Proton spin decomposition with overlap fermion

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Proton Spin decomposition

A physical gauge decomposition

R. L. Jaffe and A. V. Manohar, Nucl. Phys. B 337, 509 (1990).
X. –S. Chen et al., Phys. Rev. Lett. 100, 232002 (2008).
X. –S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

quark spin

quark OAM

$$\vec{J} = \underbrace{\begin{matrix} \frac{1}{2}\vec{\Sigma}_q \\ \int d^3x \, \frac{1}{2} \, \overline{\psi} \, \vec{\gamma} \, \gamma^5 \, \psi \end{matrix}}_{\text{quark spin}} + \underbrace{\begin{matrix} \lim_{P^z \to \infty} \langle L_q^z \rangle = \epsilon^{ij} \lim_{\Delta \to 0} \frac{\partial}{i\partial \Delta_T^i} \int dx d^2k_T \; k_T^j f(x, \vec{k}_T, \vec{\Delta}_T) \\ \int d^3x \psi^\dagger \left\{ \vec{x} \times (i\vec{D}^{pure}) \right\} \psi \end{matrix}}_{\text{quark OAM}}$$

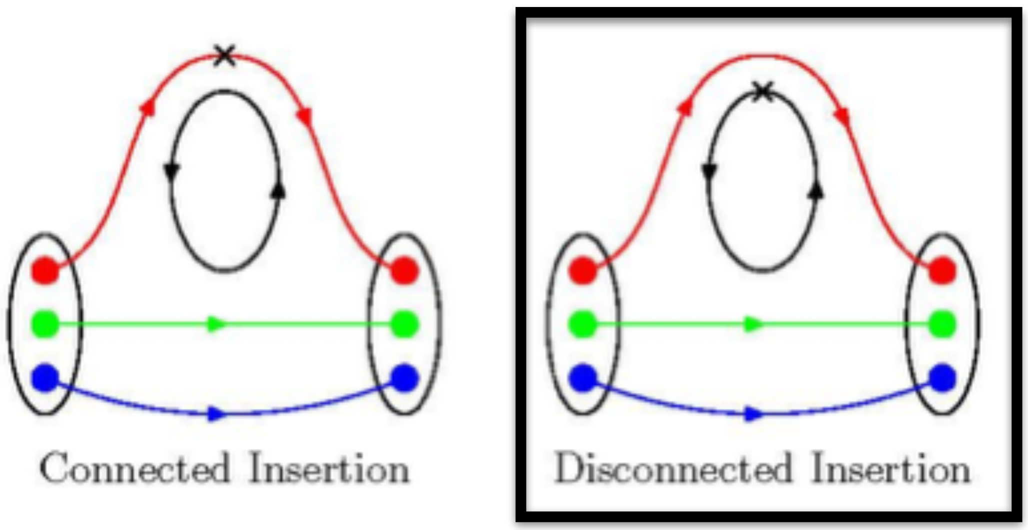
$$+ \underbrace{\begin{matrix} \int d^3x \mathrm{Tr}[\vec{E} \times \vec{A}^{phys}] \\ \lim_{P^z \to \infty} S_g^z = \Delta_G. \end{matrix}}_{\text{glue spin}} + \underbrace{\begin{matrix} \int d^3x \mathrm{Tr}[E^i \vec{x} \times \vec{D}^{pure} A^{i,phys}] \\ \lim_{P^z \to \infty} \langle L_g^z \rangle = \epsilon^{ij} \lim_{\Delta \to 0} \frac{\partial}{i\partial \Delta_T^i} \int dx d^2k_T \; k_T^j g(x, \vec{k}_T, \vec{\Delta}_T) \end{matrix}}_{\text{glue OAM}}$$

glue spin

glue OAM

Quark spin

Anomalous Ward identity



$$\bar{\psi}\partial_\mu \gamma^\mu \gamma_5 \psi = 2m_q \bar{\psi}\gamma_5\psi + \frac{1}{8\pi^2} F\tilde{F}.$$

Y.B. Yang et al., χQCD collaboration
arXiv:1504.04052

The relation between the related form factors is:

$$\frac{q_j}{2E_q}(2m_q g_P(Q^2) + g_{F\tilde{F}}(Q^2)) = \frac{q_j}{2E_q}(2m g_A^R(Q^2) - Q^2 h_A^R(Q^2)).$$

Due to AWI, the right hand of the equation are renormalization free. In order to obtain g_A, we need the information of h_A except

1. h_A is very small. The direct calculation of g_A^b/h_A^b yields very small value (O(0.001)) for the charm loop, while the contribution of $2m_q g_P$ and $g_{F\tilde{F}}$ are O(0.2).

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2. q^2 is very small. Then the contribution of h_A would negligible. A large lattice is required.

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Quark spin

Lattice setup

- Overlap valence quark on 2+1 Domain wall fermion configuration.
- 48^3x96, a=0.112(3) fm, L~5.4fm, m_π^{sea}=140MeV.
- 45 configurations with 1000 pairs low lying eigenvectors.

- 2pt: 12-12-12-32 diluted smear source, with the low mode substitution.
- loop: 4-4-4-2 e/o diluted loop with 16 Z_3 noises and the low mode average.

