# Some nucleon isovector obsesrvables from 2+1-flavor domain-wall QCD at physical mass

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RBC and UKQCD have been generating dynamical DWF ensembles

- good chiral and flavor symmetries,
- a lot of good physics in pion and kaon.

We have been at physical mass for a while now.

In nucleon: RBC and UKQCD observed puzzling and persistent deficit in the isovector axial charge, $g_A$, while vector-current form factors are well-behaved, and low structure-function moments are trending toward experiments.

Nucleon structure calculations are jointly done by LHP and RBC: Sergey Syritsyn, Michael Abramczyk, Tom Blum, Michael Engelhardt, Jeremy Green, Taku Izubuchi, Chulwoo Jung, Stefan Krieg, Meifeng Lin, SO, Stefan Meinel, Negele John, and Andrew Pochinsky.

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Nucleon form factors, measured in elastic scatterings or $\beta$ decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right] u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_5\gamma_\mu F_A(q^2) + \gamma_5 q_\mu F_P(q^2)\right] u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

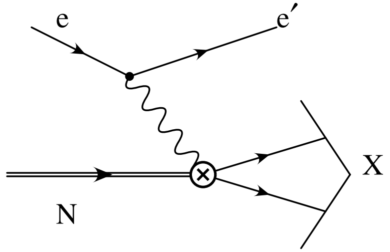
Related to mean-squared charge radii, anomalous magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2701(25)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in $t$ for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d}[e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- $c_1$, $c_2$, $e_1$, and $e_2$ are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized ($F_1/F_2$): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s\langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{trace})\right]q$$

Polarized ($g_1/g_2$): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ ($g_A$), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, $d_1$, $d_2$, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{traces})\right]q$$

and transversity ($h_1$):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu-S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, $d_1$, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + ...,$$

resulting in dependence on source-sink separation, $t_{\rm sep} = t_{\rm sink} - t_{\rm source}$,

$$\langle 0|O|0\rangle + A_1 e^{-(E_1-E_0)t_{\rm sep}}\langle 1|O|0\rangle + ...$$

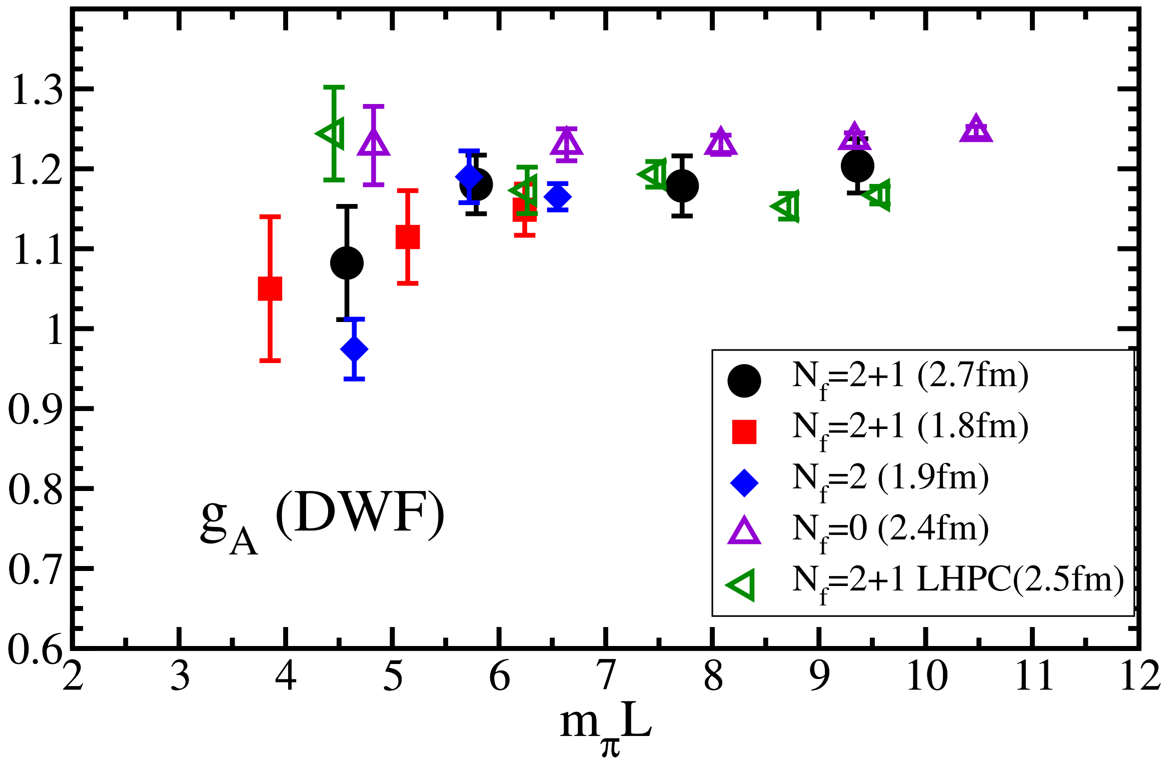
Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1|Q|0\rangle = 0$.

- $g_V$ is clean,
- $g_A$ does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

We can optimize the source so that $A_1$ is small, and we take sufficiently large $t_{\rm sep}$: Indeed with AMA we established there is no excited-state contamination present in any of our 170-MeV calculations.

Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2701(25)$, measured in neutron $\beta$ decay, decides neutron life.



