Light nuclei and nucleon form factors in $N_f = 2+1$ lattice QCD

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for PACS Collaboration

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1. Light nuclei

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, and A. Ukawa for PACS Collaboration

Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD86:074514(2012)

PRD92:014501(2015)

2. Nucleon form factors

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki and A. Ukawa

for PACS Collaboration

Outline

- Introduction
- Simulation parameters
- Results of light nuclei
  - ^{4}He and ^{3}He channels
  - NN channels
- Very preliminary result at $m_\pi \sim$ 0.145 GeV
  - Light nuclei binding energy
  - nucleon form factors
- Summary and future work

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

degrees of freedom of quarks and gluons

[Talk(Tue): N. Ukita]

$$\underbrace{\overbrace{\text{quarks and gluons} \rightarrow \text{protons and neutrons}}\rightarrow \text{nuclei}}$$

goal: quantitatively understand property of nuclei from QCD

So far not many studies for multi-baryon bound states

$\rightarrow$ Can we reproduce known binding energy in light nuclei?

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

degrees of freedom of quarks and gluons

[Talk(Tue): N. Ukita]

2nd motivation: Nucleon form factors not well understood

$$\underbrace{\overbrace{\text{quarks and gluons} \rightarrow \text{protons and neutrons}} \rightarrow \text{nuclei}}$$

goal: quantitatively understand property of nuclei from QCD

Multi-baryon bound state from lattice QCD

Extend our exploratory $N_f = 0$ study to $N_f = 2+1$ calculation

1. ^{4}He and ^{3}He
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
   - '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11, '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV, $N_f = 3$ $m_\pi = 0.81$ GeV
   - '11, '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.02 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
3. NN
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
   - '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)

Other states: ΞΞ, '12 NPLQCD; spin-2 $N\Omega$, ^{16}O and ^{40}Ca, '14 HALQCD

Calculation method of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto$ exp $\left(N_N \left[m_N - \frac{3}{2}m_\pi\right] t\right)$

2. Calculation cost

Wick contraction for ^{4}He $= p^2 n^2 = (udu)^2(dud)^2$: 518400

proton $= p = (udu)$: 2

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$

Calculation method of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\rightarrow$ heavy quark $m_\pi = 0.8$–0.3 GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400 $\rightarrow$ 1107

$\rightarrow$ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

+ block of 3 quark props(parallel) and contraction(workstation)

Multi-baryon: '12 Doi and Endres; Detmold and Orginos; '13 Günther et al.

3. Identification of bound state on finite volume

attractive scattering state $\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0$

'86,'91 Lüscher, '07 Beane *et al.*

$\rightarrow$ Volume dependence of $\Delta E_L \rightarrow \Delta E_\infty \neq 0 \rightarrow$ bound state

Spectral weight: '04 Mathur *et al.*, Anti-PBC '05 Ishii *et al.*

Simulation parameters

$N_f = 2+1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.30$ GeV and $m_N = 1.05$ GeV PRD92:014501(2015)

$m_s \sim$ physical strange quark mass

^{4}He, ^{3}He, NN(^{3}S$_1$ and ^{1}S$_0$)

		$m_\pi = 0.3$ GeV		$m_\pi = 0.5$ GeV		R
L	L [fm]	N_{conf}	N_{meas}	N_{conf}	N_{meas}	
48	4.3	400	1152	200	192	12
64	5.8	160	1536	190	256	5

$R = (N_{\text{conf}} \cdot N_{\text{meas}})_{0.3\text{GeV}} / (N_{\text{conf}} \cdot N_{\text{meas}})_{0.5\text{GeV}}$

Computational resources

PACS-CS, T2K-Tsukuba, HA-PACS, COMA at Univ. of Tsukuba

T2K-Tokyo and FX10 at Univ. of Tokyo, and K at AICS

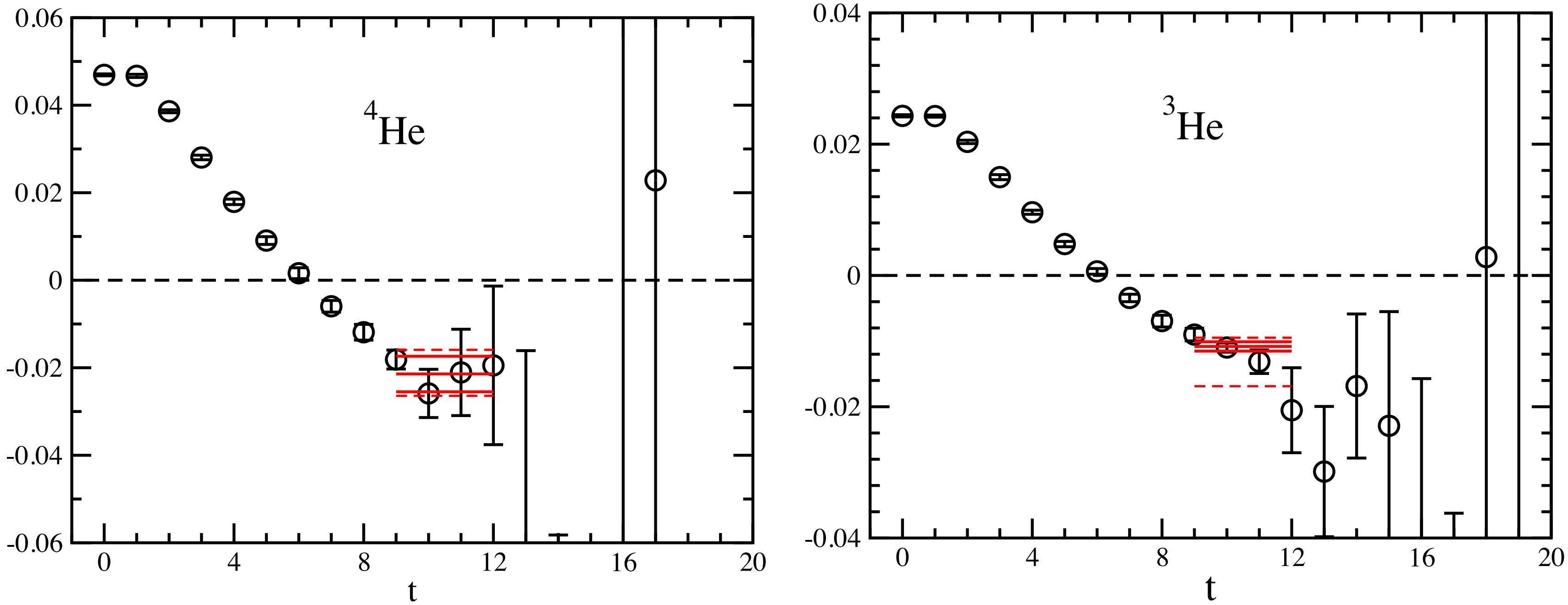
Results at $m_\pi = 0.3$ GeV

Results of $\Delta E_L = E_0 - N_N m_N$

^{4}He and ^{3}He channels at $m_\pi = 0.3$ GeV on $L = 5.8$ fm

TY *et al.*, PRD92:014501(2015)

$$\Delta E_L = \log\left(\frac{R_{^4\mathrm{He}}(t)}{R_{^4\mathrm{He}}(t+1)}\right) \text{ with } R_{^4\mathrm{He}}(t) = \frac{C_{^4\mathrm{He}}(t)}{(C_N(t))^4}$$