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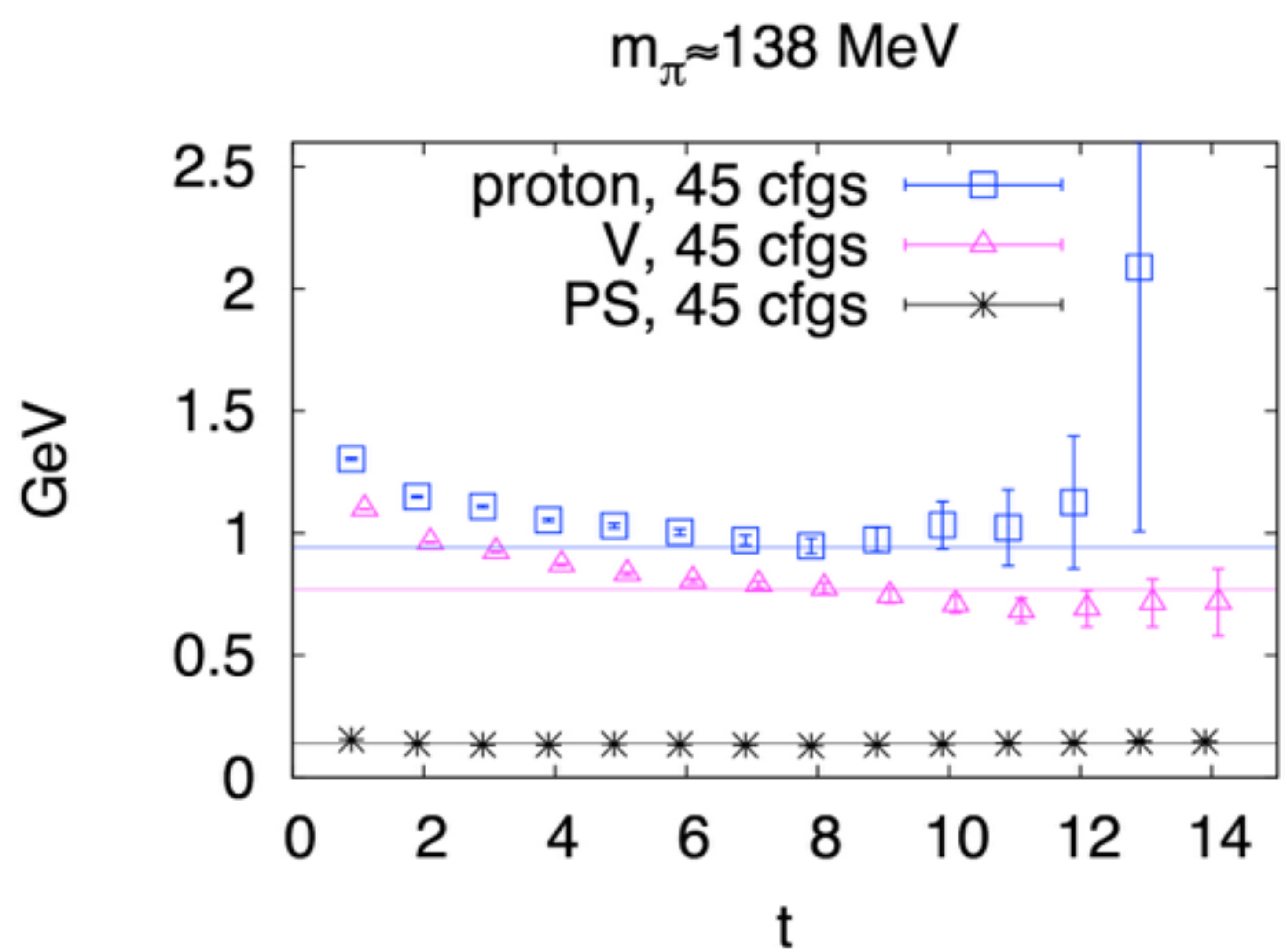


