Topological feature and phase structure of QCD at complex chemical potential

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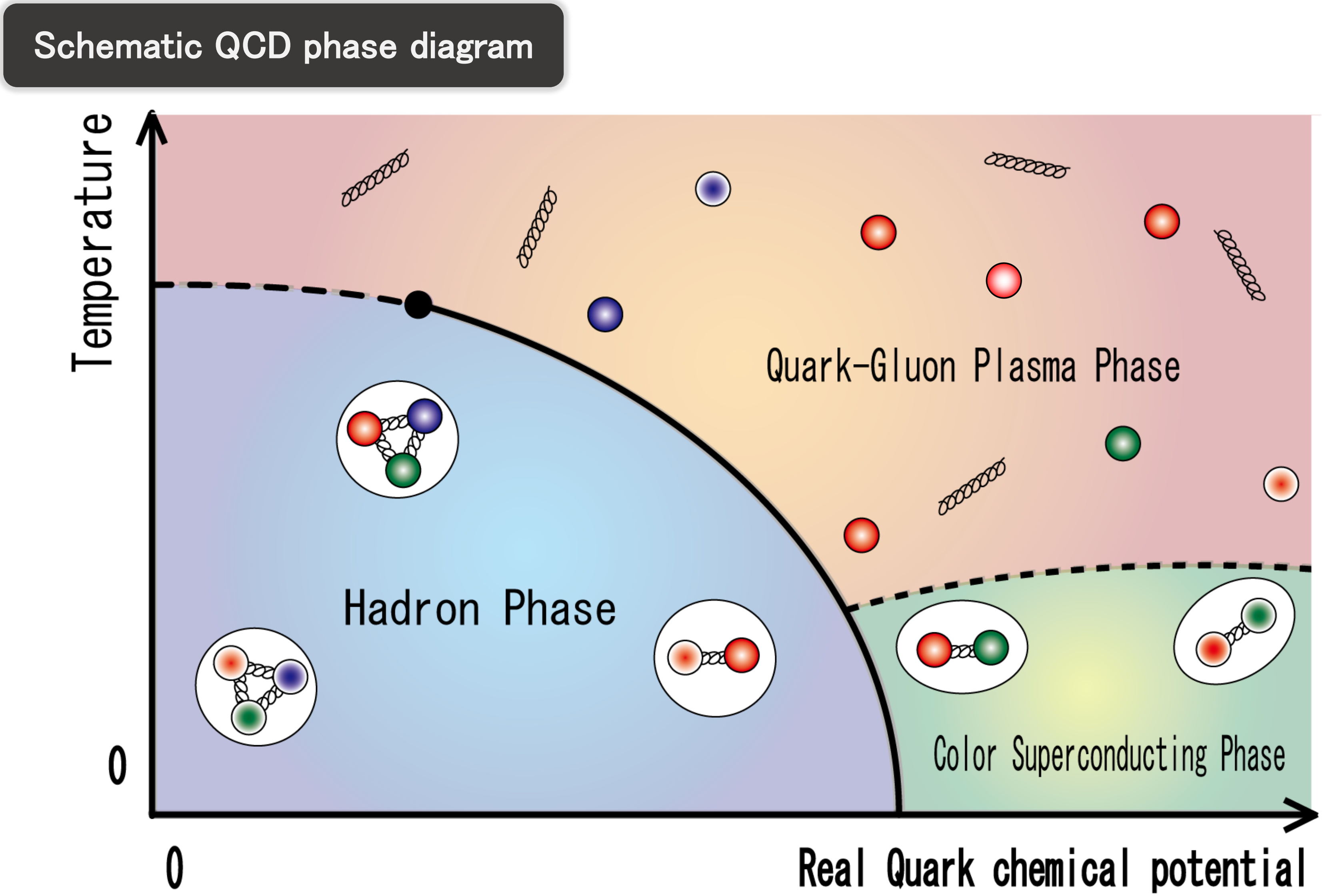