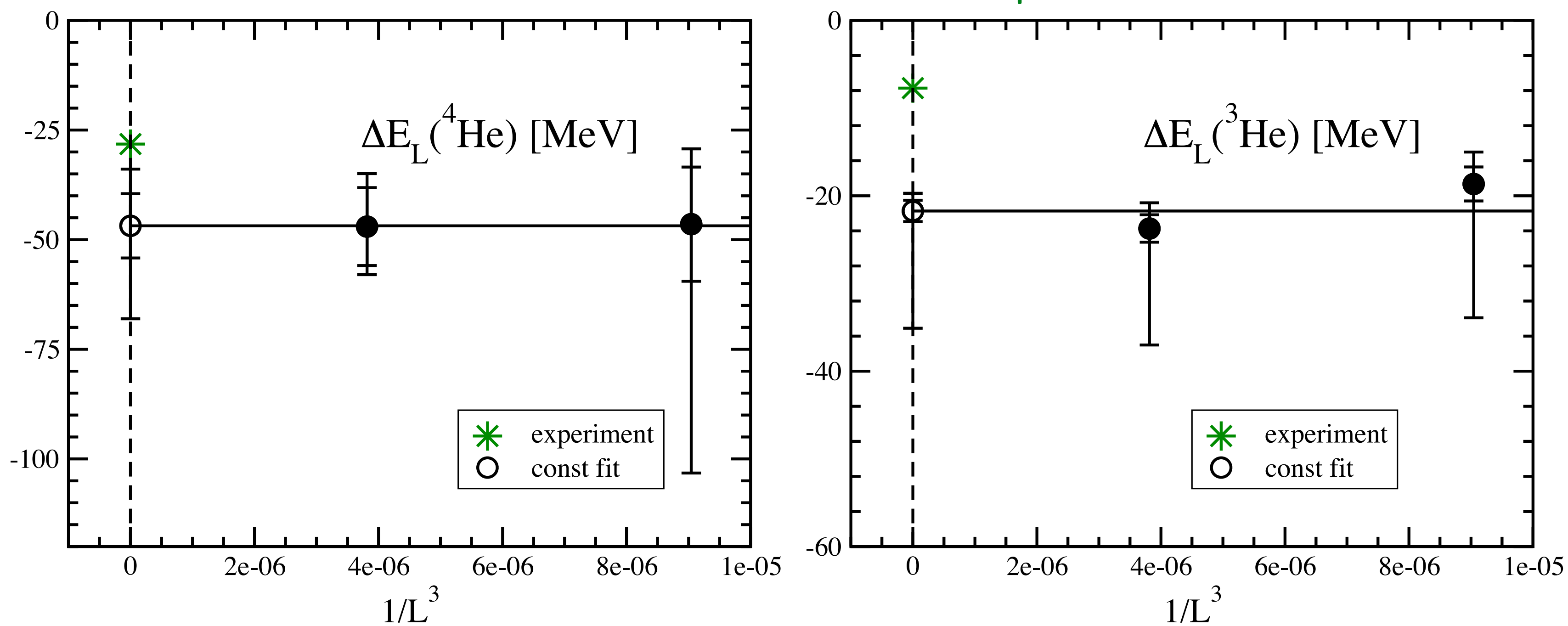


- Larger error in ^{4}He channel
- Statistical error under control in $t < 12$
- Negative ΔE_L in both channels

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.3$ GeV

TY *et al.*, PRD92:014501(2015)

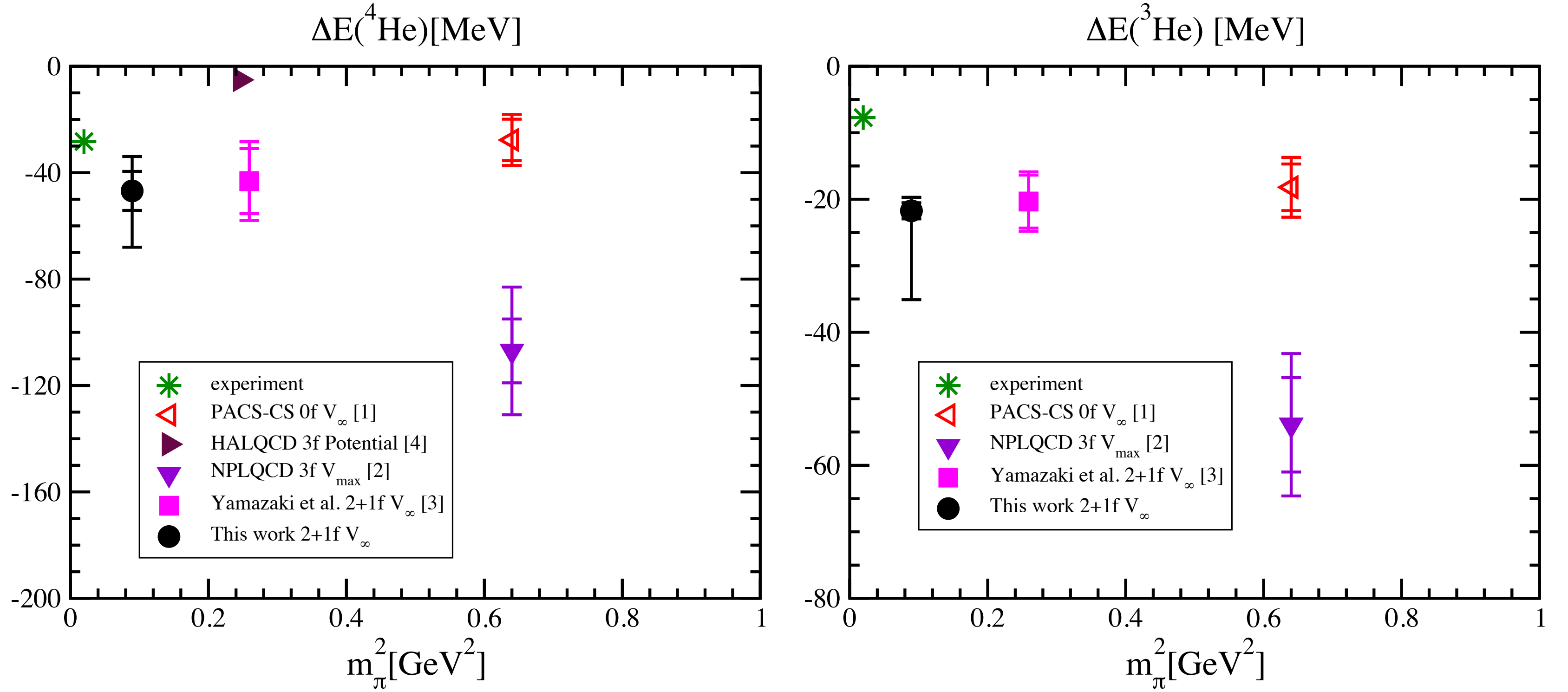
Identification of bound state from volume dependence of ΔE