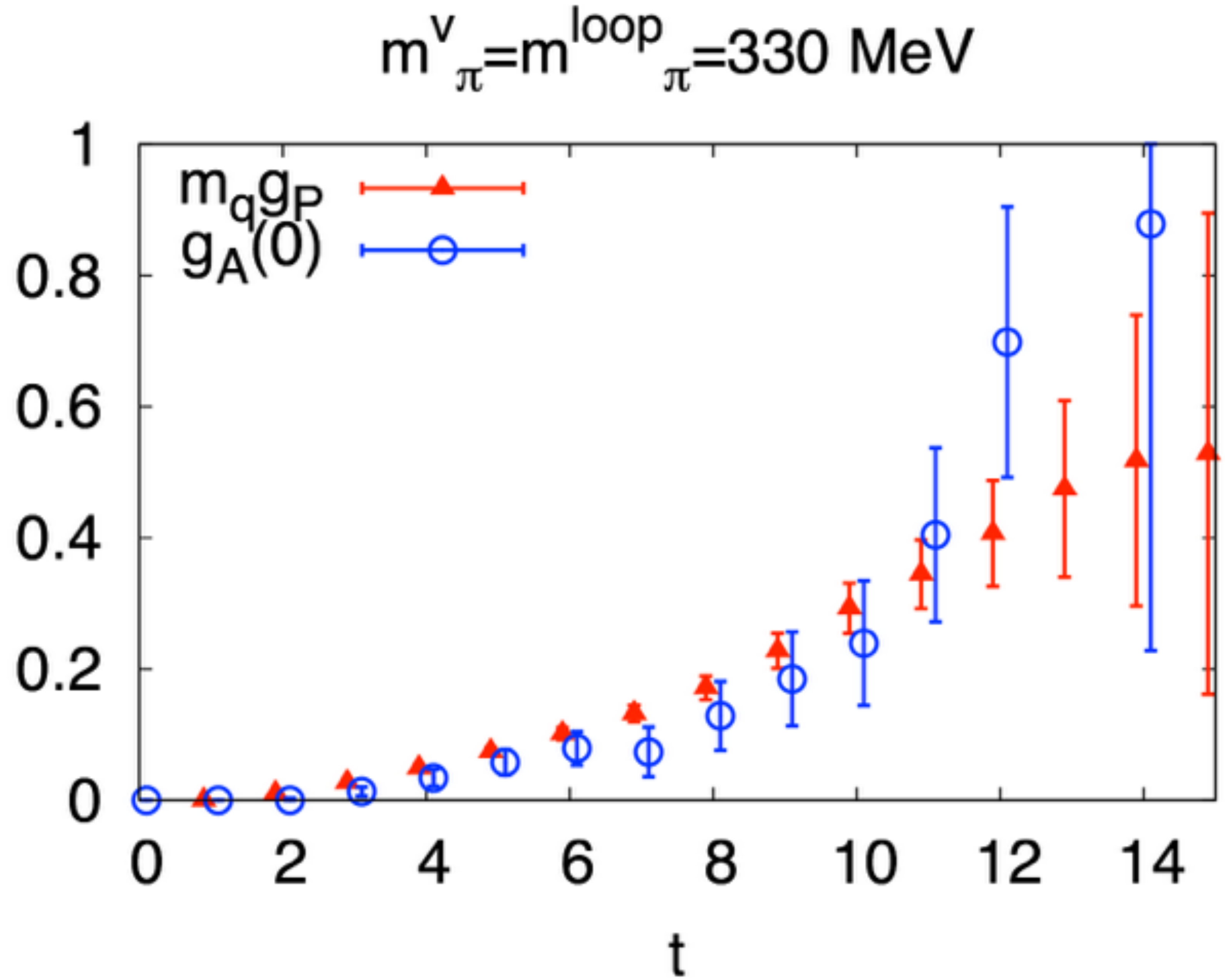
Quark spin

The signal of DI

- The rescaled plot to show the signal.

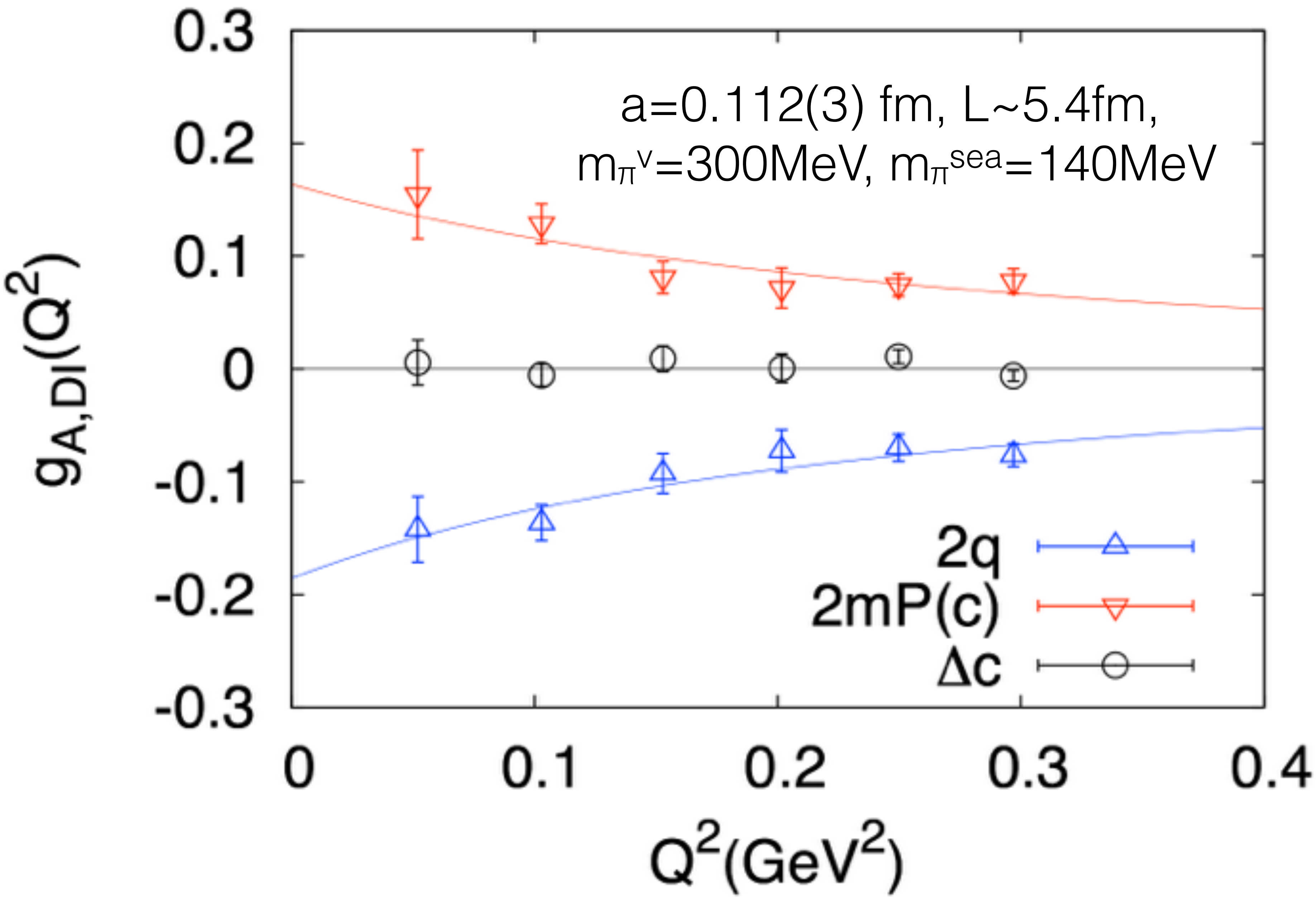
- With the same statistics:
- The axial case is noisy.
- The PS case is much better due to the low mode average.

