Rho Mass and Leptonic Decay Constant at the Physical Point

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For

(χQCD Collaboration)

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I. Introduction

- Rho resonance

$$M_{\rho^\pm} = 775.26(25) MeV, \quad \Gamma_\rho \approx 149 MeV$$

- Rho resonance are investigated in lattice QCD through $\pi\pi(I=1)$ scattering (see talks in this session).

- Rho is also involved in leptonic processes,

$$\rho^0 \to e^+ e^-;$$

$$\tau \to \rho^- \nu_\tau$$

So the loptonic decay constant f_ρ is also an important characteristic quantity of Rho.

- On the other hand, the chiral behavior of Rho mass is interesting for ChTP study of rho.

- This talk is based on the paper arXiv:1507.02541 (hep-ph)

II. Lattice formulation

- We use a mixed action formalism in the calculation

  Overlap valence on domain wall sea

- Gauge configurations

  Gauge configurations with Nf=2+1 domain wall fermions (DWF) generated by the RBC&UKQCD Collaboration

  (T. Blum et al., arXiv:1411.7017(hep-lat))

Ensemble	a^{-1}(GeV)	$L^3 \times T$	$m_\pi^{(sea)}$(MeV)	La(fm)
48I	1.730(4)	$48^3 \times 96$	139.2(4)	~ 5.5

- Overlap (OL) fermions as valence quarks

a) Also chiral fermions;

b) Multi-mass algorithm can be used in the calculation of quark propagators;

c) Very small Δ_{mix}, which measures the mass mismatch of pions made up of DWF sea quark or OL valence quarks (M. Lujan et al., PRD86(2012)014501)

$am_q^{(val)}$	0.00170	0.00240	0.00300	0.00455	0.00600	0.02030
m_π(MeV)	114(2)	135(2)	149(2)	182(2)	208(2)	371(1)

- The exact chiral symmtry of OL facilitate us to extract the decay constant of pion through the PCAC (we use 45 configurations of 48I in this study), since $Z_m Z_P = 1$ for OL

$$m_\pi^2 f_\pi = (m_u + m_d)\langle 0|\bar{u}\gamma_5 d|\pi\rangle$$

$$m_u = m_d = m_q^{(\mathrm{val})}$$

Ensemble	a^{-1}(GeV)	$m_\pi^{(sea)}$(MeV)	f_π (MeV) (RBC)	f_π (MeV) (ours)
48I	1.730(4)	139.2(4)	131.1(3)	131.3(6)
64I	2.359(7)	139.2(5)	130.9(4)	
Prediction			130.2(9)	

- This comparison can be taken as a calibration of our formulism.

