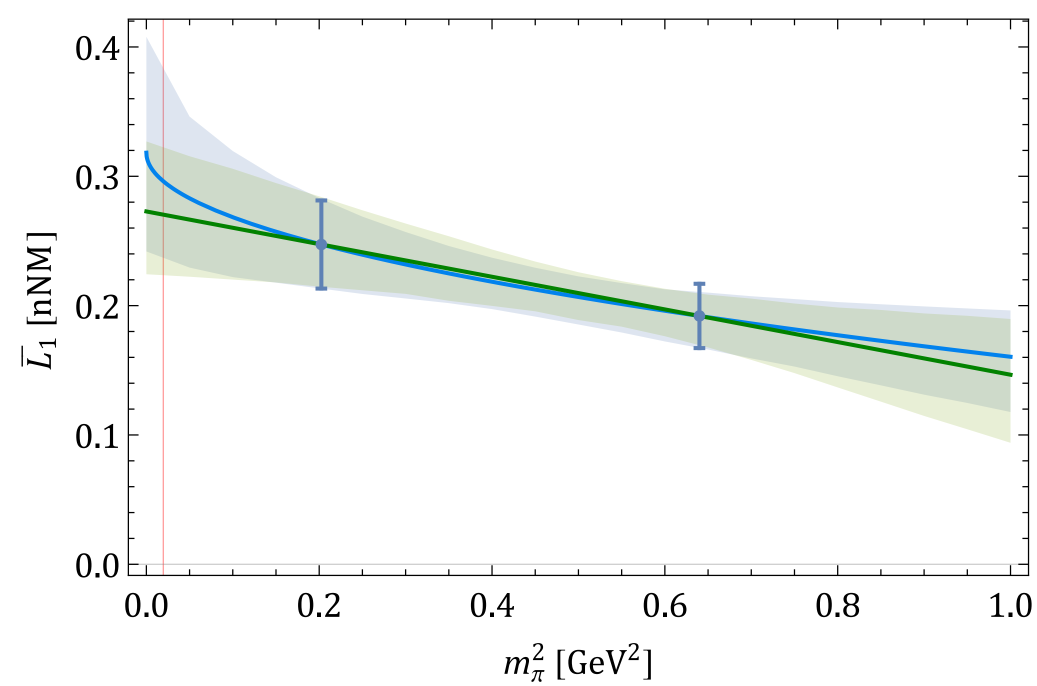
- Correlator ratios for different field strengths





- Field strength & mass dependence





[NPLQCD 1505.02422]

$np \to d\gamma$

- Extract short-distance contribution at physical mass

$$\overline{L}_1^{\text{lqcd}} = 0.285(\,^{+63}_{-60}\,)\ \text{nNM}$$

- Combine with phenomenological nucleon magnetic moment, scattering parameters at incident neutron velocity v=2,200 m/s

$$\sigma^{\text{lqcd}}(np \to d\gamma) = 307.8(1 + 0.273\ \overline{L}_1^{\text{lqcd}})\ \text{mb}$$