- The signal to noise ratio is another reason to use AWI to obtain the quark spin.



Quark spin

The heavy quark case

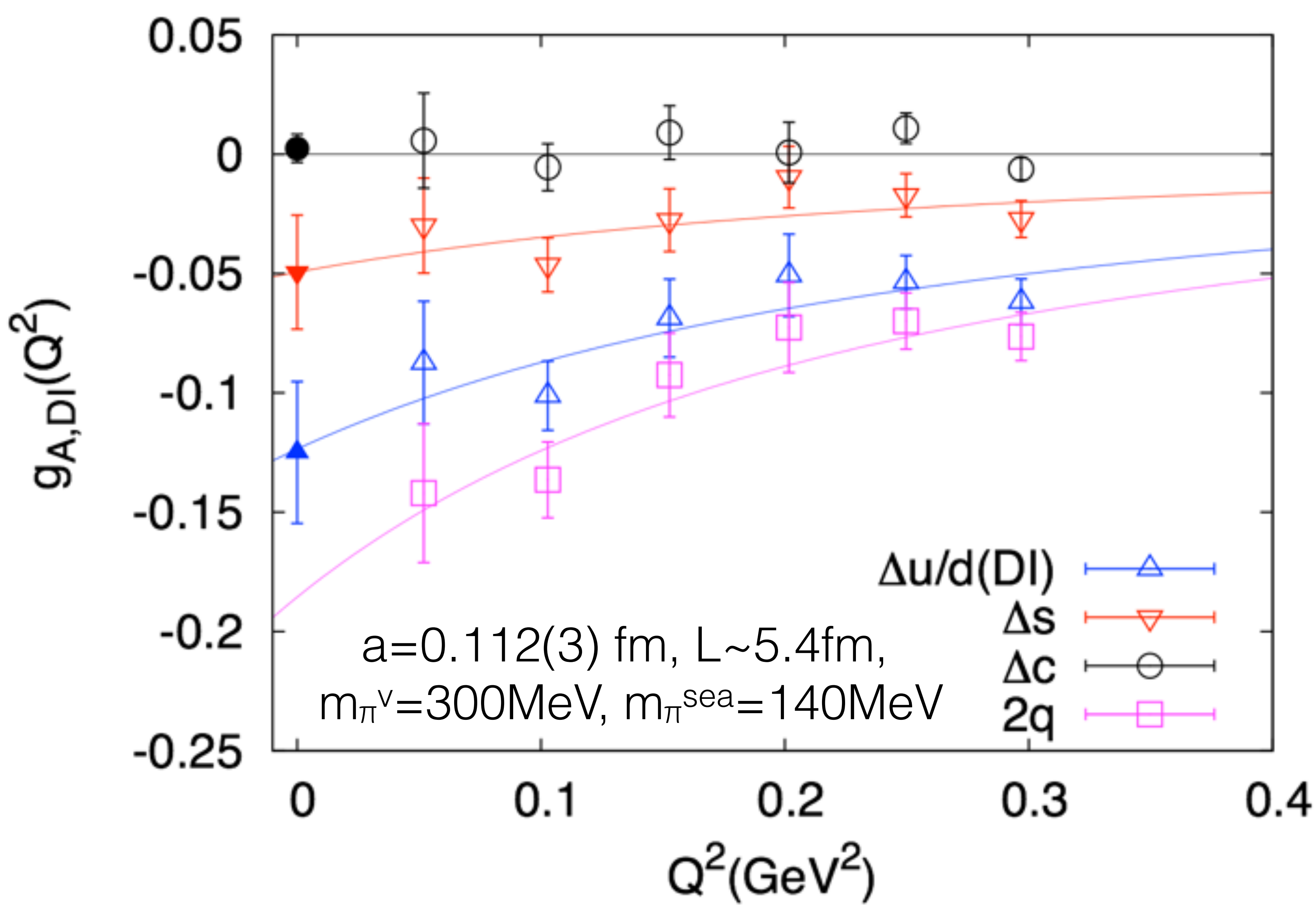


Obvious cancelation between the 2mP term and the topocharge term, the final heavy quark contribution consistent to zero.