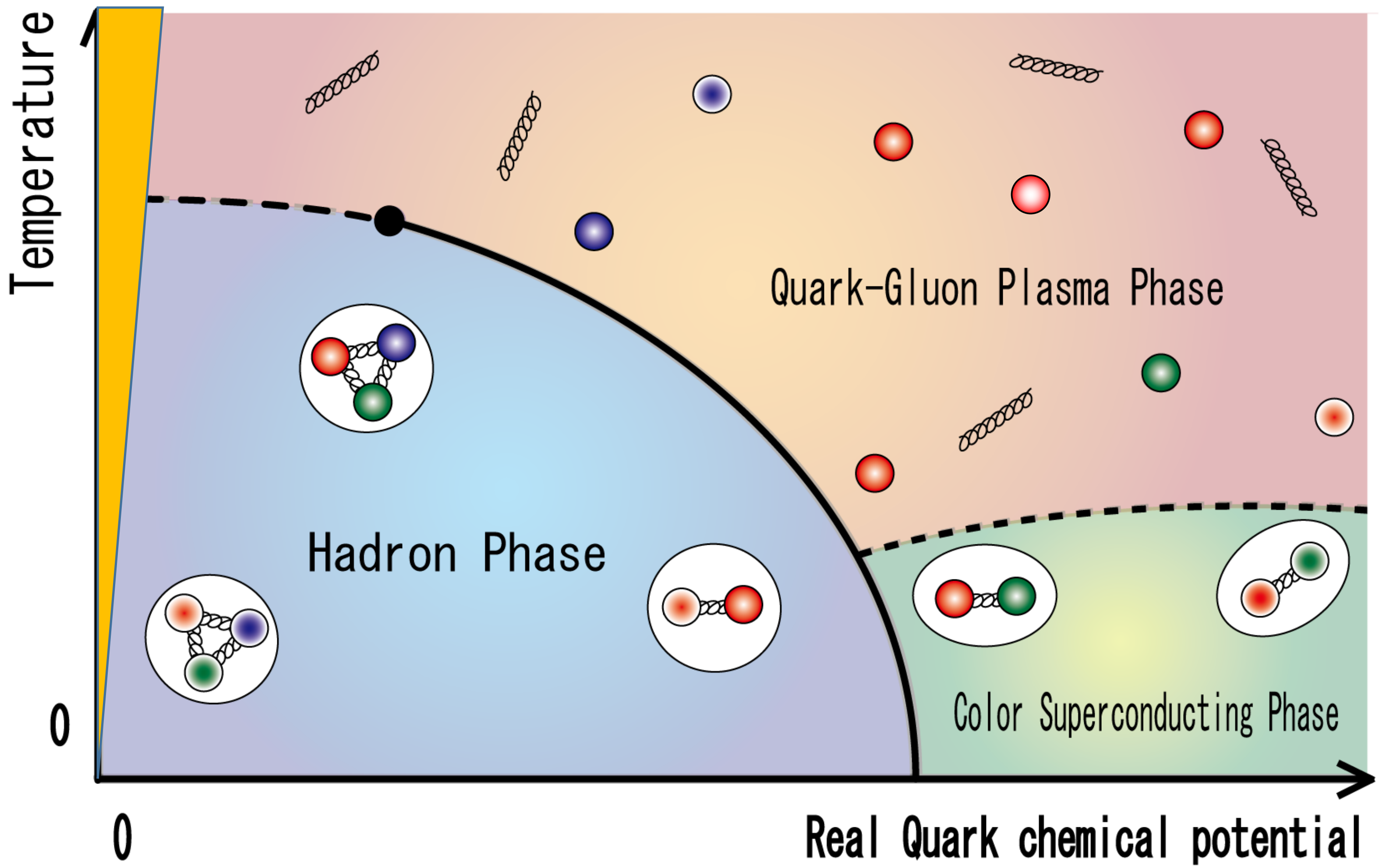
QCD phase diagram at real chemical potential



QCD phase diagram at real chemical potential

Schematic QCD phase diagram

Today's talk...