$$\sigma^{\text{lqcd}}(np \to d\gamma) = 332.4(\,^{+5.4}_{-4.7}\,)\ \text{mb}$$

c.f. phenomenological value

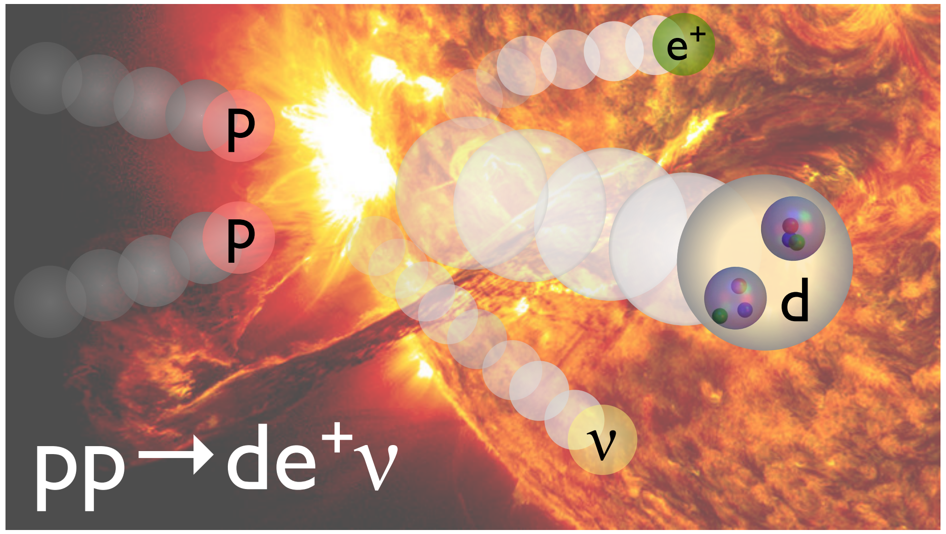
$$\sigma^{\text{expt}}(np \to d\gamma) = 334.2(0.5)\ \text{mb}$$

- NB: at m_π=800 MeV, use LQCD for all inputs (ab initio)

$$\sigma^{800\ \text{MeV}}(np \to d\gamma) \sim 10\ \text{mb}$$

[NPLQCD 1505.02422]

Further matrix elements



- Background field approach very general
  - Axial coupling to NN system
  - Quadrupole moments: requires non-constant fields [Z Davoudi, WD 1507.01908]
  - Axial form factors
  - Scalar, … matrix elements for dark matter
  - Twist-2 operators: EMC effect

QCD for nuclei

- Nuclei are under serious study directly from QCD
  - Structure: magnetic moments and polarisabilities
  - Electroweak interactions: thermal capture cross-section
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Critical role in current and upcoming particle physics experimental program
  - Learn many interesting things about nuclear physics along the way