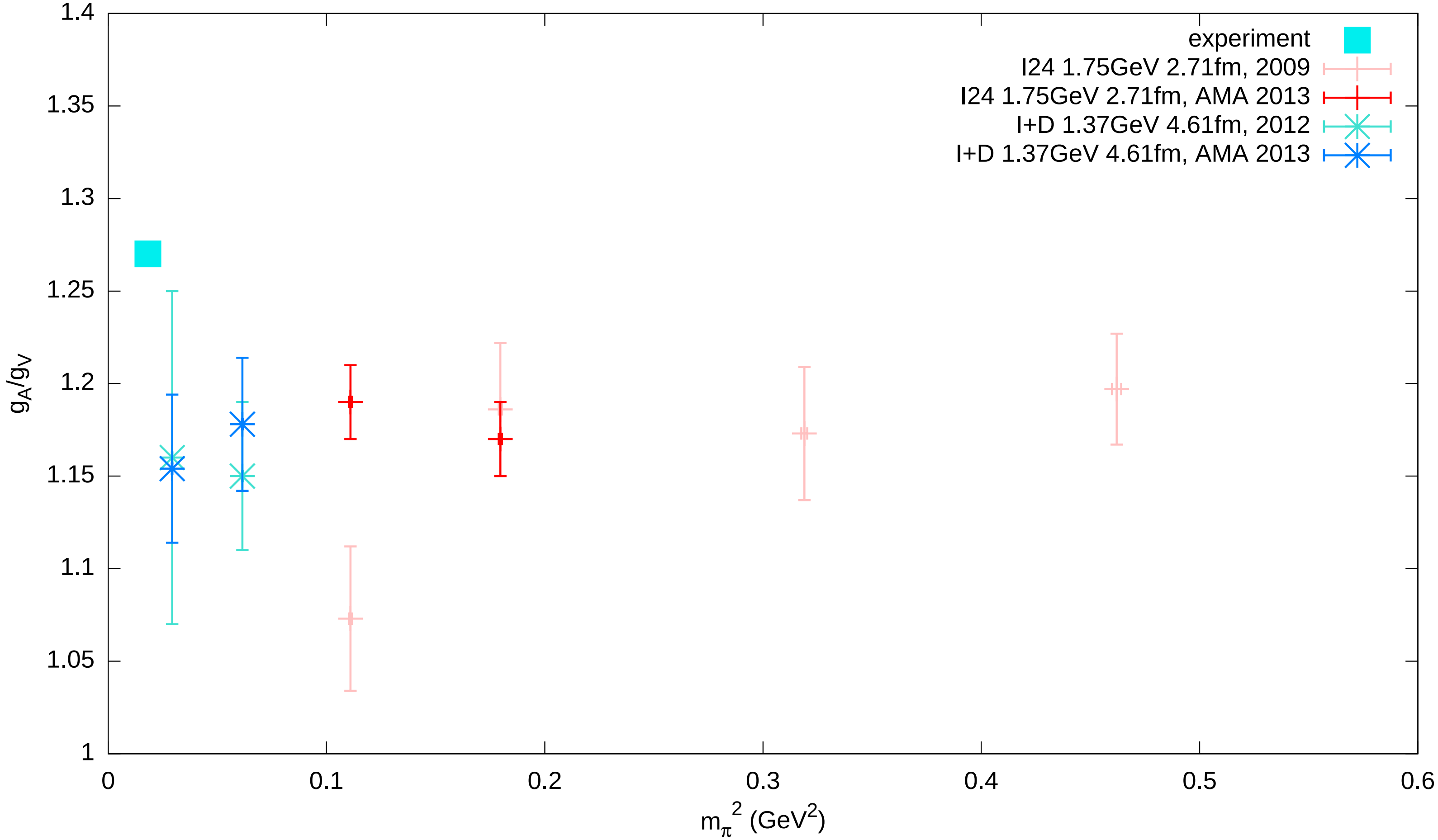

- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

Many in the past pointed out this is a fragile quantity as pion mass is set light: Adkins+Nappi+Witten, Jaffe, Kojo+McLerran+Pisarski, ...

This talk would not be possible without AMA:

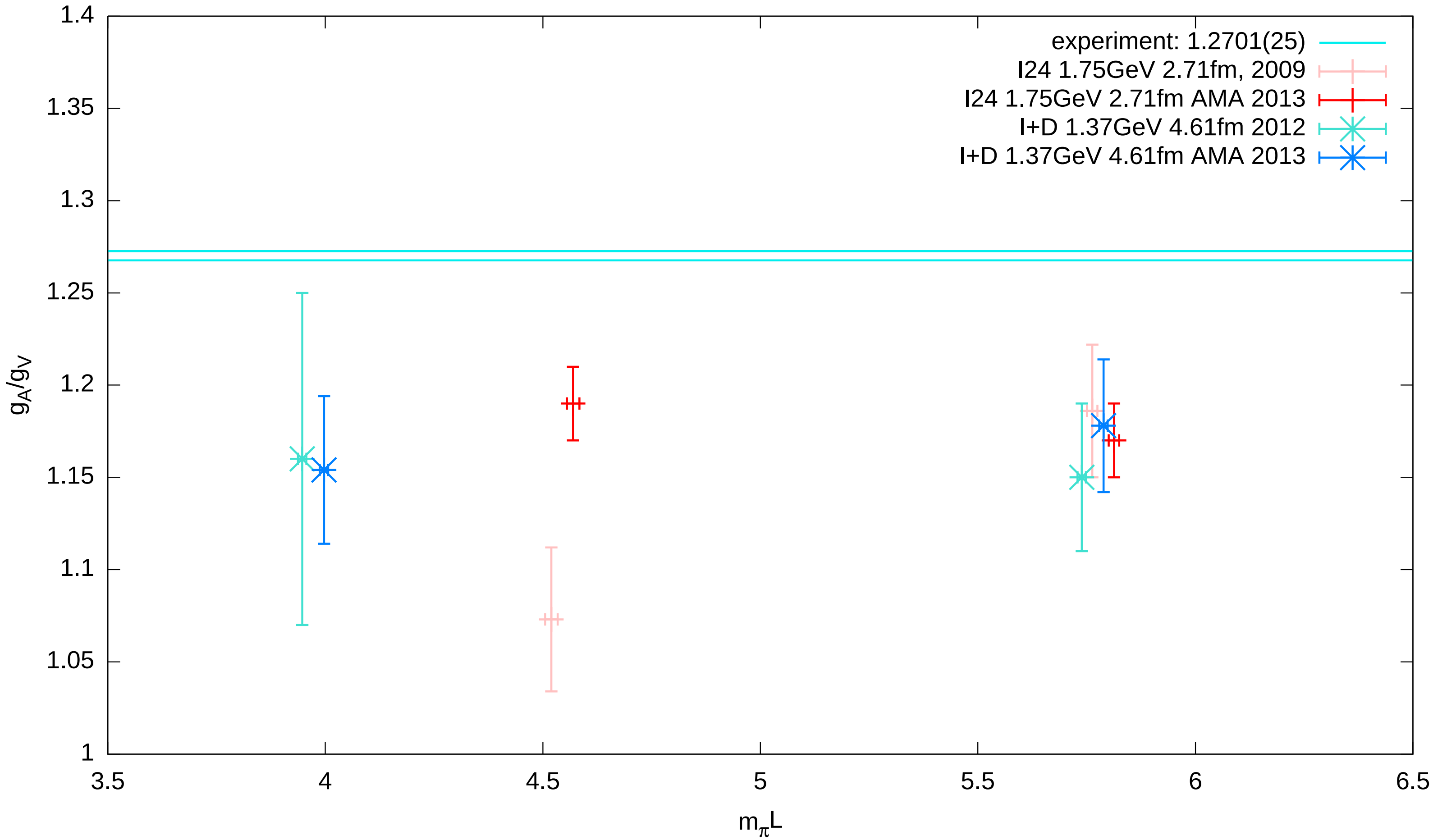
| observable | fit range | non AMA | AMA |
|---|---|---|---|
| $g_V$ | 2-7 | 1.445(14) | 1.449(8) |
| | 3-6 | 1.439(14) | 1.447(8) |
| $g_A$ | 2-7 | 1.8(2) | 1.67(5) |
| | 3-6 | 1.8(2) | 1.66(6) |
| $g_A/g_V$ | 2-7 | 1.26(13) | 1.15(4) |
| | 3-6 | 1.28(15) | 1.15(4) |
| $\langle x \rangle_{u-d}$ | 3-6 | 0.13(2) | 0.146(7) |
| | 4-5 | 0.11(3) | 0.145(8) |
| $\langle x \rangle_{\Delta u-\Delta d}$ | 3-6 | 0.19(4) | 0.165(9) |
| | 4-5 | 0.20(5) | 0.167(10) |
| $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$ | 3-6 | 0.64(13) | 0.86(5) |
| | 4-5 | 0.5(2) | 0.83(6) |
| $\langle 1 \rangle_{\delta u-\delta d}$ | 3-6 | 1.7(2) | 1.42(4) |
| | 4-5 | 1.7(2) | 1.41(5) |