- $\Delta E_L < 0$ and mild volume dependence,
  though large systematic error from choice of fit range

- Infinite volume extrapolation with constant fit
  Both ground states are bound states.
  $\Delta E_{^4\text{He}} = 47(7)(^{+20}_{-11})$ MeV $\qquad \Delta E_{^3\text{He}} = 21.7(1.2)(^{+13}_{-1.6})$ MeV

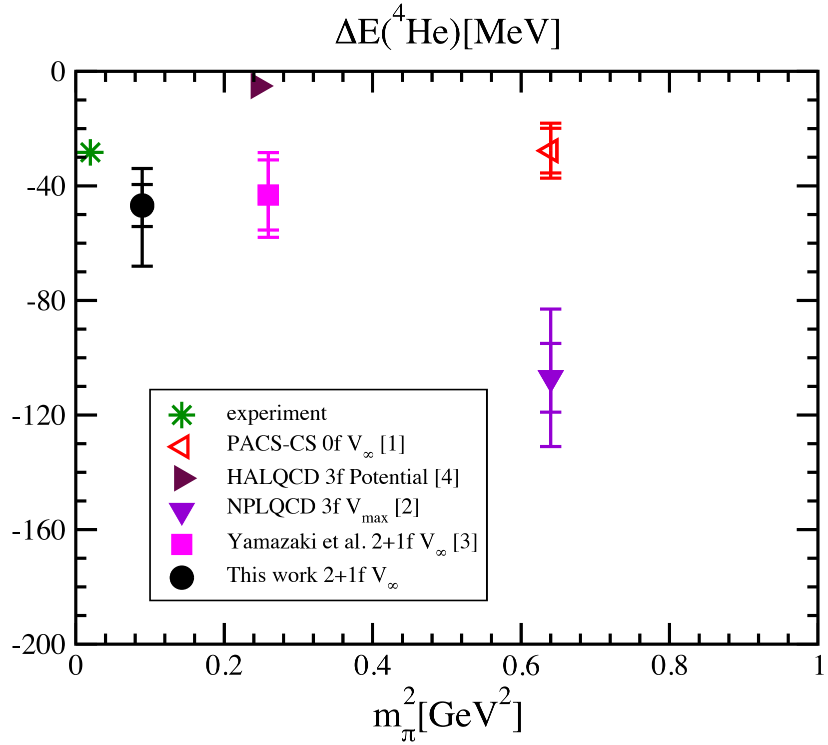
Comparison of ^{4}He and ^{3}He channels

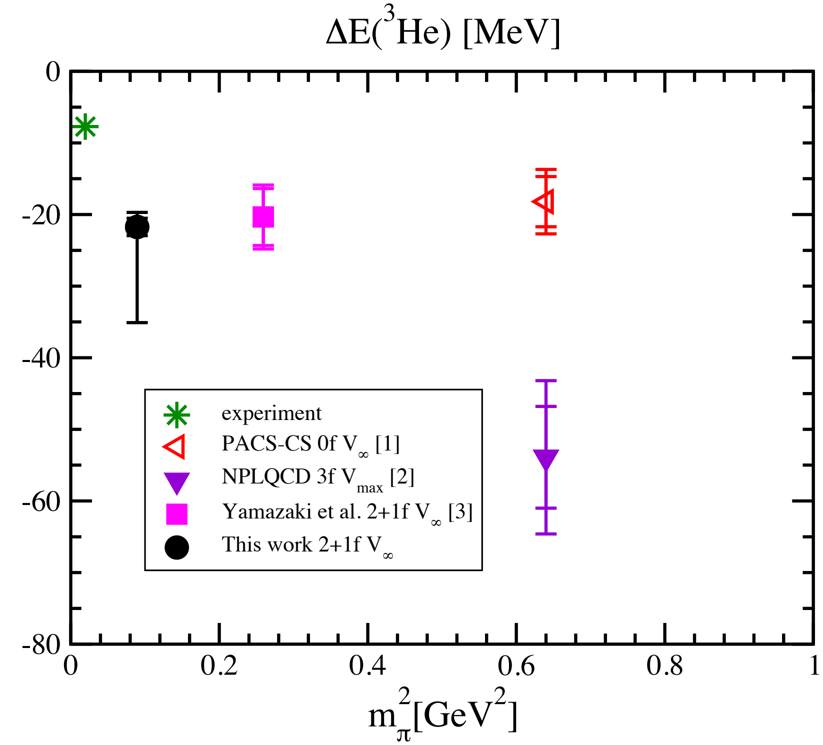


$L^3 \to \infty$ results only

- Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV
- Same order of ΔE to experiments

Comparison of ^{4}He and ^{3}He channels





$L^3 \to \infty$ results only

Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments $\to$ relatively easier than NN

large $|\Delta E|$ makes less V dependence at physical m_π

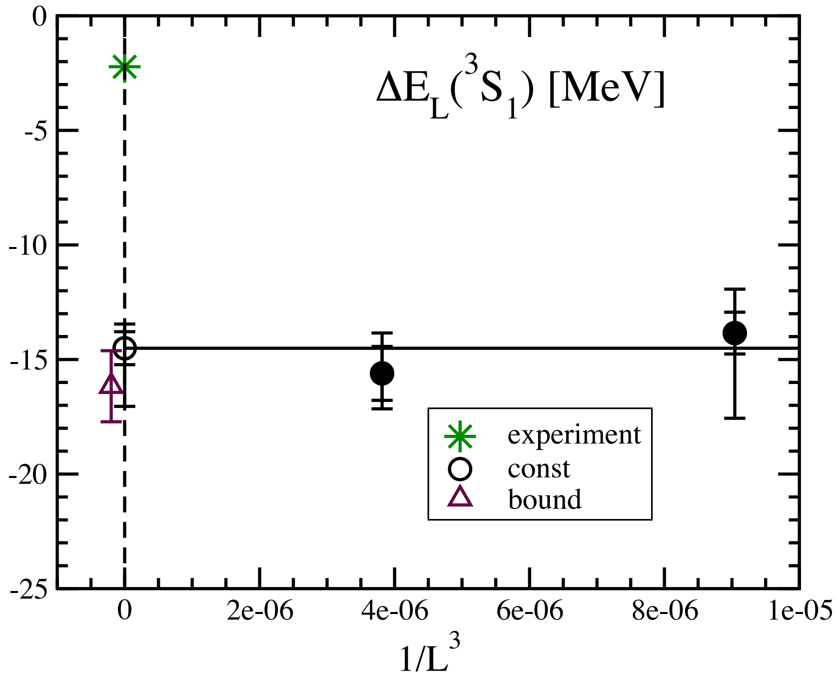
touchstone of quantitative understanding of nuclei from lattice QCD