fin

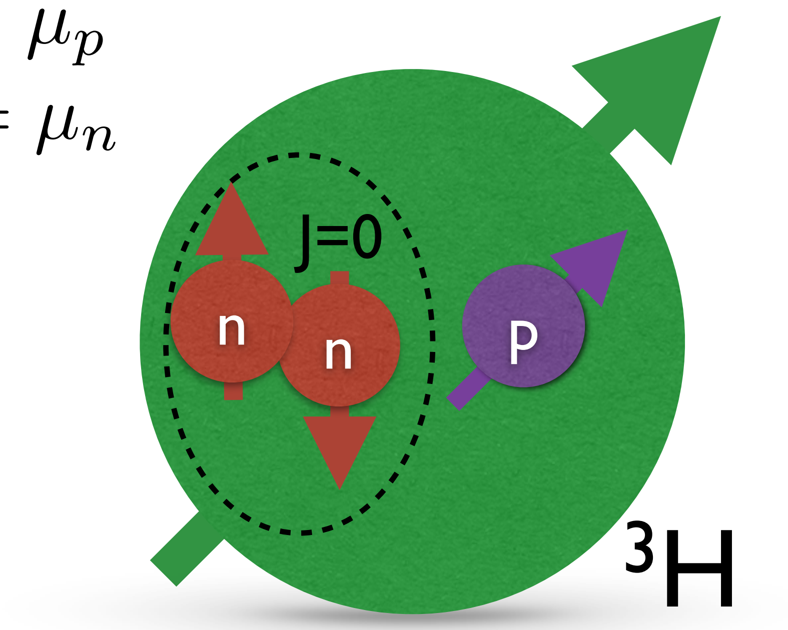
Acknowledgements

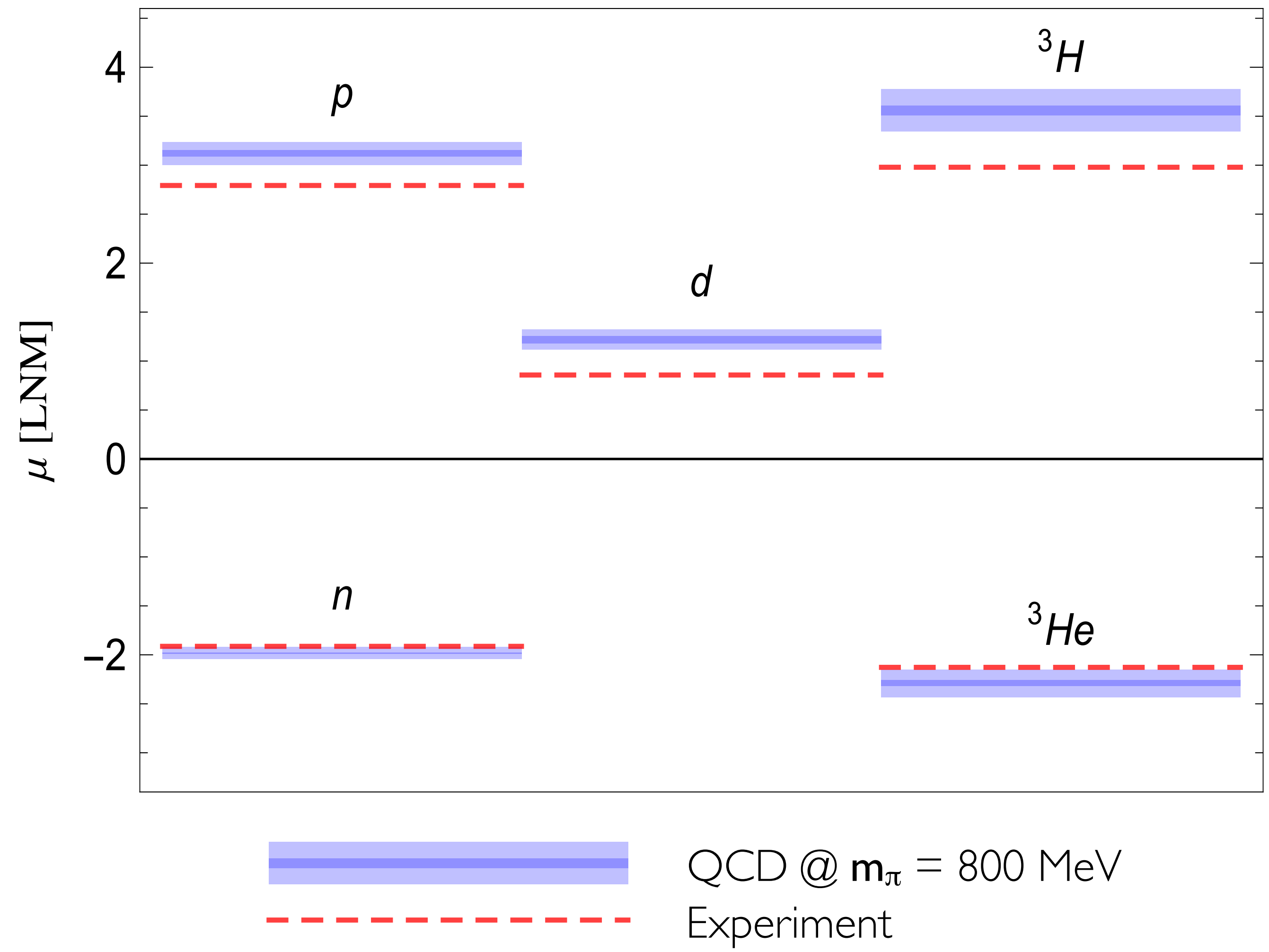
Silas Beane, Emmanuel Chang, Saul Cohen, Huey-wen Lin,
Kostas Orginos, Assumpta Parreño, Martin Savage, Brian TIburzi

Magnetic moments of nuclei

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$





- Lattice results appear to suggest heavy quark nuclei are shell-model like!

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[NPLQCD PRL 2014]