Usual phase transition

Phase transition

Ordinary phase transition ⟺ Order parameter

Spontaneous symmetry breaking

Chiral phase transition etc.

Topological order ⟺ Vacuum degeneracy

ex) X. Wen, Int. J. Mod. Phys. B4 (1990) 239

No spontaneous symmetry breaking

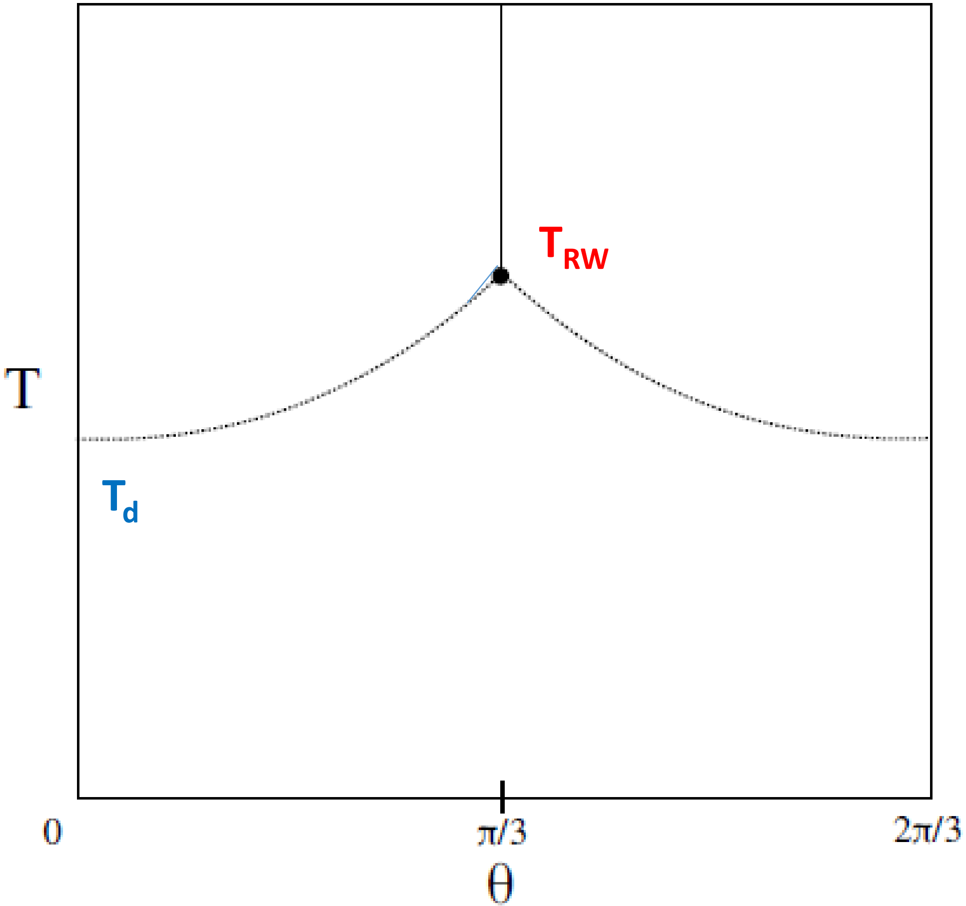
There is no order parameter

It may be related with confinement-deconfinement transition

To use analogy of Topological order, we use imaginary chemical potential at finite T