With AMA and other statistical improvements, $g_A/g_V$ vs $m_\pi^2$ then looked like the following:




Moves away from the experiment as $m_\pi$ approaches the experimental value.

About 10-% deficit in $g_A/g_V$ seems solid except perhaps for $O(a^2)$ error:




Excited-state contamination now is unlikely the cause.

Appears like monotonically decreasing with $m_\pi L$.

In agreement with the great majority of other groups.

Why?

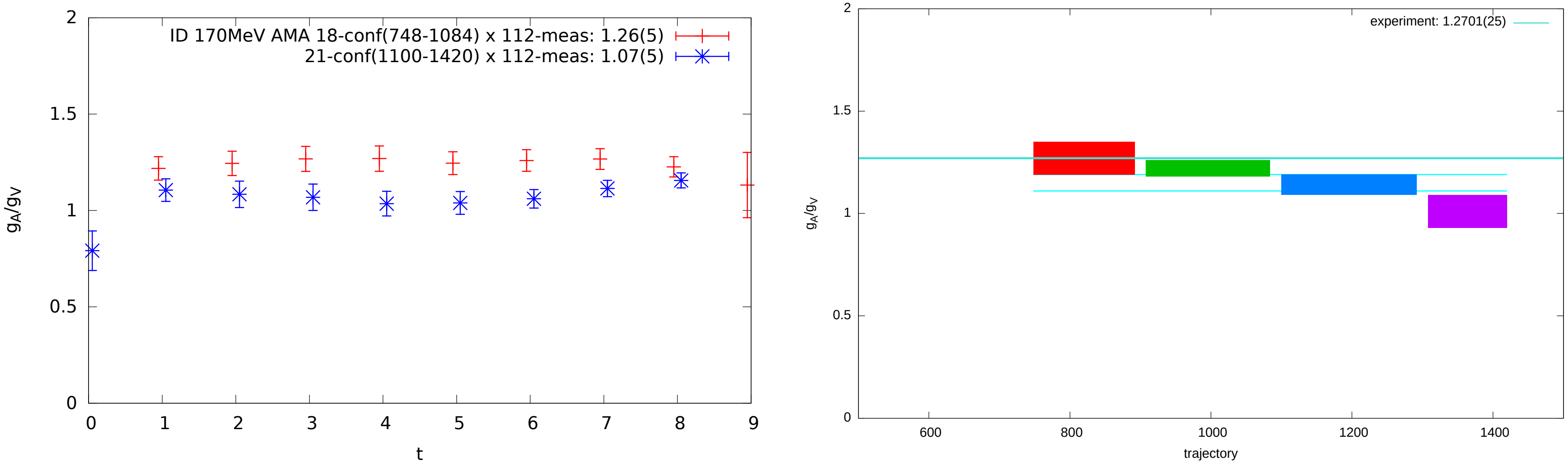
There appear long-range autocorrelations in axial charge but not in others:

Blocked jackknife analysis

| | bin size | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| $g_V$ | 1.447(8) | 1.447(6) | - | - |
| $g_A$ | 1.66(6) | 1.66(7) | 1.71(8) | 1.65(4) |
| $g_A/g_V$ | 1.15(4) | 1.15(5) | 1.15(6) | 1.14(3) |
| $\langle x \rangle_{u-d}$ | 0.146(7) | 0.146(8) | 0.146(8) | - |
| $\langle x \rangle_{\Delta u-\Delta d}$ | 0.165(9) | 0.165(11) | 0.165(10) | - |
| $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$ | 0.86(5) | 0.86(4) | - | - |
| $\langle 1 \rangle_{\delta u-\delta d}$ | 1.42(4) | 1.42(6) | 1.42(6) | 1.41(3) |

except in perhaps transversity.
But the difference may be hard to notice by standard blocked jackknife analysis.

Long-range auto-correlation seen in $g_A/g_V$:



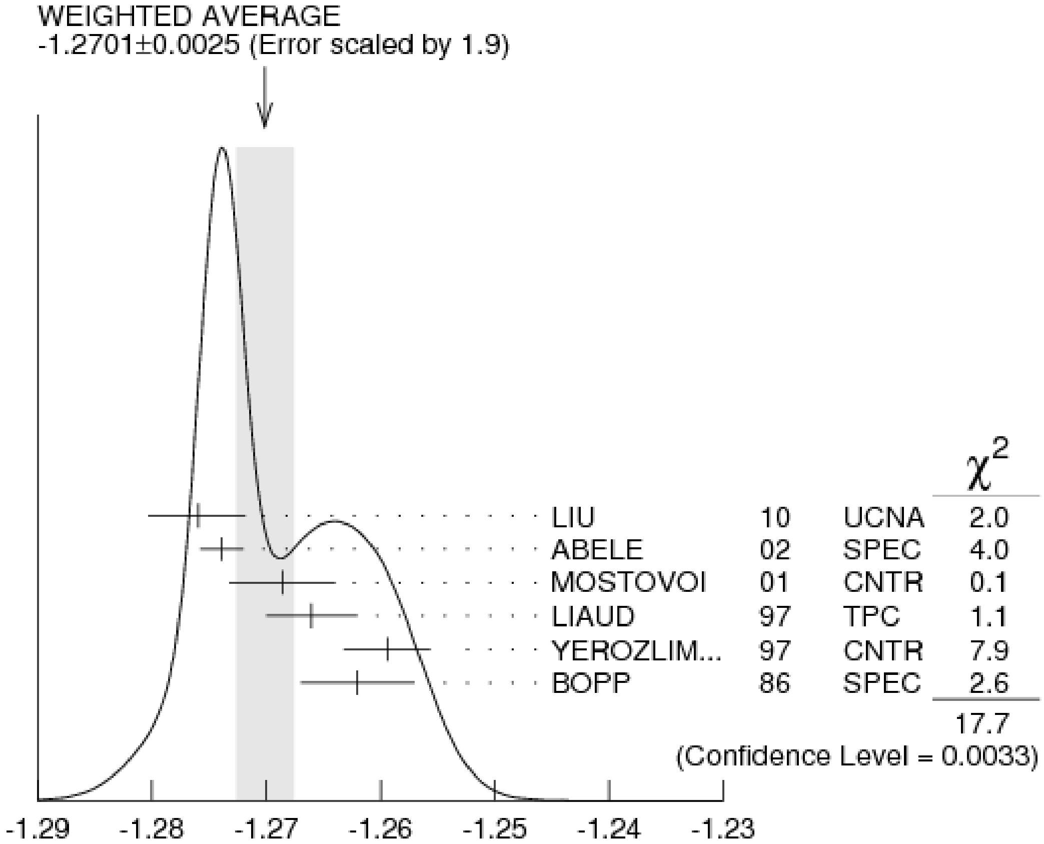

Non-AMA analyses are much noisier but not inconsistent with these:

Indicative of inefficient sampling, but only in $g_A$ and $g_A/g_V$.

Why?

## Why?

Difficult history:




Experimental value has been almost monotonically increasing since Maurice Goldhaber's first measurement.

Lattice calculations appeared to follow the same path.

Why?

Difficult history:

Non-relativistic quark model: 5/3. Very bad, but some "large-$N_c$" conform?
And with absurd "relativistic" correction: 5/4, really?

Without pion,
MIT bag model: 1.09, as good(!) as lattice but when experiment was 1.22.[1]

With only pion,
Skyrmion: 0.61(!) with a peculiar geometry but when experiment was 1.23.

Accurate reproduction of the 'pion cloud' geometry seems essential.

[1] Assuming a growth rate of 0.001 per year.

## RBC+UKQCD nucleon before Lattice 2014:

Systematics are explored in nucleon isovector observables using 2+1f dynamical DWF ensembles,

- lattice cutoff $\sim$ 1.4 GeV, $(4.6\text{fm})^3$ spatial volume,
- good chiral and flavor symmetries up to $O(a^2)$, $m_{\text{res}}a \sim 0.002$,
- $m_\pi \sim 170$ and 250 MeV, $m_N \sim 0.98$ and 1.05 GeV,

jointly generated by RBC and UKQCD Collaborations.

Serious systematics in the axial charge, about 10-% deficit in $g_A/g_V$, with long-range autocorrelation,

- but does not appear correlated with topological charge;
- appears unevenly distributed in space some of the MD time;
- does not appear affected by low-mode deflation issues.

No such serious systematics is seen in other observables,

- except perhaps in transversity, where it is at most shorter-range and milder.

If indeed accurate reproduction of the 'pion cloud' geometry is essential,
larger-volume study with high statistics is desired.

**RBC+UKQCD physical mass ensembles**

As described in 1411.7017: two Iwasaki/DWF 2+1-falvor ensembles,

- $\beta = 2.13$, $a^{-1} = 1.730(4)$ GeV, $48^3 \times 96$, $m_\pi = 139.2(4)$ MeV, $m_\pi L = 3.86$,
- $\beta = 2.25$, $a^{-1} = 2.359(7)$ GeV, $64^3 \times 128$, $m_\pi = 139.2(5)$ MeV, $m_\pi L = 3.78$.

Continuum extrapolations:

- $f_\pi = 130.2(9)$ MeV, $f_K = 155.5(8)$ MeV,
- $m_{ud}(\overline{\mathrm{MS}}, 3\mathrm{GeV}) = 3.00(5)$ MeV, $m_s(\overline{\mathrm{MS}}, 3\mathrm{GeV}) = 81.6.(1.2)$ MeV,
- $B_K$(RGI=0.750(15), $B_K(\overline{\mathrm{MS}}, 3\mathrm{GeV}) = 0.530(11)$.

Low mass and numerically large volume are challenging:

- AMA is necessary, but
- only insufficient deflation can be achieved.

Or we suffer numerous computer glitches from humongous memory demand.

**LHP+RBC nuleon:** Use the RBC+UKQCD 48I ensemble.

Source-sink separations of 8, 9, 10 and 12 lattice spacings are used.

Sergey Syritsyn optimized our measurements:

- One AMA sampling every 80 MD units,
- one accurate and 32 sloppy samples per configuration,
- sloppy samples are taken with 400 CG iterations,
- with 500 lowest eigenmodes.

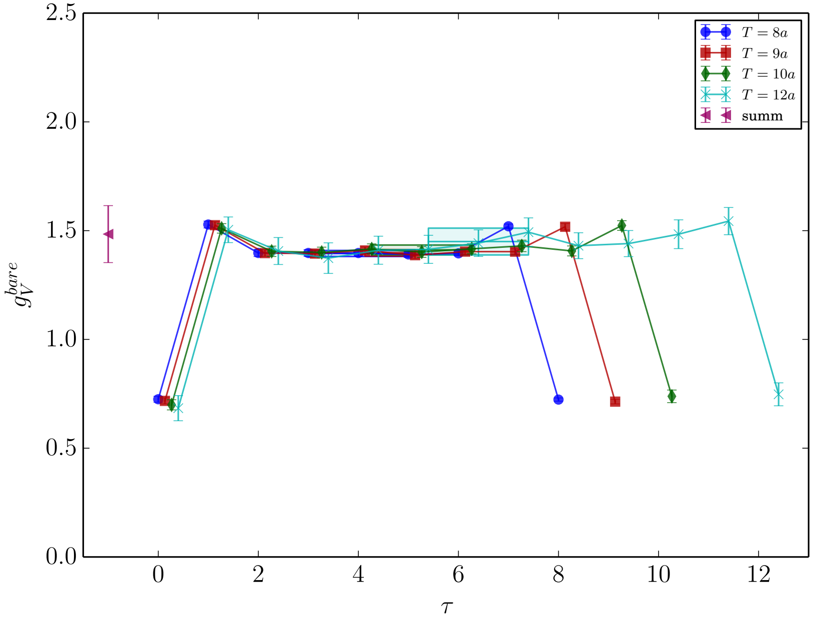
In Lattice 2014 Sergey reported results from the first 20 configurations.