The case with lighter valence quark is still noisy.

Quark spin

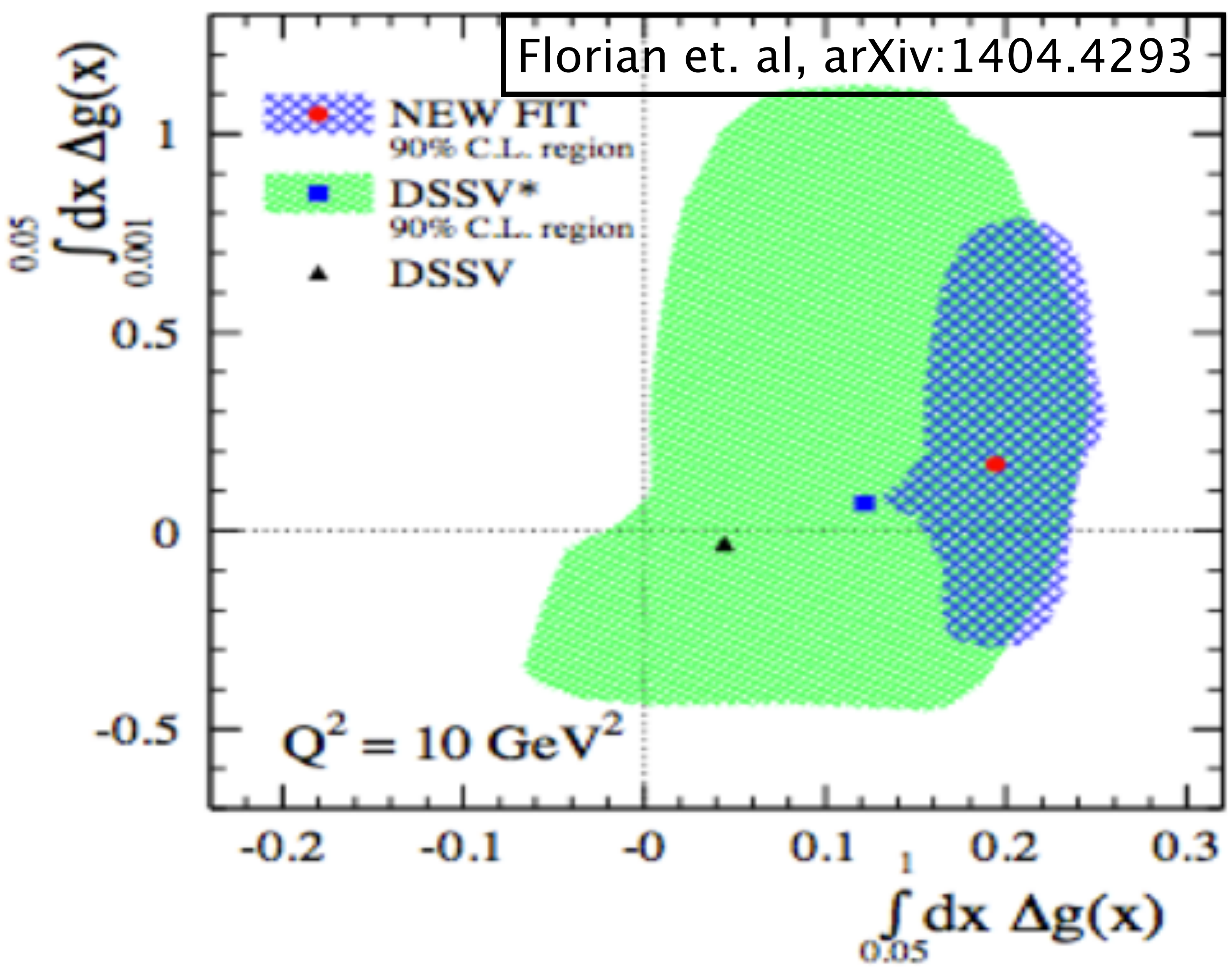
The light/strange quark case



When the quark mass becomes lighter, the contribution from the 2mP term becomes smaller and final contribution is negative due to the topocharge term.

Glue spin

The glue helicity



The global fit based on recent experimental data (2009 RHIC) shows evidence of nonzero polarization of gluon in the proton.

Carrying out integration of longitudinal momentum reduces to gauge invariant gluon spin operator

$$S_g(p^2) = \mathrm{Tr}[\vec{E}(p^2) \times \vec{A^{\perp}(p^2)}]$$

In the infinite momentum frame, it corresponds to the gluon helicity in the plot.

$$S_g(p^2) \xrightarrow[p^2 \to \infty]{} \Delta_G$$

X. Ji et al.,Phys.Rev.Lett. 111, 112002 (2013)

Glue spin

What is A_{phys}?