Phase diagram at imaginary chemical potential

$(\theta = \mu_I / T)$



Lattice QCD predictions

$T_{RW} > T_d$

T_d is usually determined from susceptibility of Polyakov-loop

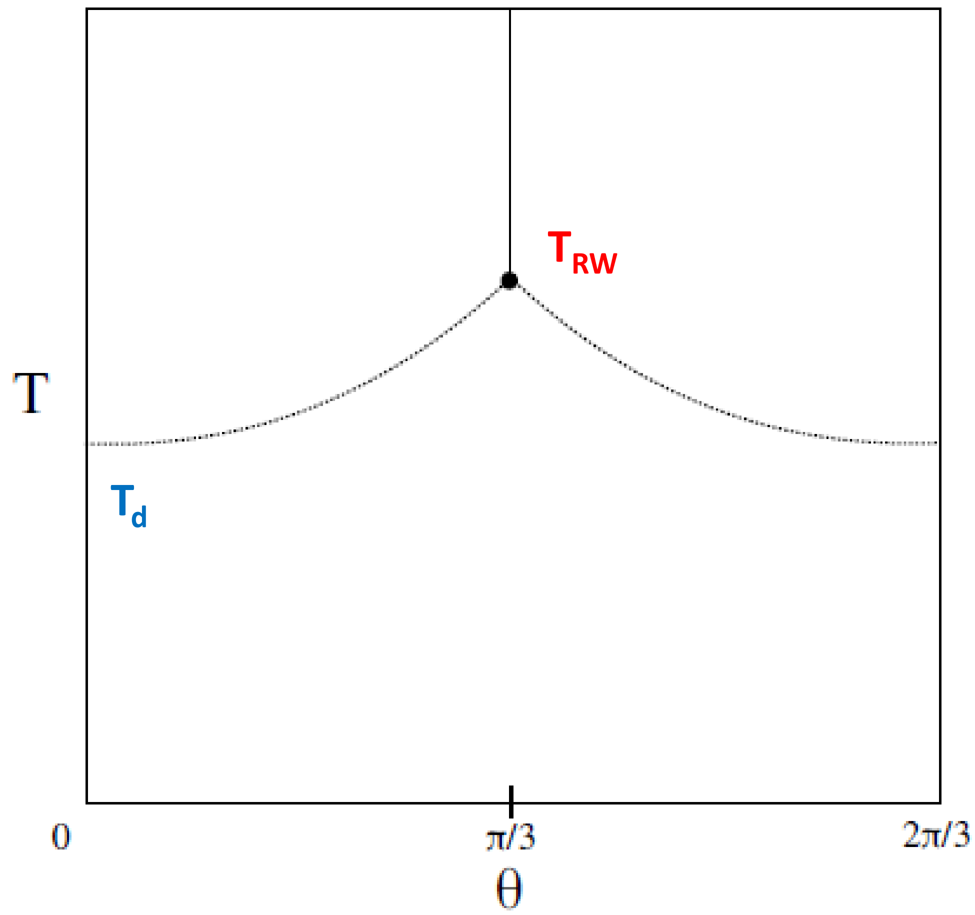
Our proposal:
From analogy of topological order, T_{RW} becomes deconfinement (pseudo)critical temperature

Details of phase structure at finite μ_I:

A. Roberge and N. Weiss, Nucl. Phys. **B** 275 (1986) 735

Phase diagram at imaginary chemical potential

$(\theta = \mu_I / T)$



In infinite quark mass limit, **Polyakov-loop** should be an **exact order parameter**

In this limit,

$$T_{RW} = T_d$$

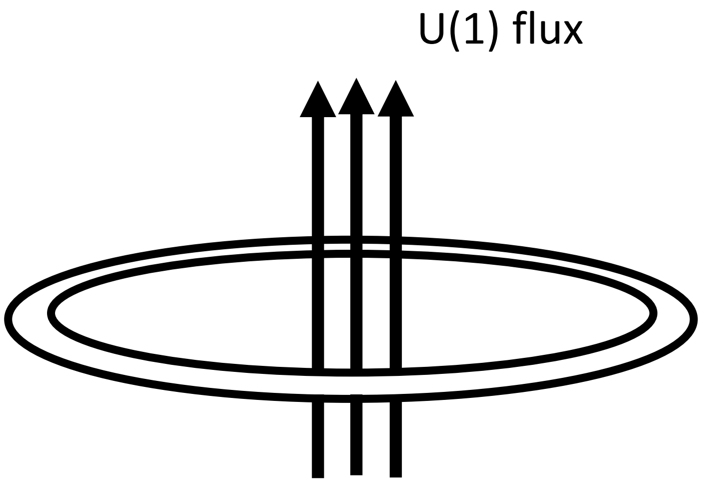
must be manifested

Actually, **this relation holds** in the limit and thus present definition is **acceptable**