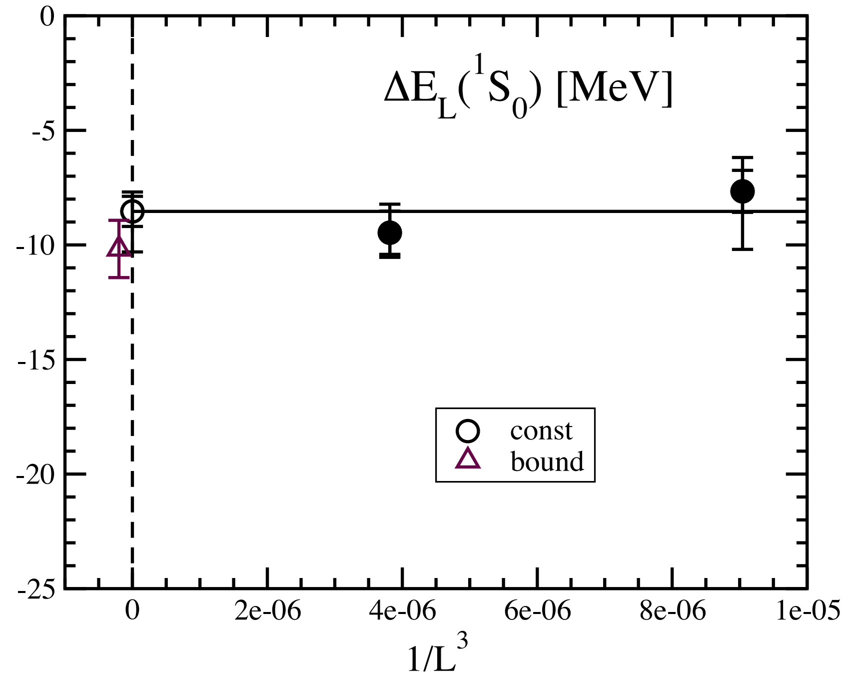
Investigations of m_π dependence $\to$ $m_\pi \sim$ 0.145 GeV on $L \sim 8$ fm

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.3$ GeV

TY *et al.*, PRD92:014501(2015)

Identification of bound state from volume dependence of ΔE





- Negative ΔE_L
- Infinite volume extrapolation of ΔE_L with constant

  Finite volume effect in two-particle system '04 Beane *et al.*, '06 Sasaki & TY

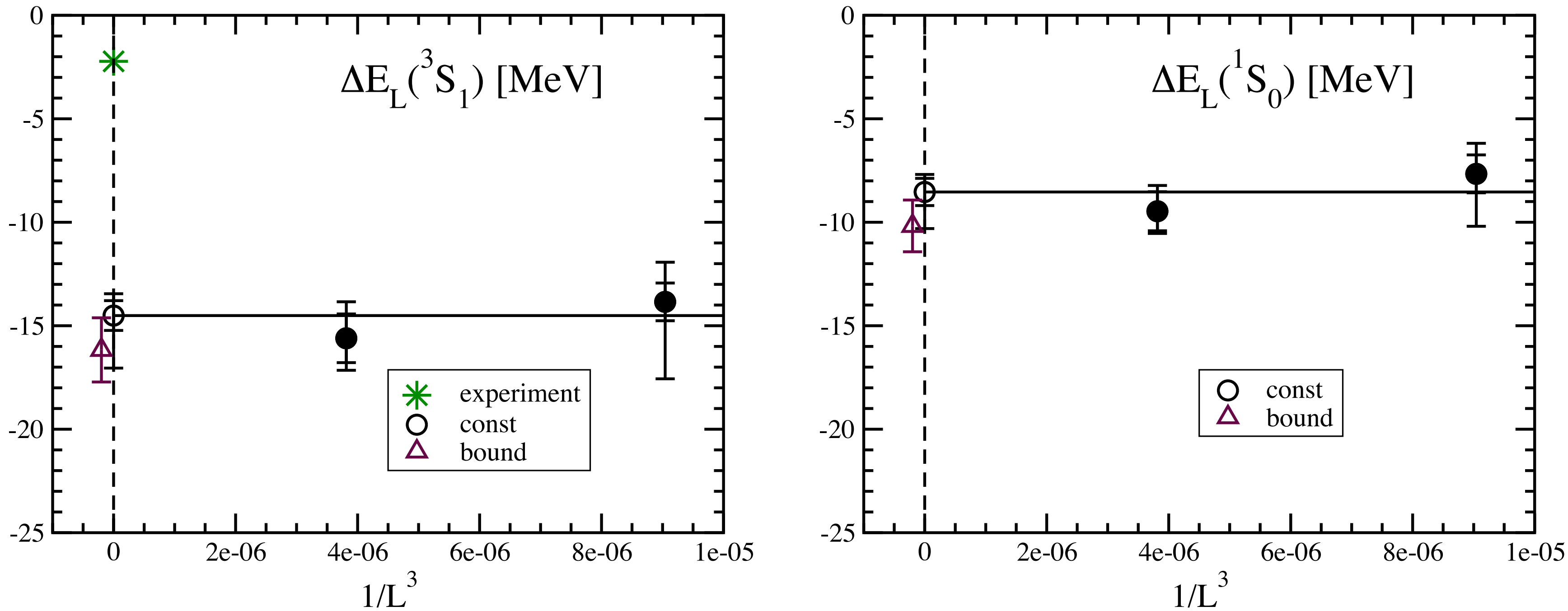
$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{|\vec{n}|\neq 0}\frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\}, \quad \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

based on Lüscher's finite volume formula

NN (^{3}S$_1$ and ^{1}S$_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.3$ GeV

TY *et al.*, PRD92:014501(2015)