III. The energy levels in the rho channel

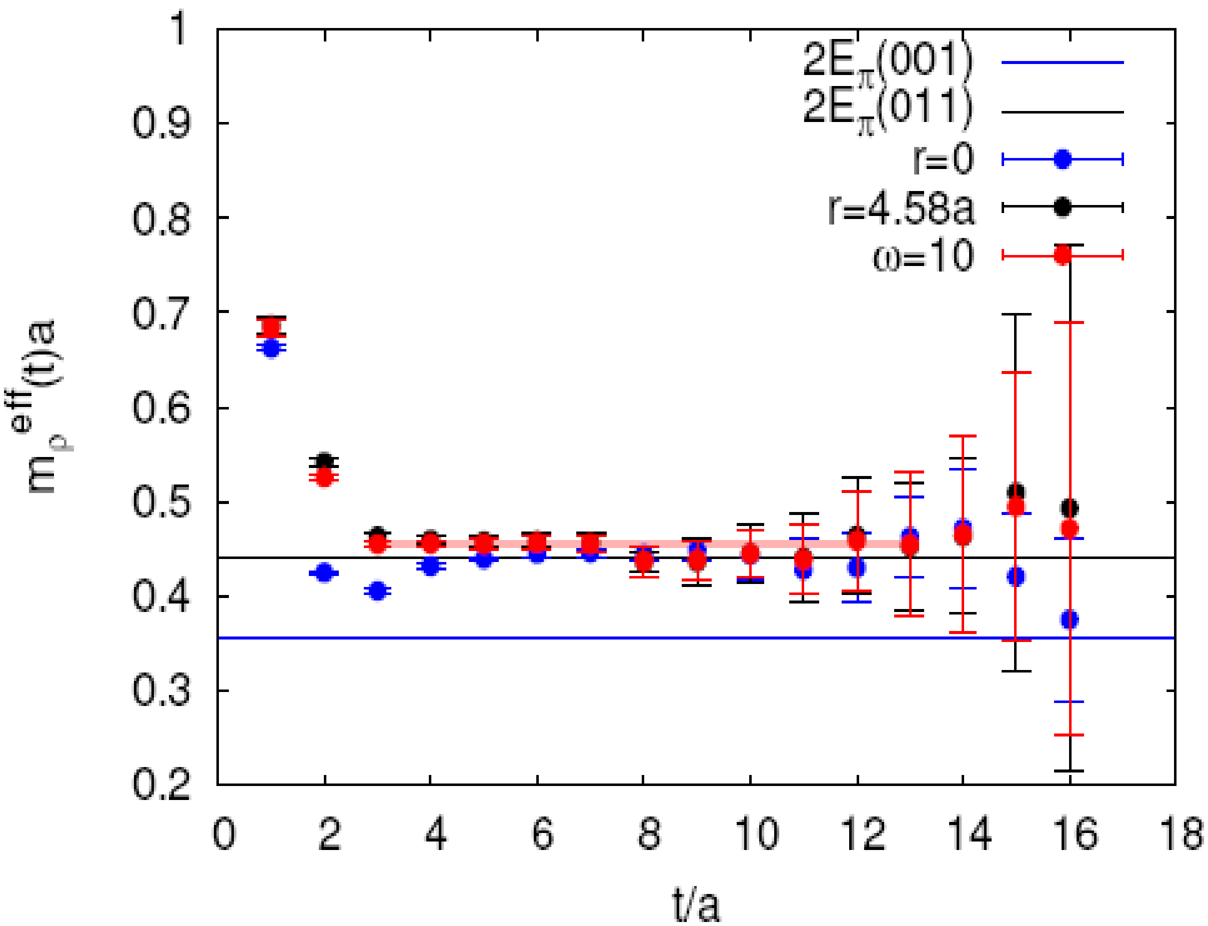
1. We use the Coulomb gauge fixed wall source to calculate two-point functions

$$O_{V,i}^{(\mathrm{w})}(t) = \sum_{\mathbf{y},\mathbf{z}} \bar{u}(\mathbf{y},t)\gamma_i d(\mathbf{z},t)$$

$$O_{V,i}(\mathbf{x},t;\mathbf{r}) = \bar{u}(\mathbf{x},t)\gamma_i d(\mathbf{x}+\mathbf{r},t)$$

$$C(r,t) = \frac{1}{3N_r} \sum_{\mathbf{x},i,|\mathbf{r}|=r} \langle 0|O_{V,i}(\mathbf{x},t;\mathbf{r})O_{V,i}^{(\mathrm{w})\dagger}(0)|0\rangle$$

2. The correlation functions with different r can be properly linearly combined to obtain better signals for the ground state (this can be seen in the following)



- This plot shows the effective plateaus in the rho channel at $m_\pi = 208MeV$
- The blueline: $\pi\pi$ threshold $2E_\pi(001) = 614MeV$
- The blackline: $\pi\pi$ threshold $2E_\pi(011) = 761MeV$
- The blue points and black points are the effective energy from C(0,t) and C(4.58a,t)
- The red points are for $C_{\rm mix}(t) = C(0,t) + \omega C(4.58a,t)$

3. There are several questions:

a) where is the lowest pion-pion scattering state?

b) what is the relation between the second lowest state and the measured plateau?

c) how can one extract useful information about rho from the measured data?

1) Coulomb gauge fixed wall source operator may strongly suppress the p-wave non-interacting $\pi\pi$ scattering states

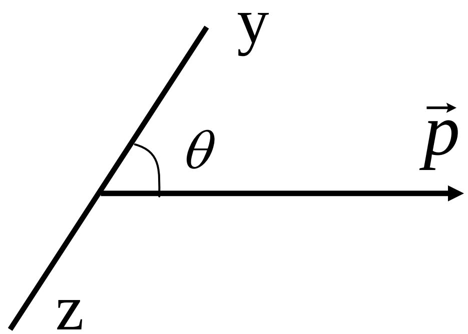
$$O_{V,i}^{(\mathrm{w})}(t) = \sum_{\mathbf{y},\mathbf{z}} \bar{u}(\mathbf{y},t)\gamma_i d(\mathbf{z},t)$$