Aharonov−Bohm phase

Imaginary chemical potential can be interpreted as the boundary condition of fermion and **Aharonov-Bohm phase**

Appearance form of vector potential is the same as imaginary chemical potential in action



Topological order

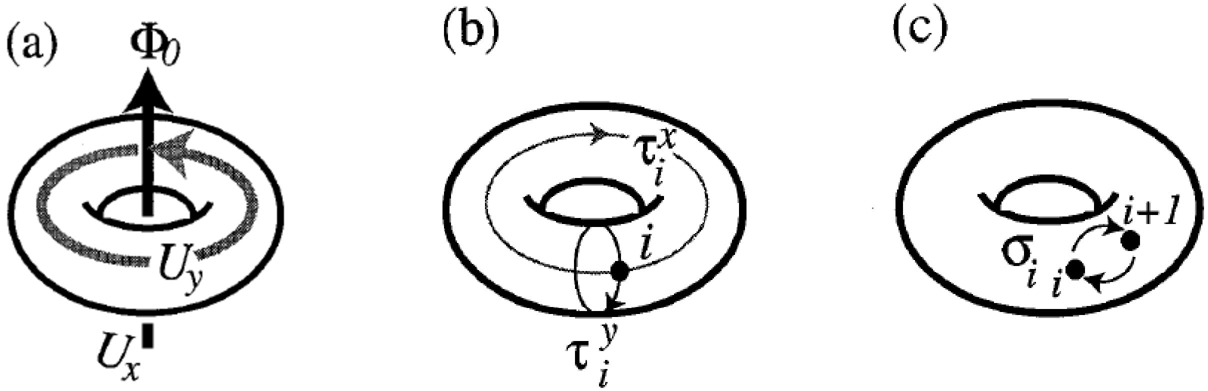
M. Sato, M. Kohmoto and Y.-S. Wu, PRL 97 (2006) 010601
M. Sato, PRD 77 (2008) 045013

Let’s consider T^3 tours

Assumption: Finite energy should be needed to create excited states

It can not be used exactly at finite T

Three **adiabatic** operations:



Braid group

Aharonov-Bohm effect

Topological order

M. Sato, M. Kohmoto and Y.-S. Wu, PRL 97 (2006) 010601
M. Sato, PRD 77 (2008) 045013

If there is fractional charge, commutation relation becomes **non-commutable**

If there is only one vacuum state, we must obtain the same state after above operations

It contradicts **non-commutable property of operations**

This idea can be used in QCD at T=0 without any unclearness

→ elementary d.o.f. : quark — Vacuum degeneracy

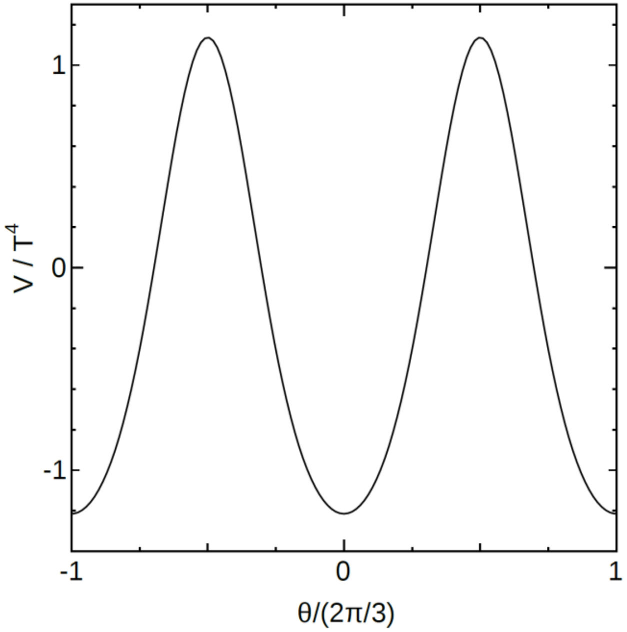
elementary d.o. f. : hadron — No vacuum degeneracy