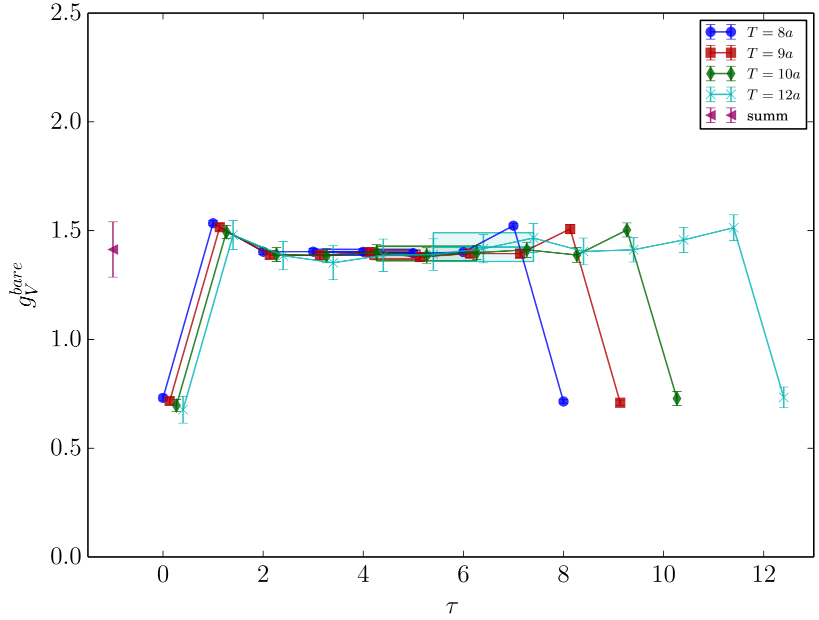
At least four time more statistics was called for, so we planned accordingly. However

- we found an error in boundary condition for some AMA sampling,
- that contaminated ten percent of our statistics.

This has been corrected, with only less than 1-% difference.

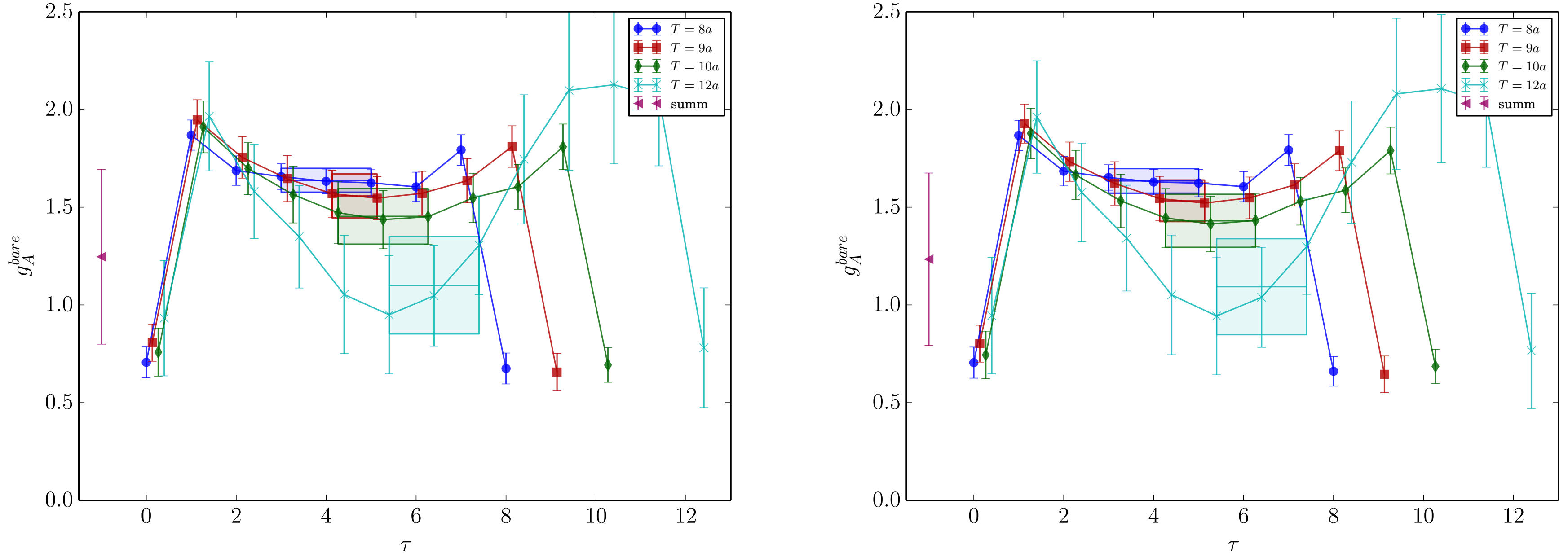
Isovector vector charge $g_V$:



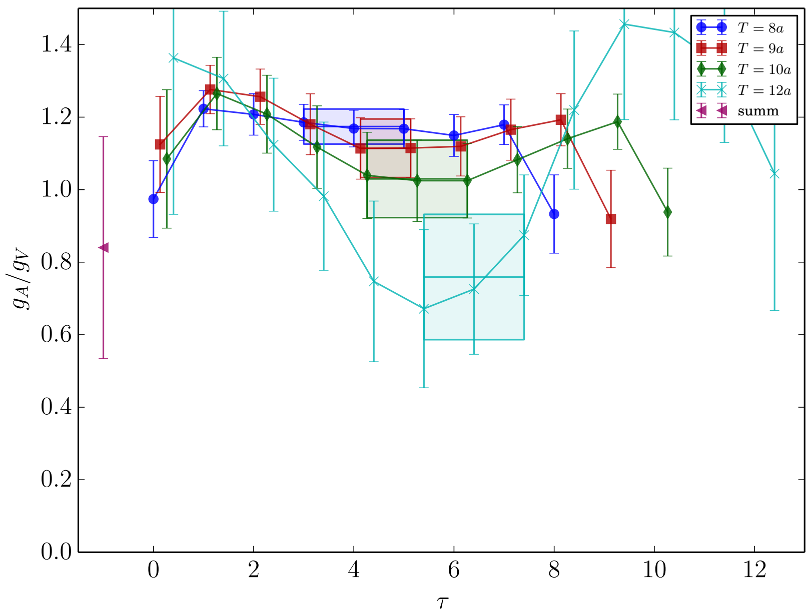

Boundary-condition error correction results in only less than 1-% deifference.