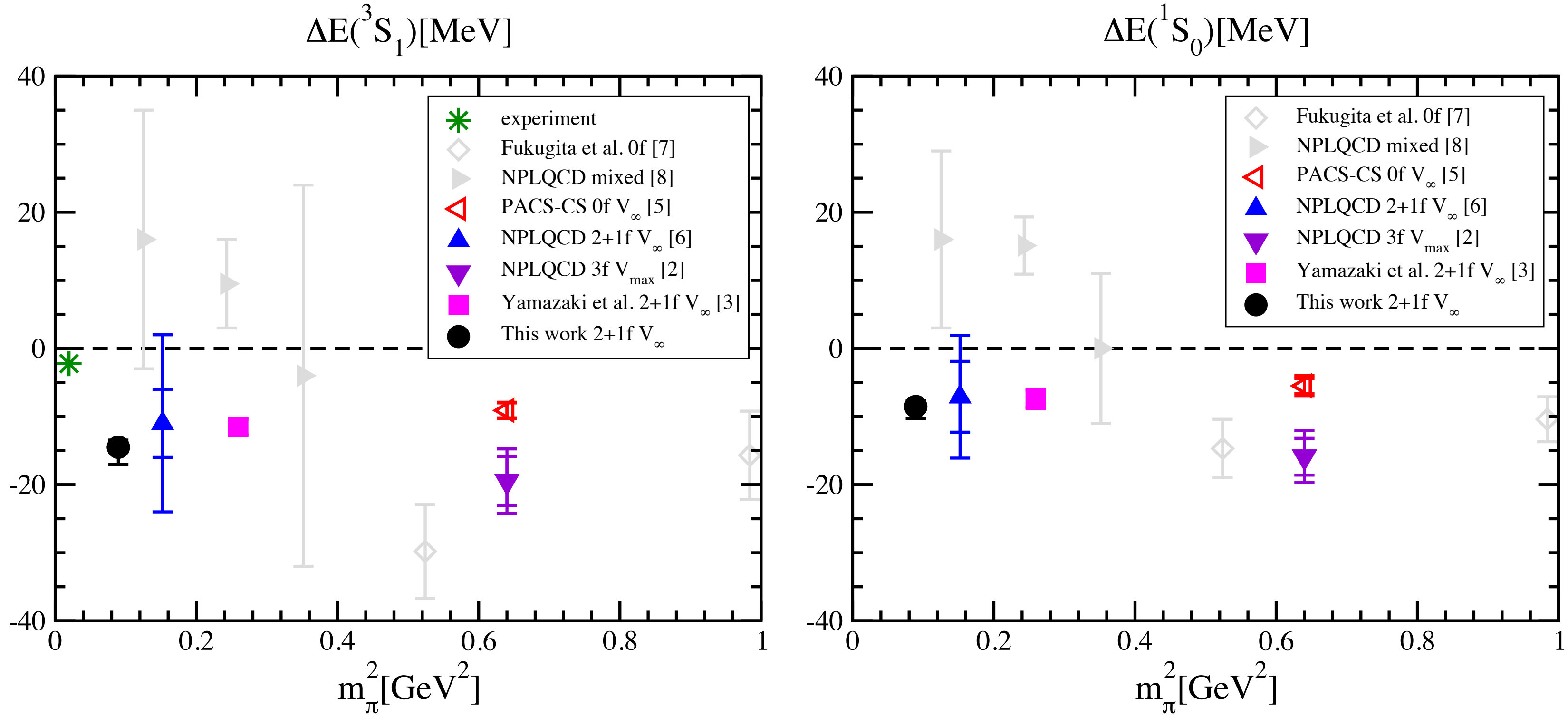
Identification of bound state from volume dependence of ΔE



Bound state in both channels $\leftarrow$ different from experiment

$\Delta E_{^3\mathrm{S}_1} = 14.5(0.7)(^{+2.4}_{-0.8})$ MeV $\quad$ $\Delta E_{^1\mathrm{S}_0} = 8.5(0.7)(^{+1.6}_{-0.5})$ MeV

Comparison of NN channels



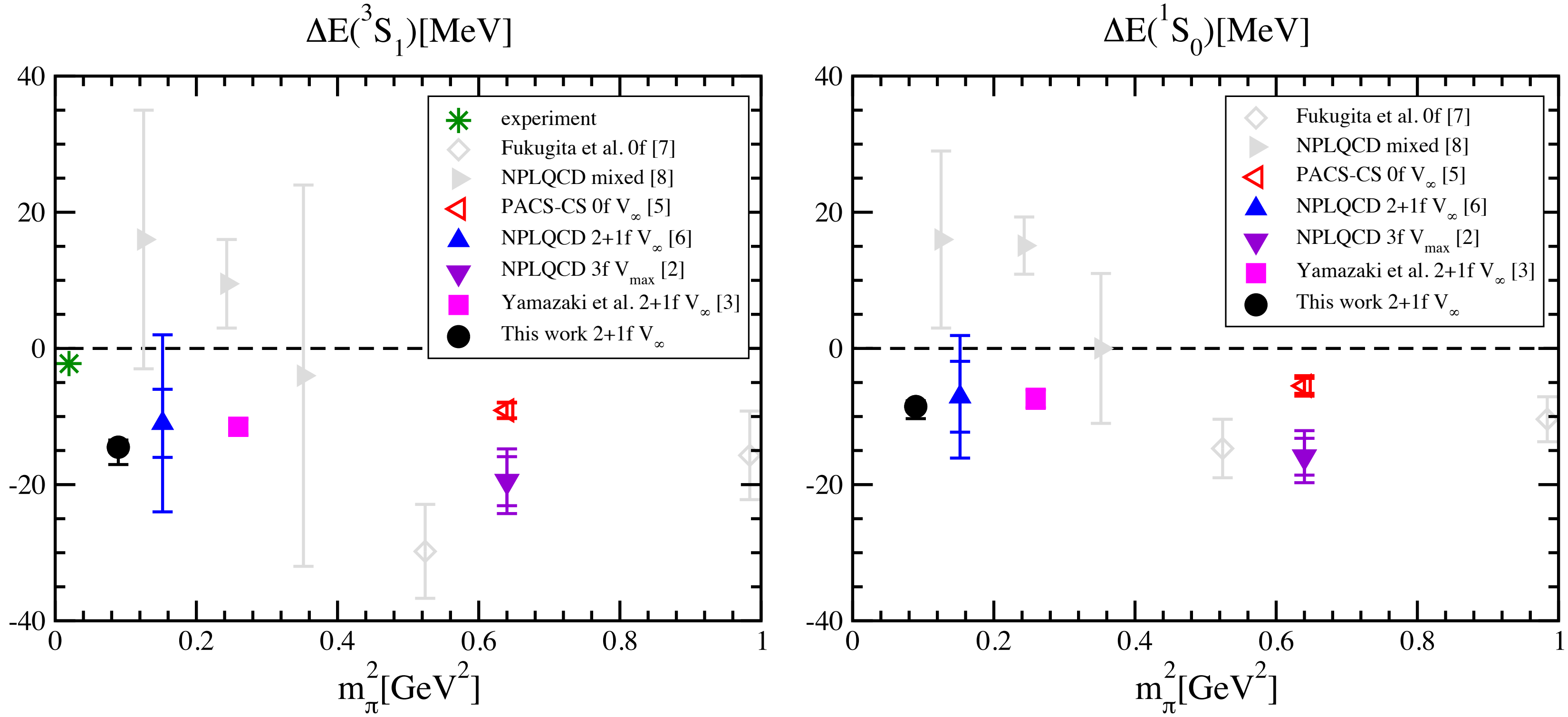
gray data: single volume calculation

$L^3 \to \infty$ data: existence of bound states in ^{3}S$_1$ and ^{1}S$_0$

inconsistent with experiment due to larger m_π(?)