RW endpoint and (pseudo) critical temperature

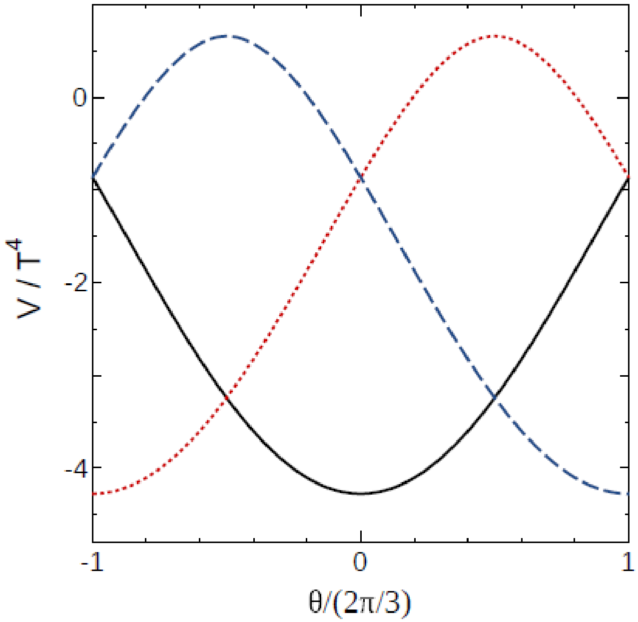
Our case Topological order can not be considered in finite T system in principle

There is Robrge-Weiss periodicity ➡ Period is the same in confined and deconfined phases