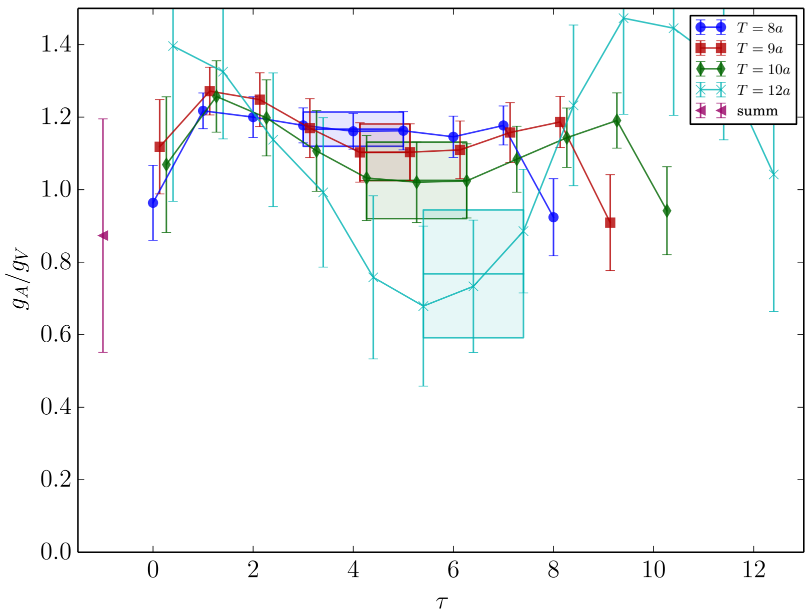
Isovector axial charge $g_A$:




Boundary-condition error correction results in only less than 1-% deifference.

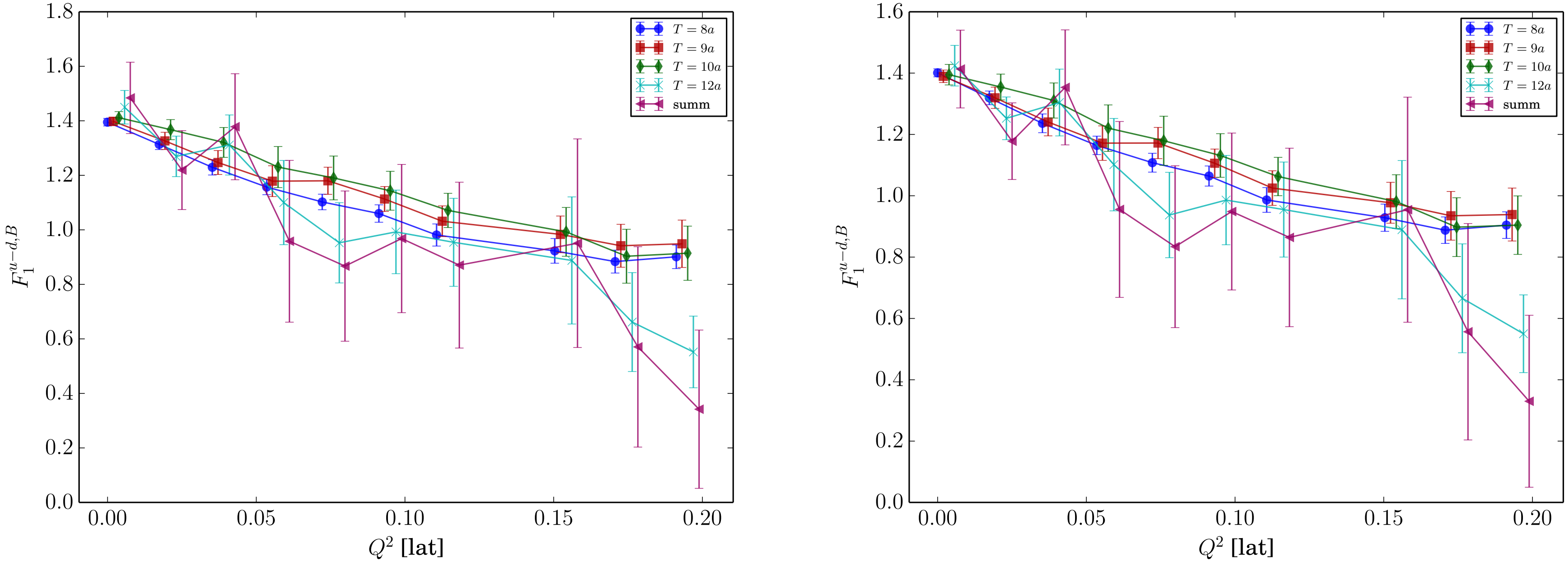
Raio of isovector axial to vector charges $g_A/g_V$:



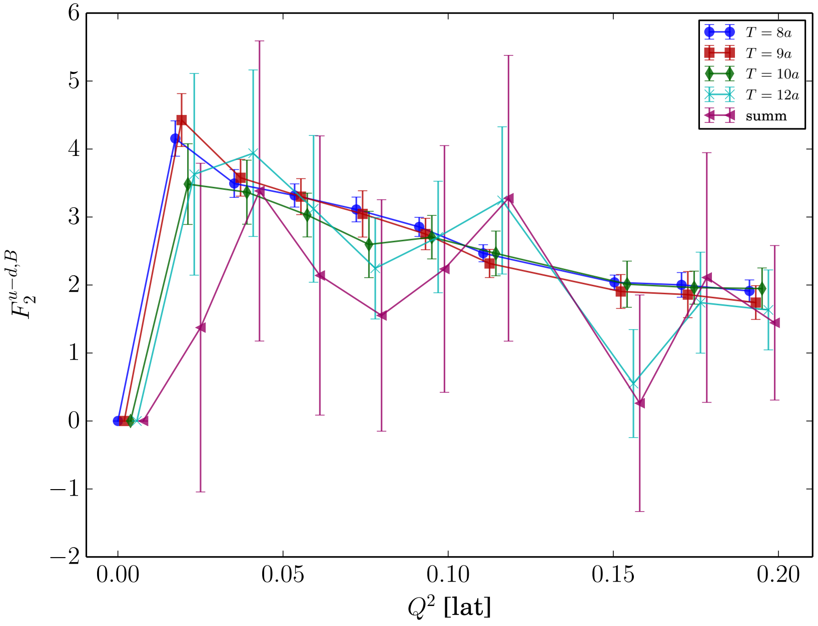

Boundary-condition error correction results in only less than 1-% deifference.