Investigation of m_π dependence $\to$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Comparison of NN channels



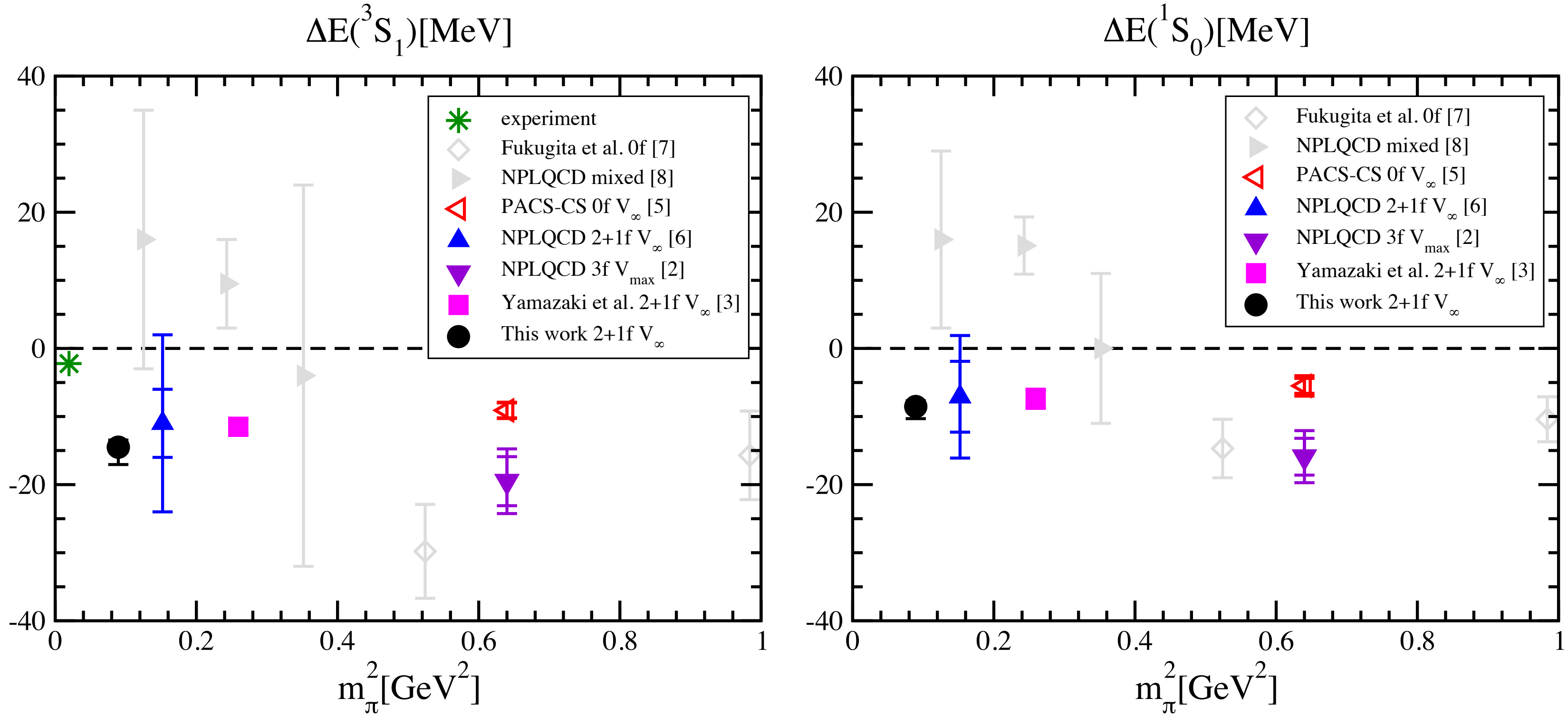
gray data: single volume calculation

Investigations of m_π dependence $\rightarrow$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 8$ fm

'86 Lüscher, '04 Beane *et al.*, '14 Briceño *et al.*

Comparison of NN channels



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Investigations of m_π dependence $\rightarrow$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 8$ fm

'86 Lüscher, '04 Beane *et al.*, '14 Briceño *et al.*

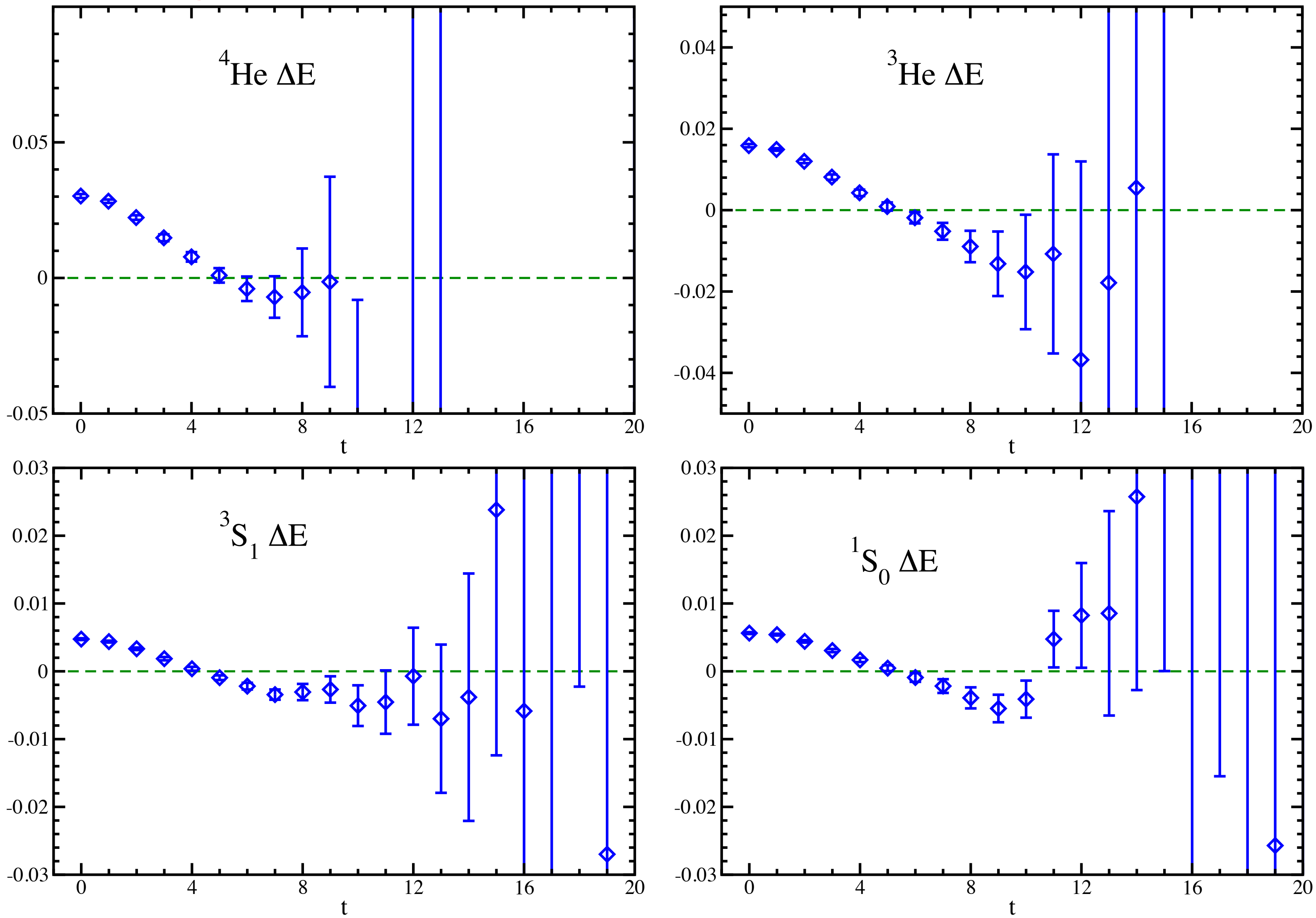
^{3}S$_1$: $\Delta E_{\rm exp} = 2.2$ MeV