Confined phase

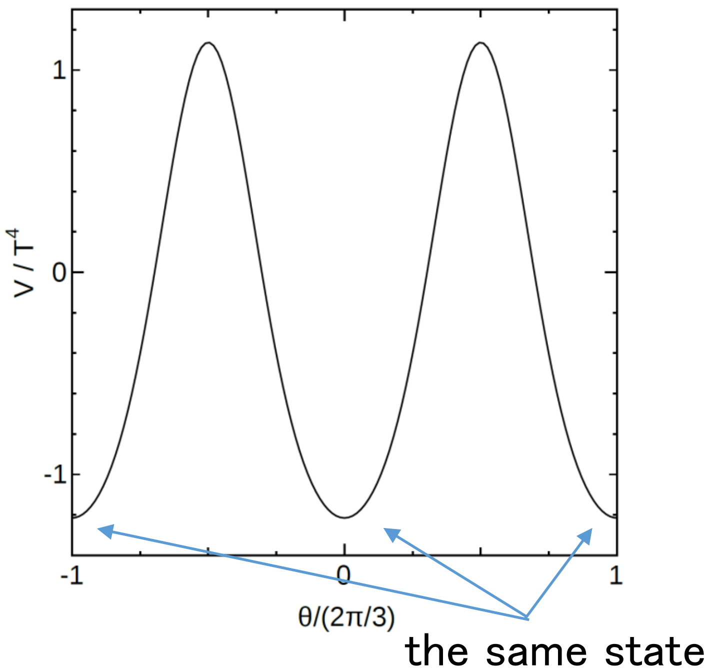


Deconfined phase

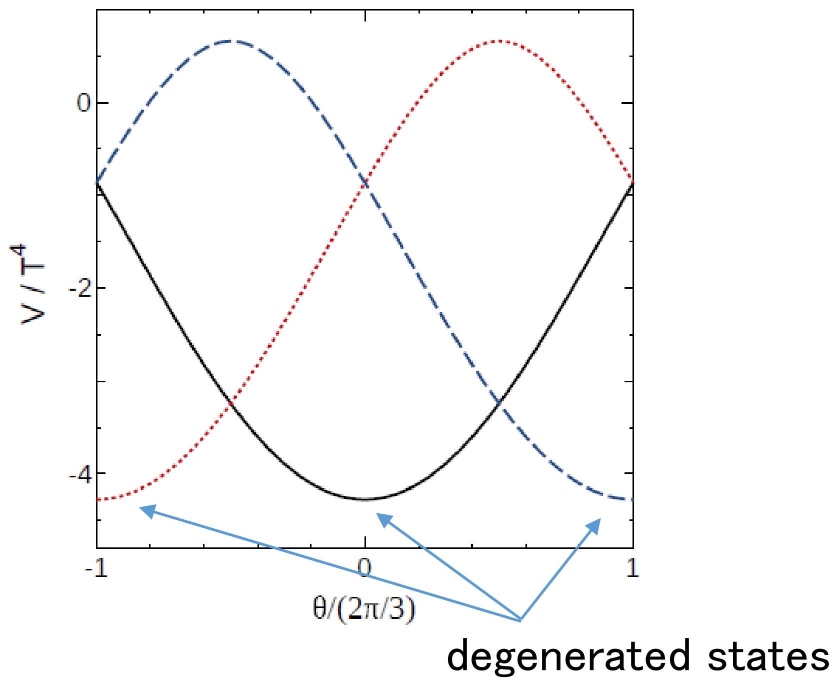


RW endpoint and (pseudo) critical temperature

Confined phase



Deconfined phase



There is significant difference between two phases in origin of RW periodicity

- Nontrivial vacuum degeneracy ➡ Nontrivial free energy degeneracy (in deconfined phase)
- No vacuum degeneracy ➡ No free energy degeneracy (in confined phase)

Expected QCD phase diagram at complex chemical potential

At finite real chemical potential,

our system with μ_I corresponds to the system with **complex chemical potential**

This calculation is very difficult at present even in effective model calculations

It may be possible after extending the study
Y. Tanizaki, H, Nishimura and **K.K.** , PRD 91 (2015) 101701

We use perturbative calculation

Effective potential:

$$\mathcal{V}(T,\mu_R,\mu_I) = \mathcal{V}(T,0,\mu_I) - \underbrace{\left(\frac{\mu_R}{T}\right)\left(Tn_q(\mu_R,\mu_I)|_{\mu_R=0}\right)} - \underbrace{\frac{1}{2}\left(\frac{\mu_R}{T}\right)^2 \frac{d[Tn_q(\mu_R,\mu_I)]}{d\mu_R/T}\Big|_{\mu_R=0}} + \mathcal{O}\left((\mu_R/T)^3\right)$$