Let us begin with the conditions of the decomposition need to satisfy:

$$A_\mu \equiv A_\mu^{phys} + A_\mu^{pure},$$

with the gauge transformation laws,

$$A_\mu^{phys} \rightarrow A_\mu^{\prime phys} \equiv g A_\mu^{phys} g^{-1},$$
$$A_\mu^{pure} \rightarrow A_\mu^{\prime pure} \equiv g A_\mu^{pure} g^{-1} + \frac{i}{g_0} g \partial_\mu g^{-1},$$

X. -S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

In addition, $A^\mu_{\rm pure}$ is constrained by a null-field-strength condition,

$$F_{\mu\nu}^{pure} \equiv \partial_\mu A_\nu^{pure} - \partial_\nu A_\mu^{pure} - i g_0 [A_\mu^{pure}, A_\nu^{pure}] = 0.$$

To find a solution, it is suggested that $A^\mu_{\rm phys}$ satisfies the non-Abelian Coulomb condition,

$$\mathcal{D}_i A_i^{phys} \equiv \partial_i A_i^{phys} - i g_0 [A_i^{pure}, A_i^{phys}] = 0,$$

Then in the infinite momentum limit, Chen's decomposition condition $\mathcal{D}_i A_i^{phys} \equiv \partial_i A_i^{phys} - i g_0 [A_i^{pure}, A_i^{phys}] = 0$ becomes to the light-cone decomposition condition $A_+^{phys} = 0$, and then the gluon spin operator corresponds to the glue helicity in this limit,

$$\lim_{P^z \to \infty} S_g^z = \Delta_G.$$

X. Ji, J. H. Zhang and Y. Zhao Phys.Rev.Lett. 111, 112002 (2013)

Glue spin

How to obtain A_{phys} on the lattice?

$$A_\mu \equiv A_\mu^{phys} + A_\mu^{pure},$$

$$A_\mu^{phys} \to A_\mu'^{phys} \equiv g A_\mu^{phys} g^{-1}, \qquad A_\mu^{pure} \to A_\mu'^{pure} \equiv g A_\mu^{pure} g^{-1} + \frac{i}{g_0} g \partial_\mu g^{-1},$$

$$\mathcal{D}_i A_i^{phys} \equiv \partial_i A_i^{phys} - i g_0 [A_i^{pure}, A_i^{phys}] = 0, \quad F_{\mu\nu}^{pure} \equiv \partial_\mu A_\nu^{pure} - \partial_\nu A_\mu^{pure} - i g_0 [A_\mu^{pure}, A_\nu^{pure}] = 0.$$

One can starts from the gauge link $U^\mu(x) = \exp(-i a g_0 A^\mu(x))$ that connects x to $x + a\hat{\mu}$. Under a gauge transformation $g(x)$,

$$U^\mu(x) \to U'^\mu(x) = g(x) U^\mu(x) g(x + a\hat{\mu})^{-1} .$$

By finding a gauge transformation g_c that makes

$$U^\mu(x) = g_c(x) U_c^\mu(x) g_c(x + a\hat{\mu})^{-1}$$

where $U_c^\mu(x)$ is fixed in the Coulomb gauge, one can define a new gauge link $U_{\rm pure}^\mu$ and obtain the solution for $A_{\rm phys}^\mu$,

$$U_{\rm pure}^\mu \equiv g_c(x) g_c(x + a\hat{\mu})^{-1},$$

$$A_{\rm phys}^\mu \equiv \frac{i}{a g_0} \left(U^\mu(x) - U_{\rm pure}^\mu(x)\right) = \frac{i}{a g_0} g_c(x) (U_c^\mu(x) - 1) g_c(x)^{-1} + O(a) = g_c(x) A_c^\mu(x) g_c(x)^{-1} + O(a) .$$

Glue spin

The glue spin operator E×A_{phys}

One can check that $A^\mu_{\rm phys}$ so defined satisfies the gauge transformation law with U^μ_c being unchanged and g_c transforming as $g'_c = g g_c$,

$$\begin{aligned} U'^\mu(x) &= g'_c(x) U^\mu_c(x) g'_c(x+a\hat{\mu})^{-1} = g(x) U^\mu(x) g(x+a\hat{\mu})^{-1}, \\ A'^\mu_{\rm phys} &= g'_c(x) A_c(x) g'_c(x)^{-1} + O(a) = g(x) A^\mu_{\rm phys}(x) g(x)^{-1} + O(a). \end{aligned}$$

A short calculation confirms that the decomposition conditions are also satisfied up to $O(a)$ corrections which vanish in the continuum limit,

$$\begin{aligned} F^{\mu\nu}_{\rm pure}(x) &= \frac{i}{a^2 g_0}\left(U^\mu_{\rm pure}(x) U^\nu_{\rm pure}(x+a\hat{\mu}) U^{\dagger\mu}_{\rm pure}(x+a\hat{\nu}) U^{\dagger\nu}_{\rm pure}(x) - 1\right) + O(a) &= O(a)\ , \\ \mathcal{D}^i A^i_{\rm phys}(x) &= \frac{i}{a^2 g_0} g_c(x)\Big(U^i_c(x) - U^i_c(x-a\hat{i})\Big) g_c^{-1}(x) + O(a) &= O(a)\ . \end{aligned}$$

Then the glue operator $\mathrm{Tr}[E\times A_{\rm phys}]$ could be rewritten into

$$\mathrm{Tr}[E\times A_{\rm phys}] = \mathrm{Tr}[E_c \times A_c].$$

Yong Zhao, Keh-Fei Liu, Yibo Yang, arXiv:1506.08832

Glue Spin

Lattice setup

- Overlap valence quark on 2+1 Domain wall fermion configuration.