$$\Delta E_L = -\left(\Delta E_{\rm exp} + \mathcal{O}(\exp(-L\sqrt{m_N \Delta E_{\rm exp}}))\right) \lesssim -4 \text{ MeV}$$

^{1}S$_0$: $a_0^{\rm exp} = 23.7$ fm

$$\Delta E_L = -\frac{4\pi a_0^{\rm exp}}{m_N L^3} + \mathcal{O}(1/L^4) \lesssim -2 \text{ MeV}$$

Very preliminary results of ΔE at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm



c.f. [Talk(Tue) for hadron spectrum: N. Ukita]

Computational resources

HA-PACS, COMA @Univ. of Tsukuba, K @AICS, FX100 @RIKEN

Nucleon form factors at almost physical m_π

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, and A. Ukawa

for PACS Collaboration

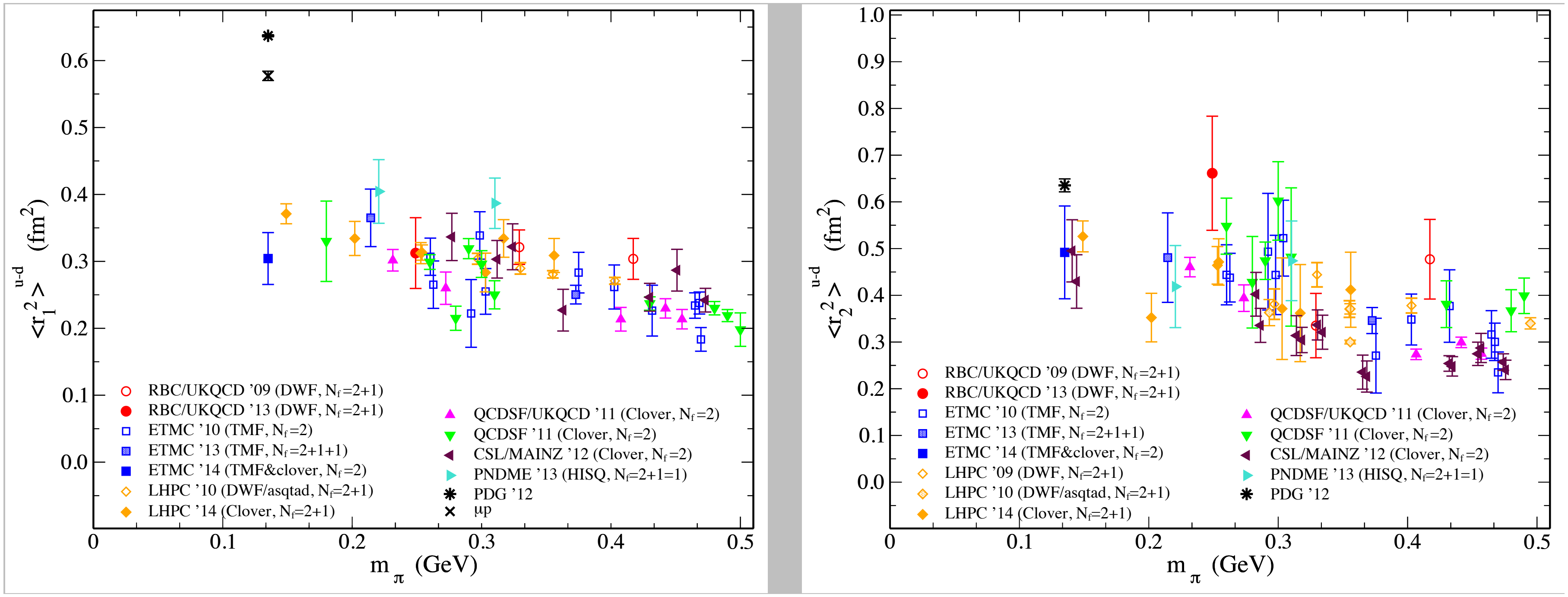
Computational resources

COMA @Univ. of Tsukuba, FX10 @Univ. of Tokyo,

K @AICS, FX100 @RIKEN, System E @Kyoto Univ.

Example of large quark mass dependence near $m_\pi \to 0$
Isovector radii from form factors F_1 and F_2