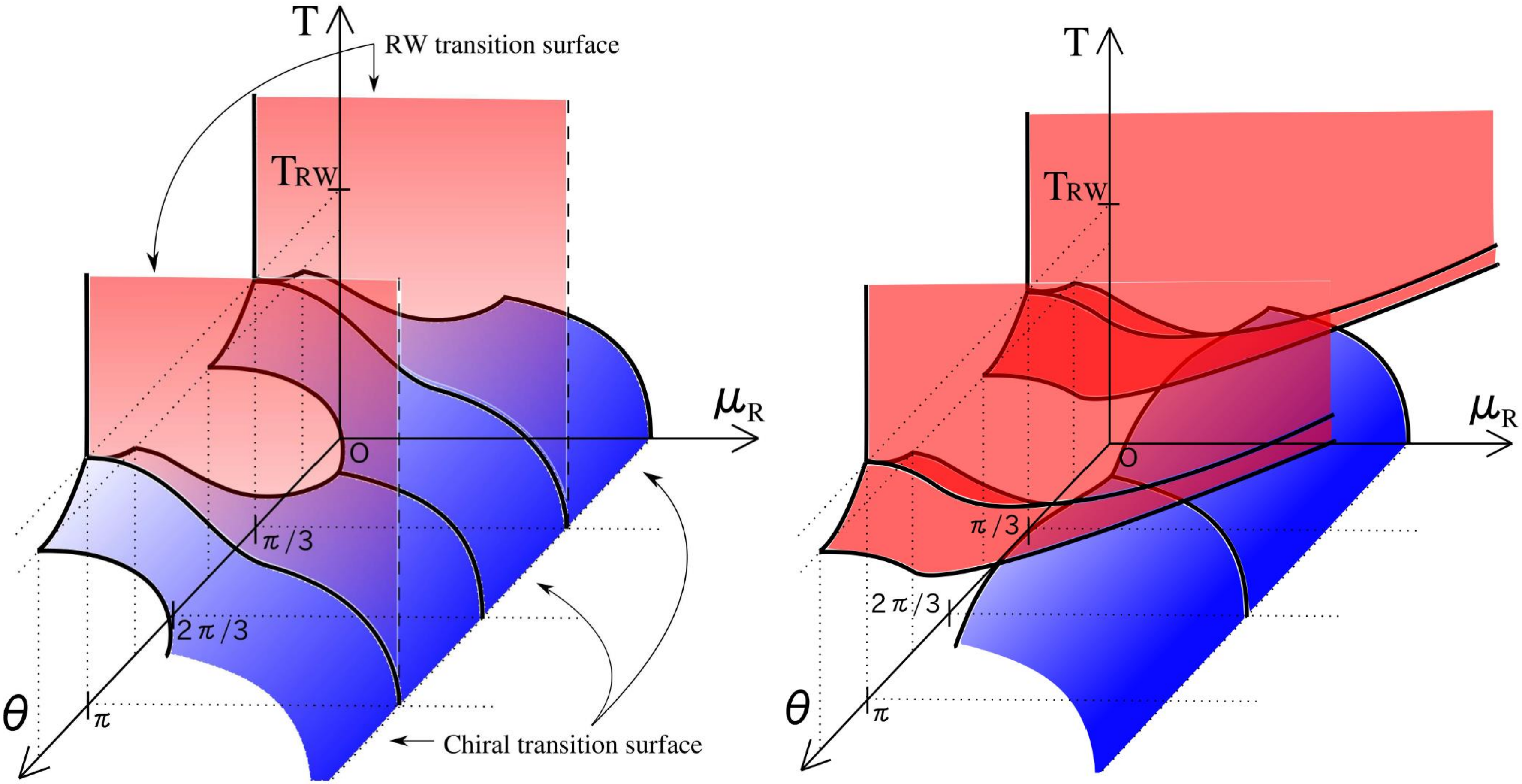
Pure imaginary

This term make effective potential
with confinement configuration higher
than that with deconfinement configuration

T_{RW} should decrease at least in very small μ_R

Expected QCD phase diagram at complex chemical potential

Two possible expected phase diagram



Summary

Imaginary chemical potential can be interplayed as **Aharonov-Bohm phase**

We may determine deconfinement (pseudo)critical temperature from RW endpoint

It is not exact discussion…

$\theta=0$ → θ_{AB} (additional or dynamical) = 0 and $\theta_{AB} = \pm 2\pi/3$ have the same (minimum) energy

$\theta=\pi/3$ → $\theta_{AB} = \pm\pi/3$ and $\theta_{AB} = \pi$ have the same (minimum) energy

These states are degenerated different vacua at $T>T_{RW}$ but the same states at $T < T_{RW}$

We investigate QCD phase structure at finite **complex chemical potential**

As a first step, we use perturbative calculation

Two possible expected QCD phase diagrams are presented

Topological order may have deep relation with **entanglement entropy**

It is quite interesting to investigate deconfinement transition from present viewpoint