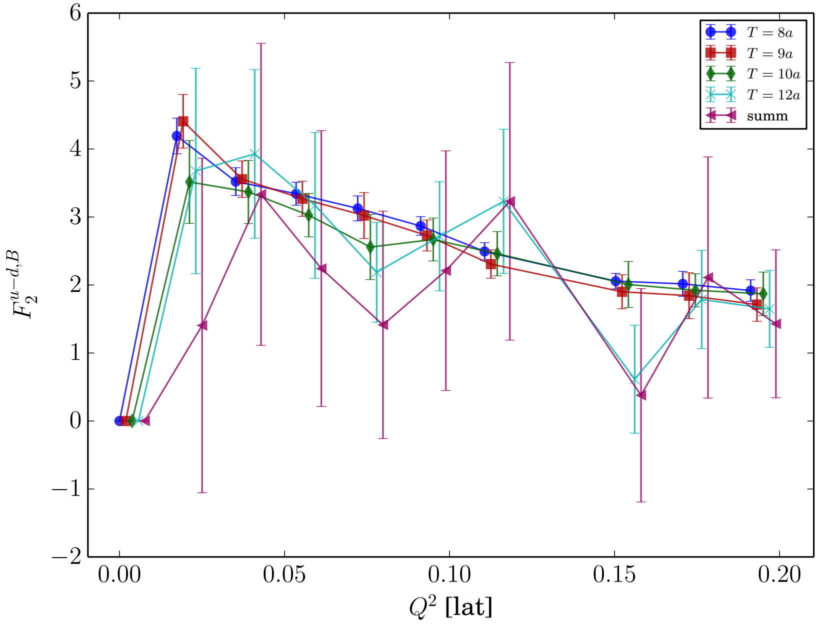
Dirac form factor $F_1$:




Boundary-condition error correction results in only less than 1-% deifference.

Pauli form factor $F_2$:

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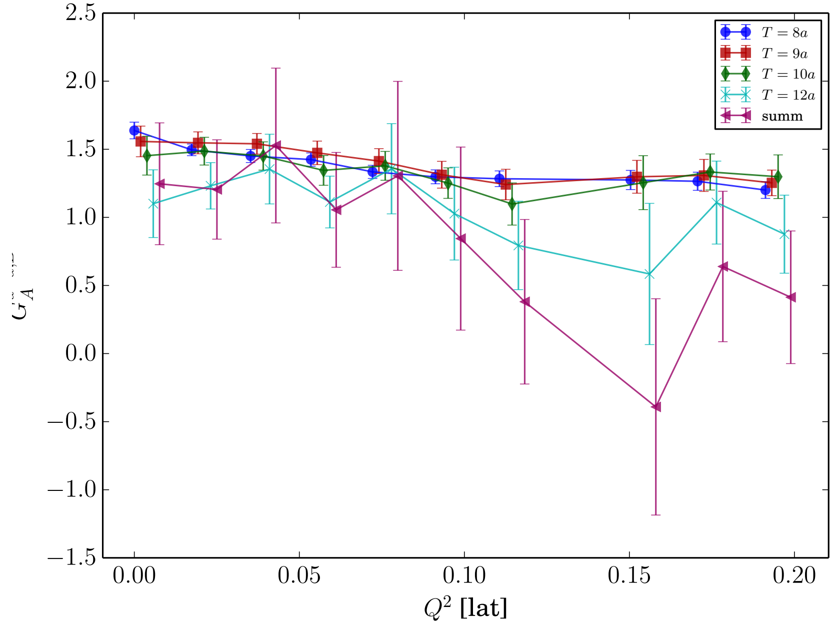
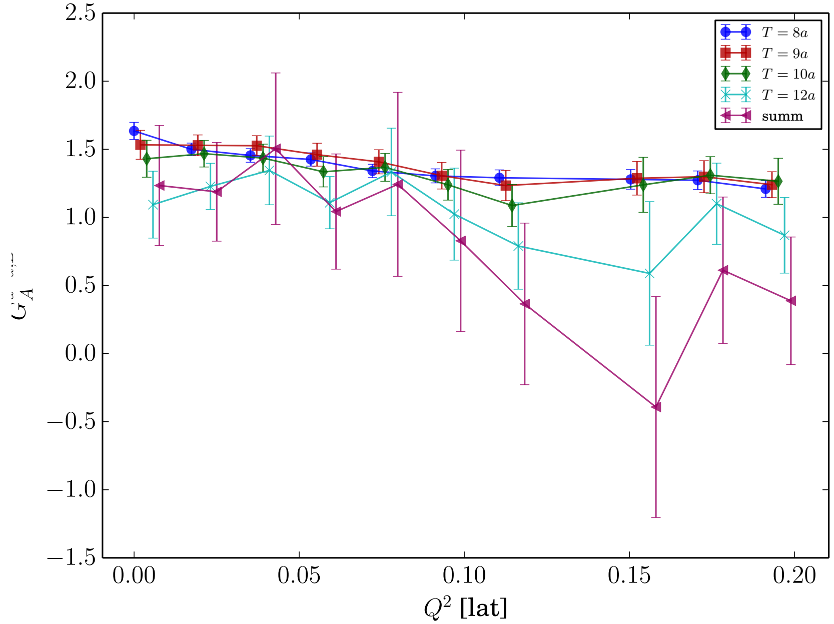
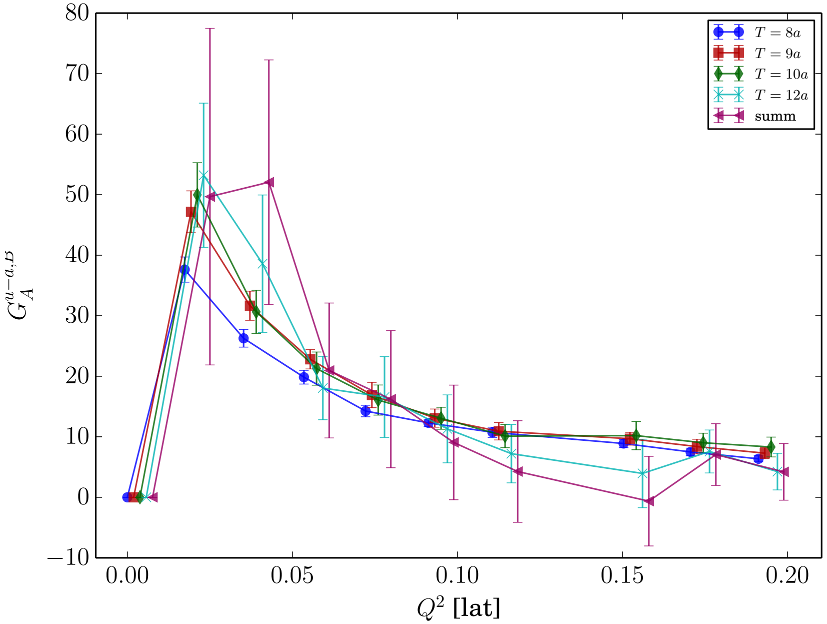
Axialvector form factor $G_A$:

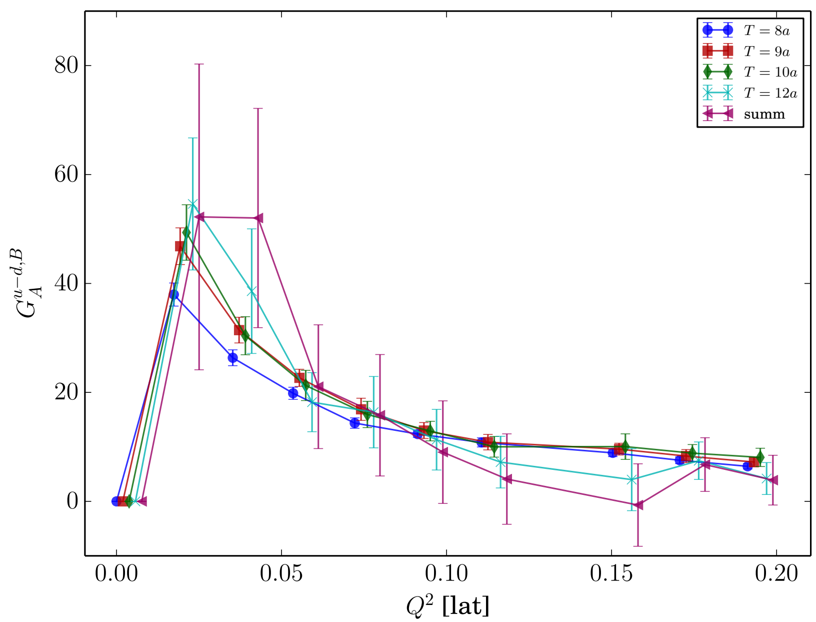



Boundary-condition error correction results in only less than 1-% deifference.

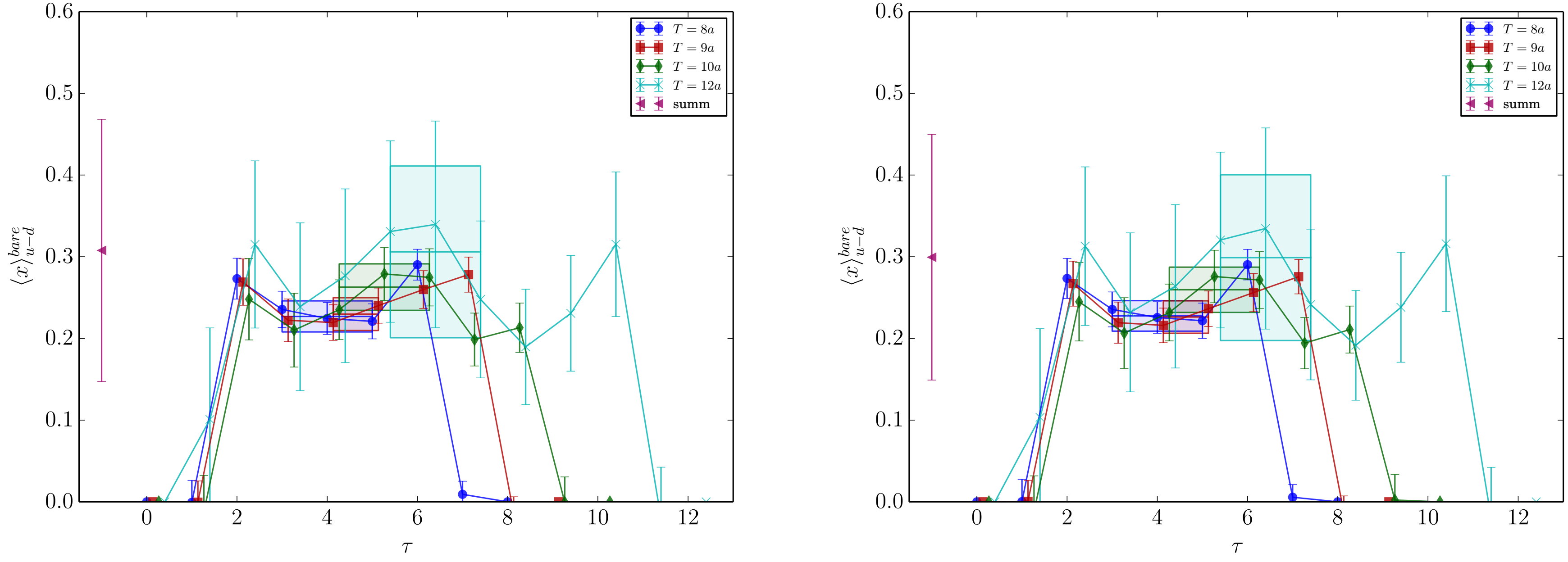
Pseudoscalar form factor $G_P$:




Boundary-condition error correction results in only less than 1-% deifference.

Quark momentum fraction $\langle x \rangle_{u-d}$:




Boundary-condition error correction results in only less than 1-% deifference.

## Summary

LHP and RBC continue to work on nucleon structure using RBC+UKQCD physical mass DWF ensembles.

We made less-than-1-% corrections to our initial report.

We are back in order and ready to increase our statistics.

However our deflation memory demand is so large that not many computers can accommodate it.

All such computers are oversubscribed. Yet we should double our statistics this year.