Qualitatively,

$$\langle 0|\bar{u}(\mathbf{y})\gamma_i d(\mathbf{z})|\pi(\mathbf{p})\pi(-\mathbf{p})(L=1)\rangle \sim Z_i(|\mathbf{p}|,r)\cos\theta$$

$$\cos\theta = \frac{\mathbf{r}\cdot\mathbf{p}}{|\mathbf{r}||\mathbf{p}|}, \quad \mathbf{r} = \mathbf{y} - \mathbf{z}$$

$$\langle 0|O_{V,i}^{(w)}|\pi(\mathbf{p})\pi(-\mathbf{p})(L=1)\rangle = 0$$



2) Through large N_c analysis, it is suggested by Jaffe that ρ is an ordinary meson like a Feshbach resonance which exists as a $q\bar{q}$ confined state in the continuum with zero width in the large N_c limit. (R.L. Jaffe, AIP Conf. Proc. 964, 1 (2007); Prog. Theor. Phys. Suppl. 168, 127 (2007) [arXiv:hep-ph/0701038]) .

3) The eigenstates of the lattice Hamiltonian in the rho channel can be taken as linear superpositions of non-interacting $\pi\pi$ states and would be $q\bar{q}$ confined states.

A complete state set:
$$\begin{cases} \left|\pi(p)\pi(-p)\right\rangle, p = 2\pi n / L, n = (001),(011),\ldots \\ \left|\rho_i\right\rangle \end{cases}$$

4) In the resonant energy region $E_{\pi\pi} \sim m_\rho$ the lattice energy levels should show avoiding level crossings, which can be taken as the mixing effects of the would-be confined state and the nearest scattering state. (for references, see for example K. Rummukianen and S. Gottlieb, Nucl. Phys. B450 (1995) 397.)

Considering the mixing of a two-state system,

$$H_0|\pi\pi\rangle = E_1|\pi\pi\rangle, \qquad H_0|\rho\rangle = E_2|\rho\rangle,$$

$|\pi\pi\rangle$: non-interacting $\pi\pi$ state

$|\rho\rangle$ would-be $q\bar{q}$ confined state

When the coupling of $|\pi\pi\rangle$ and $|\rho\rangle$ is considered, the Hamiltonian is

$$H = H_0 + H_{\rm I} = \begin{pmatrix} E_1 & x \\ x & E_2 \end{pmatrix},$$

whose eigen values are

$$\begin{aligned} E_\pm &= \frac{1}{2}\left(E_1 + E_2 \pm \sqrt{(E_2 - E_1)^2 + 4x^2}\right) \\ &\equiv M \pm \frac{1}{2}\Delta\delta, \end{aligned}$$

$$M = \tfrac{1}{2}(E_1 + E_2)$$

$$\Delta = E_2 - E_1$$

$$\delta = \sqrt{1 + 4x^2/\Delta^2}$$

and the corresponding eigenstates are

$$\begin{aligned} |\alpha\rangle &= a_-|\pi\pi\rangle + b_-|\rho\rangle, \\ |\beta\rangle &= a_+|\pi\pi\rangle + b_+|\rho\rangle \end{aligned} \qquad H|\alpha\rangle = E_-|\alpha\rangle, \quad H|\beta\rangle = E_+|\beta\rangle,$$

with the combination coefficients,

$$\begin{pmatrix} a_- & b_- \\ a_+ & b_+ \end{pmatrix} = \frac{1}{\sqrt{2\delta}} \begin{pmatrix} \sqrt{\delta+1} & -\sqrt{\delta-1} \\ \sqrt{\delta-1} & \sqrt{\delta+1} \end{pmatrix}.$$

These states contribute to the correlation function as

$$\begin{aligned} C(t) &= \langle O_P(t) O_W^+(0) \rangle \\ &= \langle 0|O_P|\alpha\rangle\langle\alpha|O_W^+|0\rangle e^{-E_- t} + \langle 0|O_P|\beta\rangle\langle\beta|O_W^+|0\rangle e^{-E_+ t} \end{aligned}$$