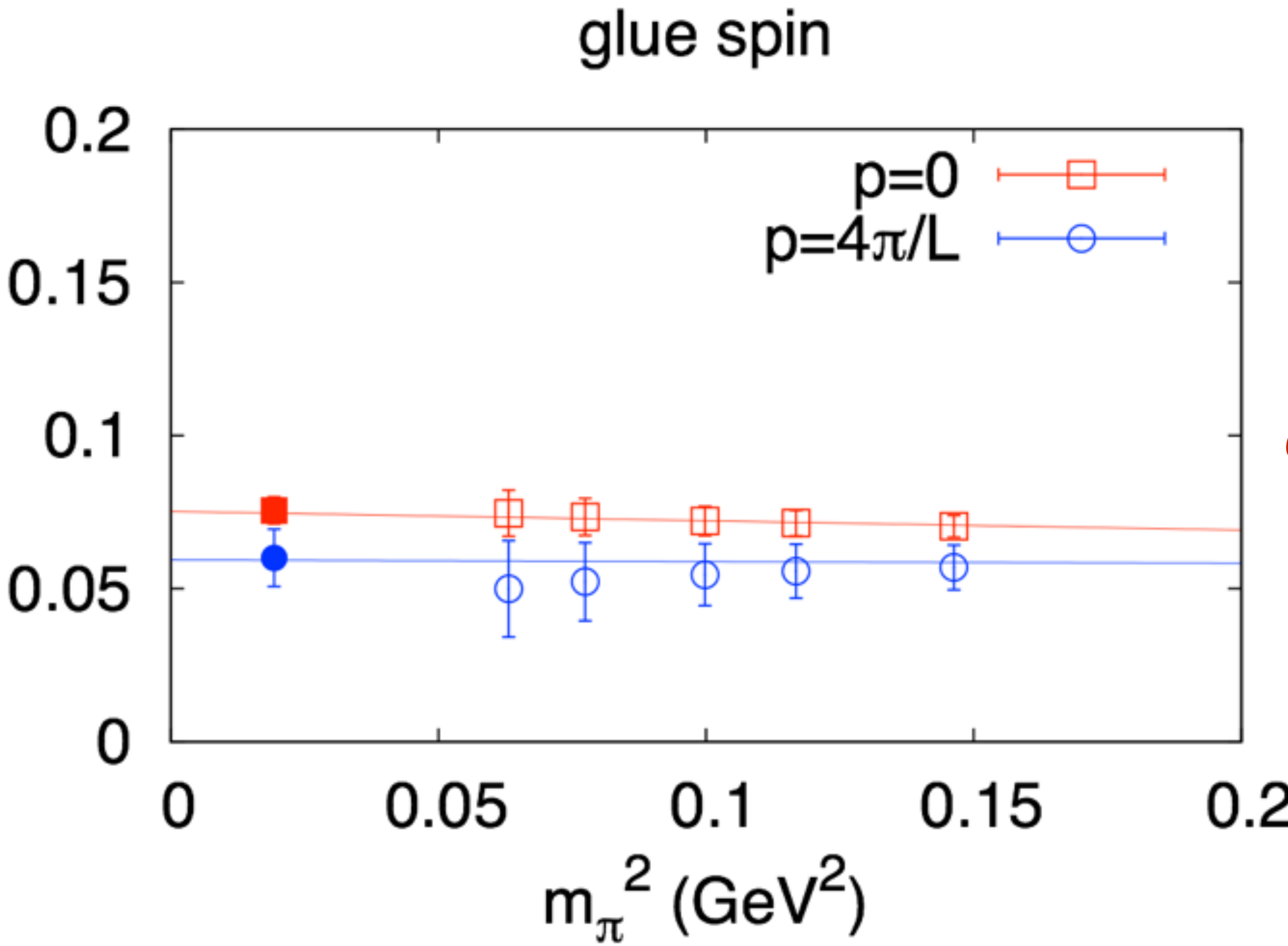
- 24^3x64, a=0.112(3) fm, L~2.7fm, m_π^{sea}=330MeV. 203 configurations with 200 pairs low lying eigenvectors.
- 32^3x64, a=0.084(2) fm, L~2.7fm, m_π^{sea}=300MeV. 304 configurations with 300 pairs low lying eigenvectors.

- 2pt: L/2-L/2-L/2-T/2 diluted smear source, loop over t, with the low mode substitution.
- glue: clover operator based on HYP smeared configuration.