Constantinou, Lat14 plenary

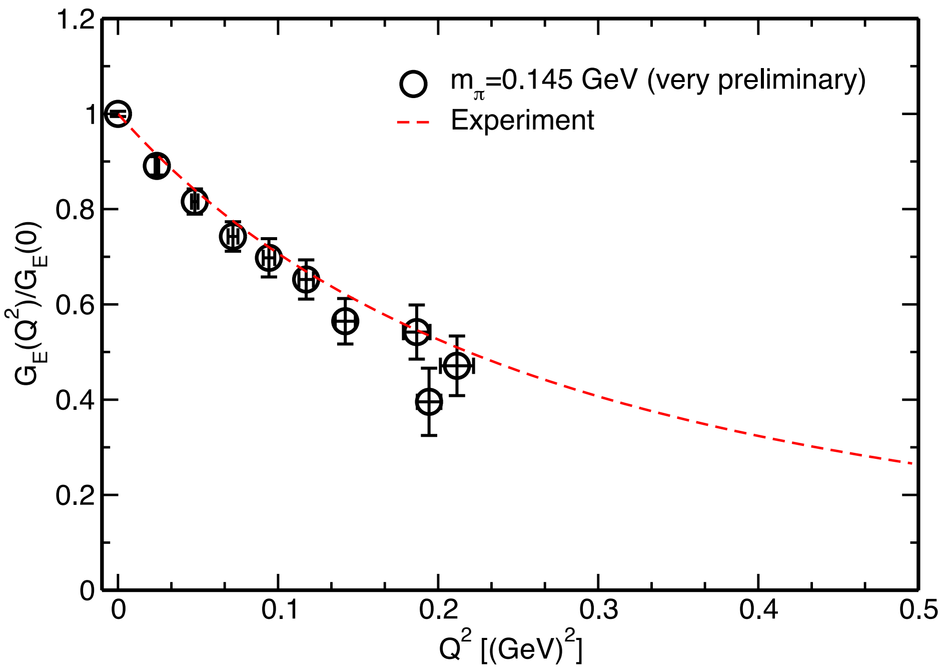


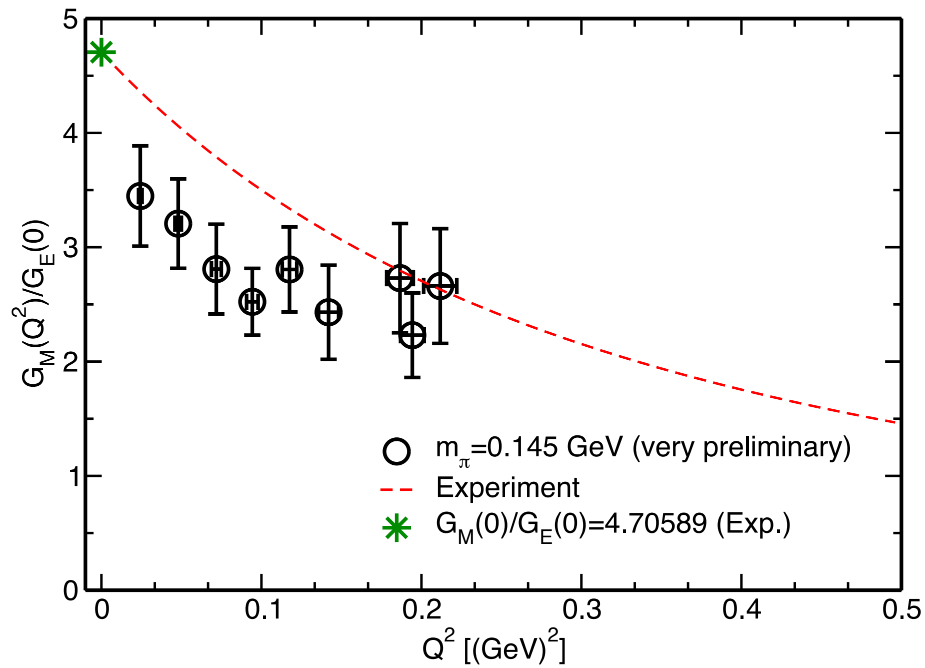
important for understanding of nucleus property
Can we reproduce experiment at physical m_π?

c.f.) '15 Capitani *et al.*,'15 ETM

Isovector electric and magnetic Sachs form factors

Very preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm





Need much more statistics

but encouraging signal in G_E

Axial charge $g_A = Z_A g_A^{\text{bare}}$

Very preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

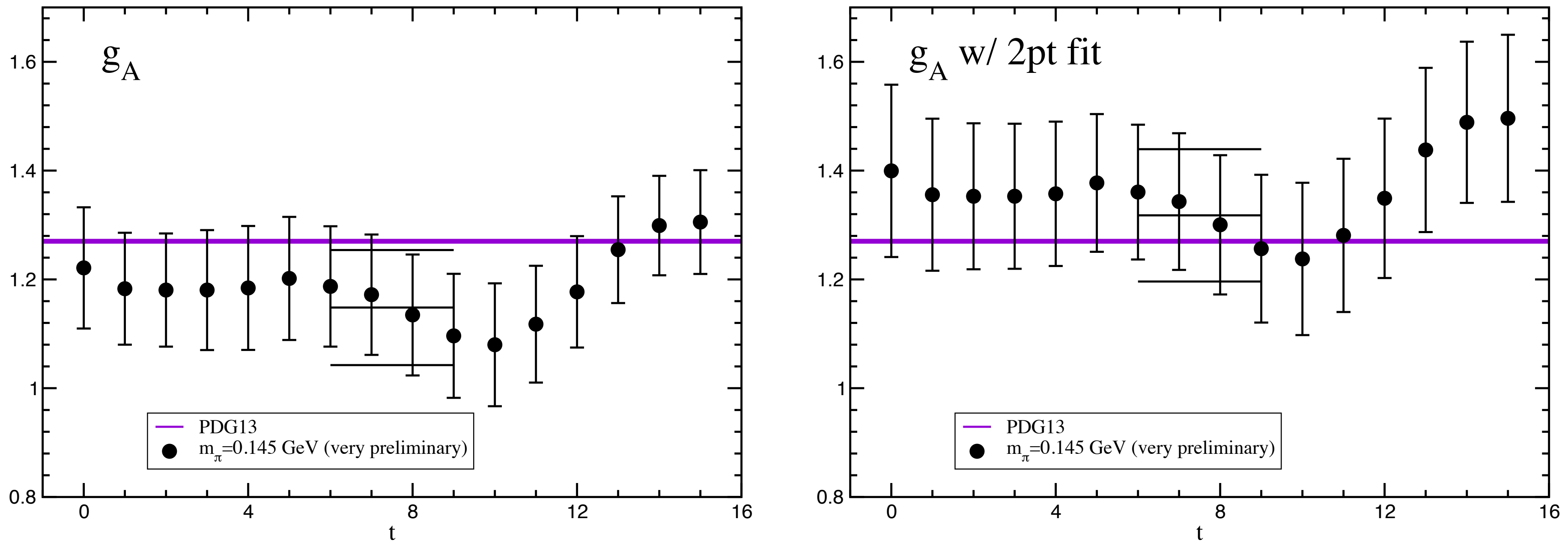
$Z_A \sim 0.965$ from SF scheme [Poster: K.-I. Ishikawa]

consistent with $Z_V = 1/G_E(0)$ within 1–2%

$g_A^{\text{bare}} = C_{A_3}(t)/C_N(t_{\text{sink}})$

$g_A^{\text{bare}} = C_{A_3}(t)/(Z_N^2 \exp(-M_N t_{\text{sink}}))$

Z_N and M_N from fit of $C_N(t)$



Discrepancy of two analyses → systematic error

roughly consistent with experiment,

but need much more statistics for stringent test

Summary

$N_f = 2 + 1$ lattice QCD at $m_\pi = 0.5$ and 0.3 GeV

- Volume dependence of ΔE

  $\Delta E \neq 0$ of 0th state in infinite volume limit

  $\rightarrow$ bound state in ^{4}He, ^{3}He, $^3S_1$ and $^1S_0$ at $m_\pi = 0.5$ and 0.3 GeV

- ΔE larger than experiment and small m_π dependence

- Bound state in $^1S_0$ not observed in experiment

  Deep bound state in $N_f = 3$ at $m_\pi = 0.8$ GeV ('12 NPLQCD)

- No bound state in HALQCD method

Need further investigations

e.g. systematic error from large m_π and finite lattice spacing

$N_f = 2 + 1$ $m_\pi \sim 0.14$ GeV on $L \sim 8$ fm

ΔE for nuclei and Isovector form factors of nucleon