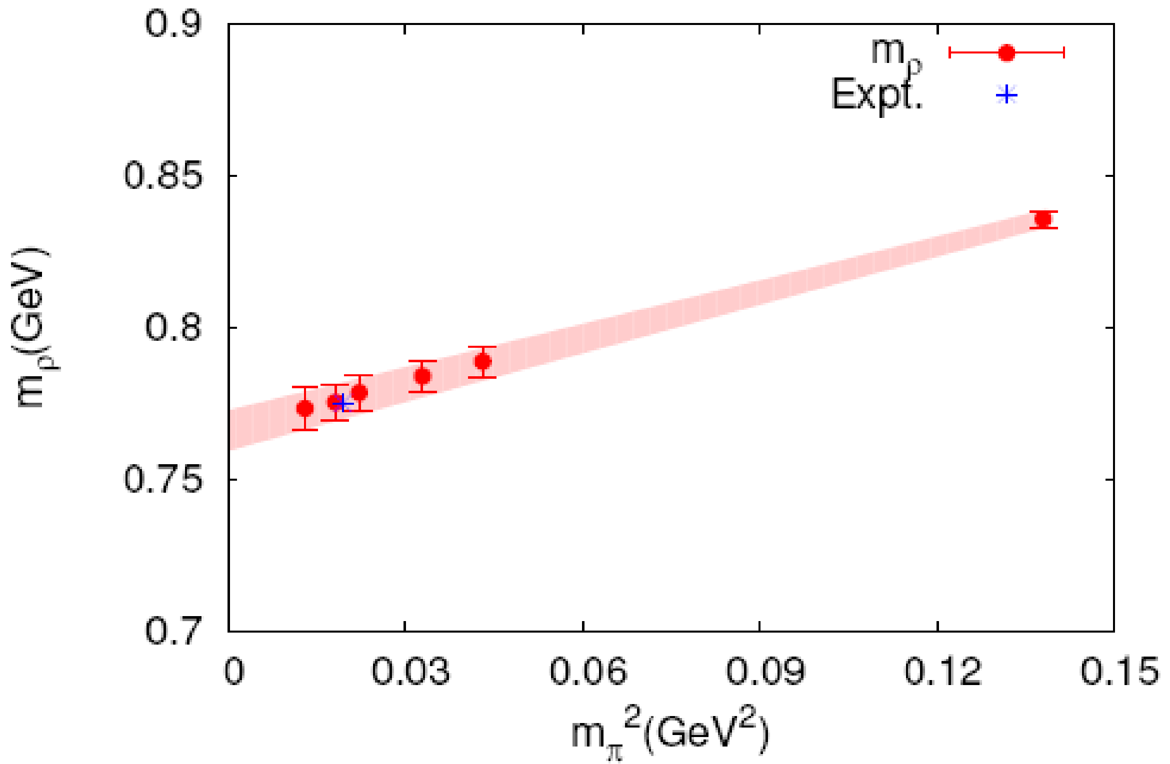
According to our previous discussion, if $\langle 0|O_W|\pi\pi\rangle = 0$

$$
\begin{aligned}
C(t) &= \langle 0|O_P|\rho\rangle\langle\rho|O_W^+|0\rangle(b_-^2 e^{-E_- t} + b_+^2 e^{-E_+ t}) \\
&= \langle 0|O_P|\rho\rangle\langle\rho|O_W^+|0\rangle\frac{1}{2\delta}\left((\delta-1)e^{-(M-\frac{1}{2}\Delta\delta)t} + (\delta+1)e^{-(M+\frac{1}{2}\Delta\delta)t}\right) \\
&= \langle 0|O_P|\rho\rangle\langle\rho|O_W^+|0\rangle e^{-E_2 t}\left(1+\frac{1}{8}((\Delta\delta)t/2)^2+\ldots\right)
\end{aligned}
$$

Even though we cannot resolve the two states numerically in this study, when we use one exponential to fit the correlatio function we actually obtain the mass of the would-be $q\bar{q}$ confined state $|\rho\rangle$, which we take as an approximation to the mass of rho meson.