Glue spin

The chiral extrapolation

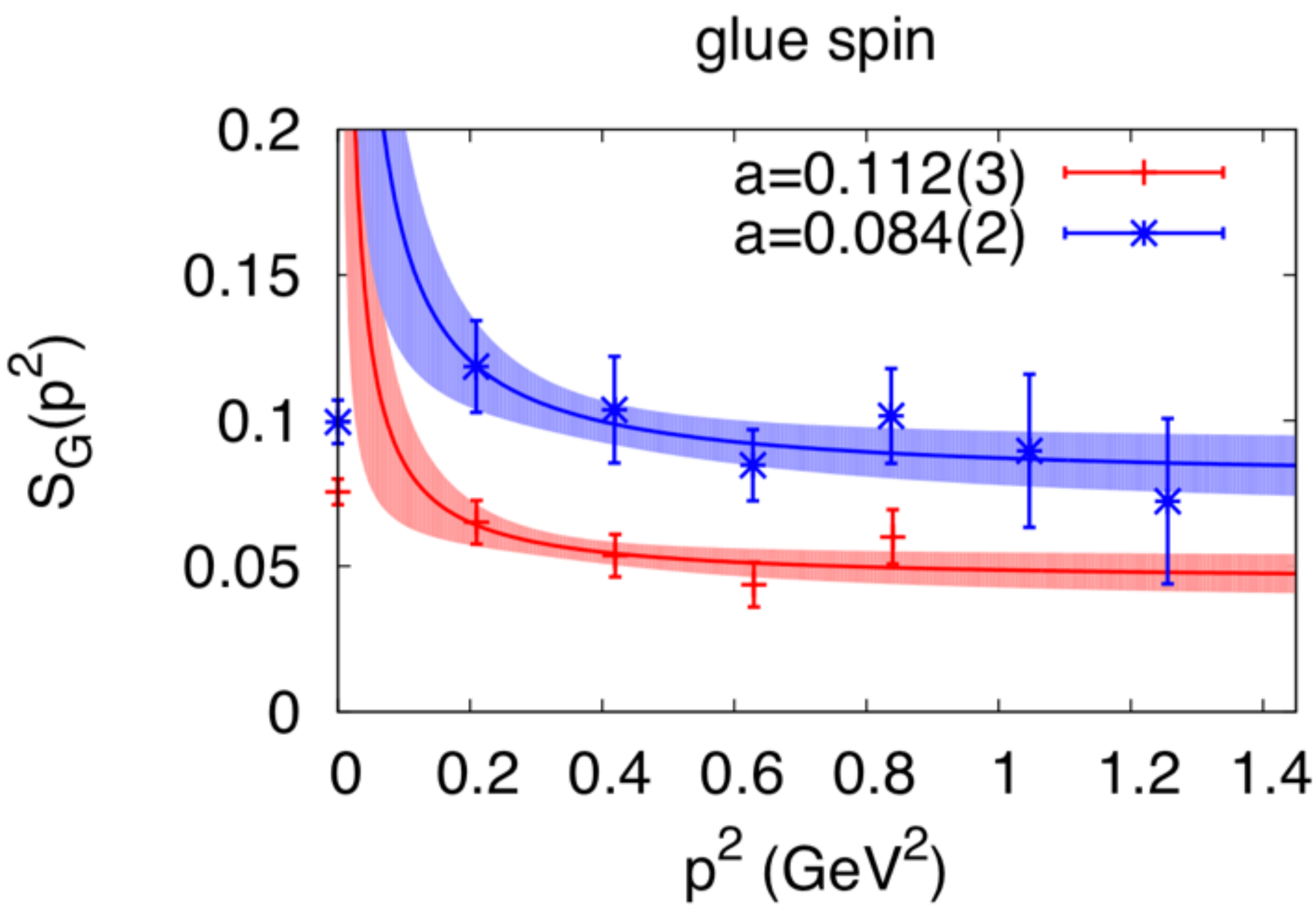
a=0.112(3) fm, L~2.7fm,
m_π^v=330MeV, m_π^{sea}=330MeV



The chiral dependences are mild.

Glue spin

The preliminary bare result



The continuum limit with the linear extrapolation for the bare value at p=2GeV: S_G~0.13(3)

Glue spin

Comments for the matching to IMF

- The perturbative renormalization matching to the MS-bar in the continuum.

1. We have proven that all the calculation could be done under the coulomb gauge, while the iwasaki coulomb gauge fixed glue propagator is non-trivial.

2. The calculation on the HYP smeared configuration is hard to calculate by hand and requires numerical tools.

- The matching and mixing from a finite momentum to the infinite momentum frame (has been performed in the one loop level).

The coefficients in the one loop level are large at 2 GeV. High order calculation would be required.

Summary

- **The anomalous Ward identity provides a new way to obtain the renormalization free quark spin, while the calculation will be much more complicated than the direct calculation.**
- ***1. The cancelation of the 2mP and the topocharge term has been confirmed on the 5.6 fm lattice.***
- ***2. The final Di contribution would be large, more statistics is required.***

- **The glue spin in the finite momentum has been obtained at two lattice spacing, with the chiral extrapolation.**
- ***1. The linear continuum extrapolation of the bare value at 2 GeV is***
- ***$\Delta g \sim 0.13(3)$.***
- ***2. The perturbative matching is ongoing.***