4. We carry out similar analysis at all the pion masses and get the result,

m_π(MeV)	114(2)	135(2)	149(2)	182(2)	208(2)	371(1)
m_ρ(MeV)	773(7)	775(6)	779(6)	784(5)	789(5)	836(3)



$$m_\rho(m_\pi) = m_\rho(0) + c_1 m_\pi^2$$

The ChPT study gives the chiral behavior of rho mass like

(P.C. Bruns and U.-G. Meissner, Eur. Phys. J. C 40, 97 (2005))

$$m_\rho(m_\pi) = m_\rho(0) + c_1 m_\pi^2 + c_2 m_\pi^3 + c_3 m_\pi^4 \ln \frac{m_\pi^2}{m_\rho^2}$$

But the linear form in terms of m_π^2 can well describe our data ,

$$m_\rho(m_\pi) = m_\rho(0) + c_1 m_\pi^2$$

wiith the best-fit parameters

$$m_\rho(0) = 766(7)\,\mathrm{MeV}, \quad c_1 = 0.505(3)\,\mathrm{GeV}^{-1}$$

and the rho mass at the physical pion mass

$$m_\rho = 775.9 \pm 6.0 \pm 1.8 \;\mathrm{MeV}$$

It should be noted that since we do the chiral interpolation, the precise slope parameter c_1 is reliable and can potentially serve as a contraint to the ChPT study of rho.

Furthermore, the (valence) pion-rho sigma term can be obtained directly from the Feynman-Hellman theorem,

$$\sigma_{\pi\rho}^{(\mathrm{val})} = m_\pi^2 \frac{dm_\rho}{dm_\pi^2} = c_1 m_\pi^2 = 9.82(6)\,\mathrm{MeV}$$

III. The leptonic decay constant of rho f_{ρ^-}

$$\langle 0|J_\mu^{(-)}(0)|\rho^-(\vec{p},\zeta)\rangle = m_\rho f_{\rho^-}\epsilon_\mu(\vec{p},\zeta)$$

$$J_\mu^{(-)}(x) = (\bar{u}\gamma_\mu d)(x)$$

Recalling the definition

$$O_{V,i}(\mathbf{x},t;\mathbf{r}) = \bar{u}(\mathbf{x},t)\gamma_i d(\mathbf{x}+\mathbf{r},t)$$

It is easily seen that the decay constant can be extracted through the joint fit of

Bare decay constant

$$C(0,t) = \sum_n 2m_n L^3 f_n Z_n^{(\mathrm{w})} e^{-m_n t}$$

$$C^{(\mathrm{w})}(t) \approx \sum_n 2m_n L^3 (Z_n^{(\mathrm{w})})^2 e^{-m_n t}$$

However, $C^{(\mathrm{w})}(t)$ is notoriously noisy. So we propose a technique to improve the quality of it signal, …….

1. Why $C^{(\mathrm{w})}(t)$ so noisy?

Actually,

$$C^{(\mathrm{w})}(t) \equiv \frac{1}{3}\sum_i \langle 0|O_{V,i}^{(\mathrm{w})}(t)O_{V,i}^{(\mathrm{w}),\dagger}(0)|0\rangle$$

$$= \frac{1}{3}\sum_{\mathbf{x},\mathbf{r},i} \langle 0|O_{V,i}(\mathbf{x},t;\mathbf{r})O_{V,i}^{(\mathrm{w}),\dagger}(0)|0\rangle$$

$$= \sum_r N_r C(r,t)$$

$$C(r,t) = \sum_n \Phi_n(r)e^{-E_n t}$$

Since $\Phi_n(r)$ damps exponentially with r, a $C(r,t)$ with large r only contributes noise.

2 An approximate $C^{(\mathrm{w})}(t)$ with a cutoff of r

$$C^{(\mathrm{w})}(r_c, t) = \sum_{r \le r_c} N_r C(r, t)$$

and the approximation quality is measured by the ratio,

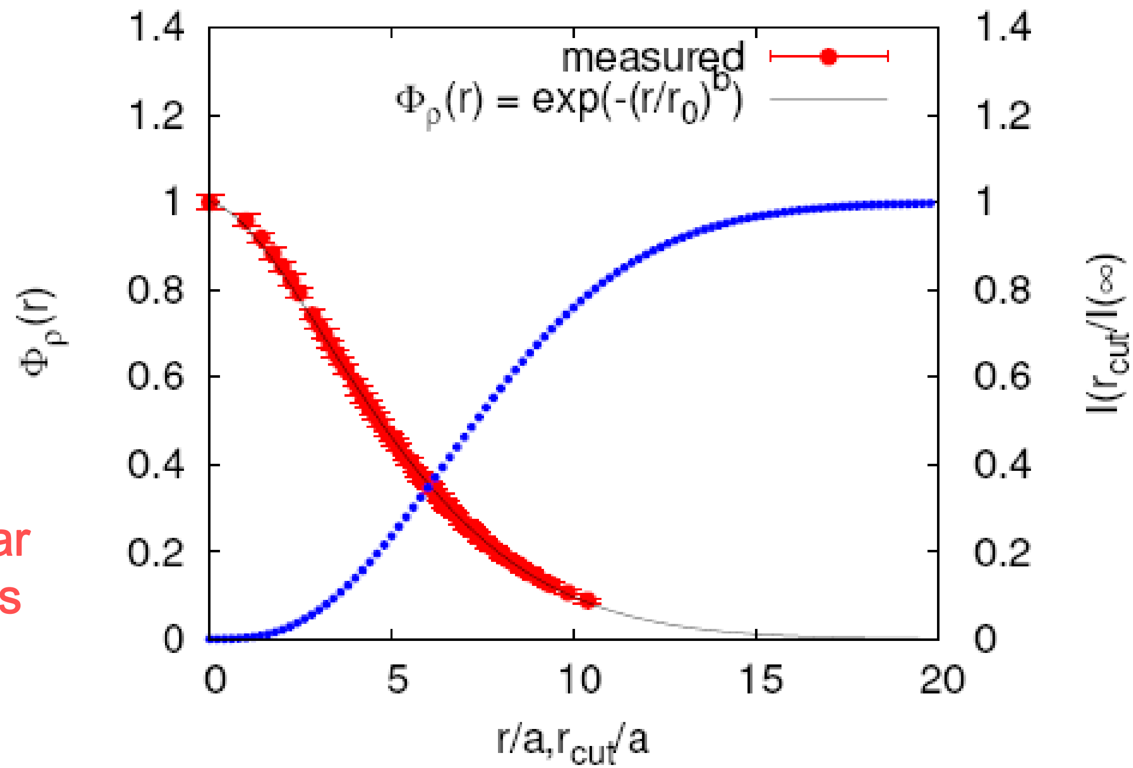
$$\frac{C^{(\mathrm{w})}(r_c, t)}{C^{(\mathrm{w})}(t)} \approx \frac{\int_0^{r_c} dr r^2 \Phi_1(r)}{\int_0^{r_c} dr r^2 \Phi_1(r)} \equiv \frac{I_1(r_c)}{I_1(\infty)}$$

$$\Phi_1(r) = \Phi_1(0) e^{-\left(\frac{r}{r_0}\right)^{\alpha}}$$

$\alpha = 1.60$ and $r_0 = 5.88a$

$$\frac{I_1(20a)}{I_1(\infty)} \approx 0.995$$

The situations are similar for different pion masses in this work.



c) The non-perturbative renormalization of the vector current

This can be easily done for overlap fermions. Since

$$Z_m = Z_P^{-1} \qquad Z_V = Z_A$$

The Zv can be derived directly from $Z_A = \frac{2m_q\langle 0|P|\pi\rangle}{m_\pi \langle 0|A_4|\pi\rangle}$ $\quad A_\mu = \bar{u}\gamma_5\gamma_\mu d$

m_π(MeV)	114(2)	135(2)	149(2)	182(2)	208(2)	371(1)
Z_A	1.103(4)	1.103(3)	1.104(2)	1.104(2)	1.105(1)	1.105(1)
f_ρ(MeV)	206(7)	208(7)	211(6)	215(5)	217(5)	223(3)

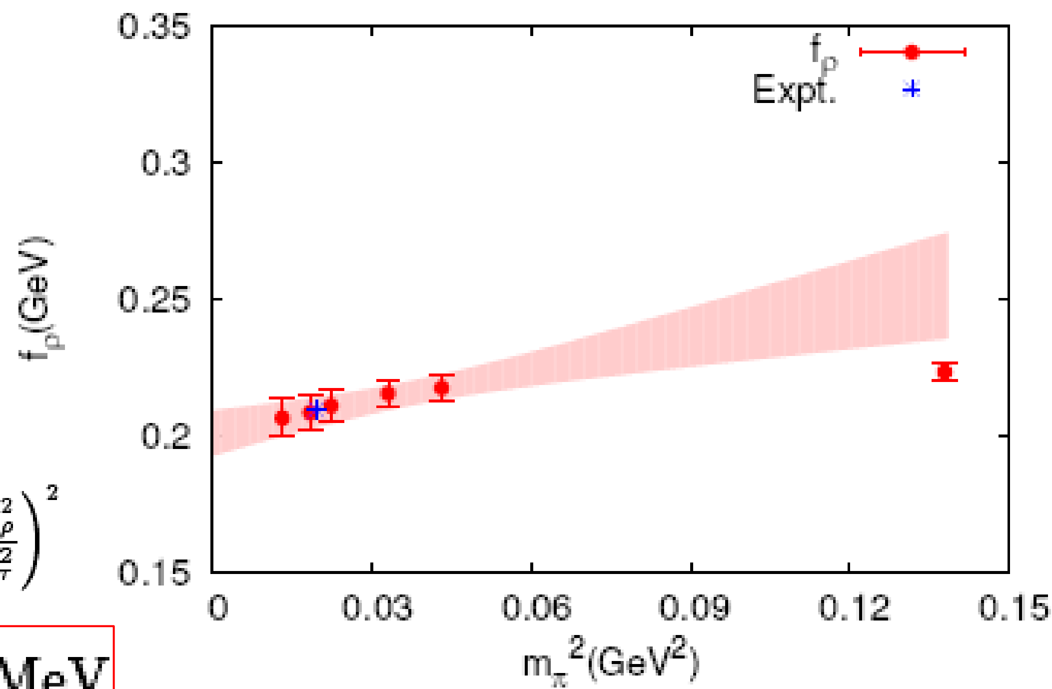
The linear interpolation
To the physical point,

$$f_{\rho^\pm} = 208.5 \pm 5.5 \pm 0.9 \ \ \mathrm{MeV}$$

Experimentally, from

$$\mathrm{Br}(\tau \to \rho^- \nu_\tau) = \frac{G_F^2 m_\tau |V_{ud}|^2}{8\pi} \tau_\tau m_\rho^2 f_{\rho^-}^2 \times \left(1 + \frac{m_\tau^2}{2m_\rho^2}\right)\left(1 - \frac{m_\rho^2}{m_\tau^2}\right)^2$$

$$f_\rho^{\pm,\mathrm{exp}} = 209.4 \pm 1.5 \ \ \mathrm{MeV}$$



V. Summary

- By using chiral fermions for both sea and valence quark, we can study at the physical point.

- The mass of rho meson is derived at the physical point

$$m_\rho = 775.9 \pm 6.0 \pm 1.8 \text{ MeV}$$

. The chiral behavior of the rho mass is very clear seen.

. We obtain the leptonic decay constant of the charge rho,

$$f_{\rho^\pm} = 208.5 \pm 5.5 \pm 0.9 \text{ MeV}$$

which is in very good agreement wit the experimental value

$$f_\rho^{\pm,\text{exp}} = 209.4 \pm 1.5 \text{ MeV}$$

Thanks!

We check this using the new gauge configurations with 2+1 flavor domain-wall sea quarks on a large lattice.

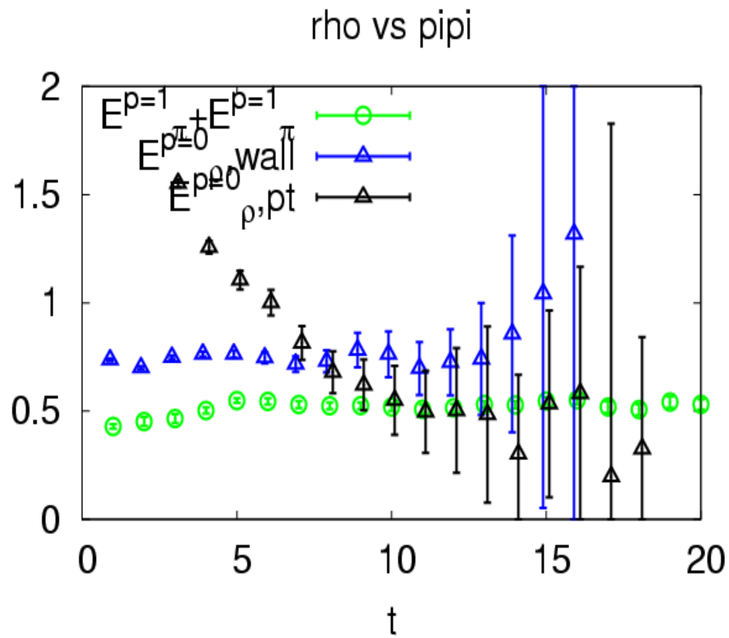
$1/a(\mathrm{GeV})$	label	$am^{(s)}$	$L^3 \times T$	N_{conf}
1.76(1)	48I	0.00078/0.0362	$48^3 \times 96$	45

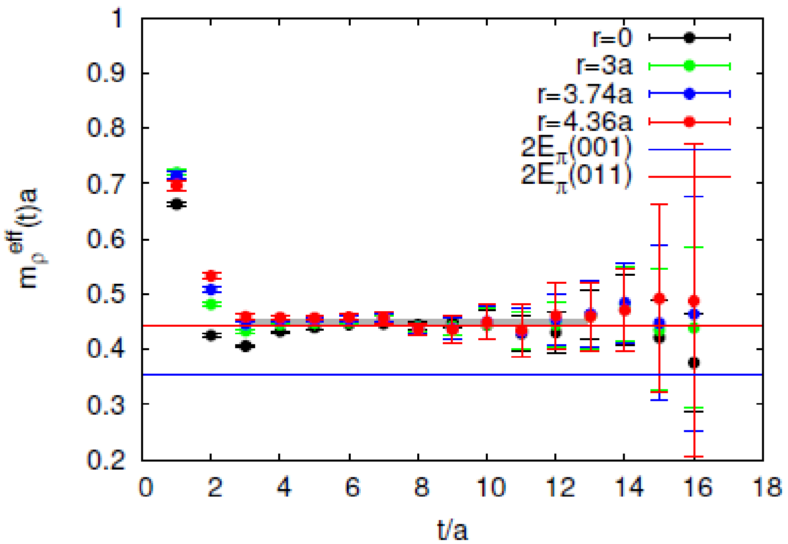
$$m_\pi^{(ss)} \approx 140 MeV, \quad m_\pi^{(vv)} \approx 117, 140, 150, 170, 210 MeV$$

$$La = 5.5\, fm \qquad p_{\min} \approx 230 MeV$$

We take the the rho-channel for example, because

$$m_\rho > 2E_\pi(p_{\min}) = 2\sqrt{m_\pi^2 + p_{\min}^2}$$





Left panel-Point source

- green points: $2E_\pi(p_{\min}) = 540MeV$
- blue triangle:

$$m_\rho = 780MeV$$

- black triangle: effetive masses of the point-source correlation functions.

Right panel-Coulomb wall source

- points: effective masses of wall-source correlation functions.

$$m_\rho \approx 780MeV$$

- blue line: $2E_\pi(p_{\min})$
- red line : two-pion threshold with the next smallest lattice momentum.

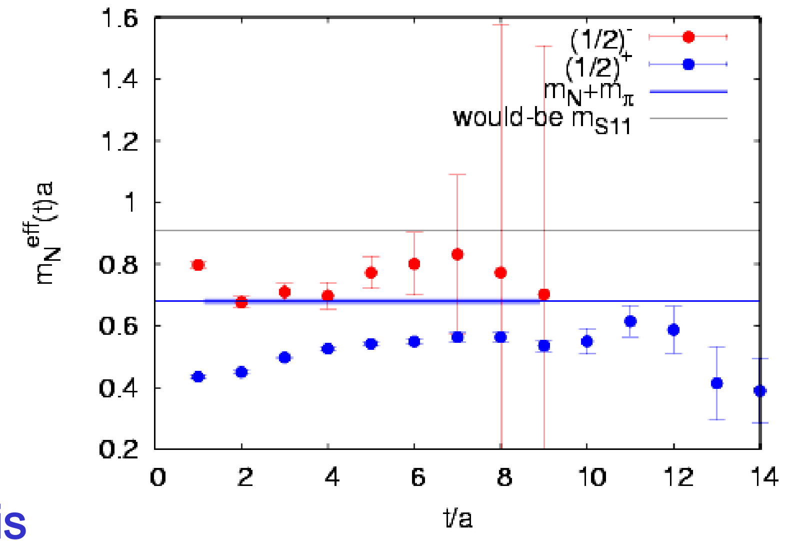
2. Pion-N scattering states in S11 channel

- Pion-N S-wave scattering states.
- The enengy of the lowest pion-N state is

$$E_{\pi N}^{\min}(L=0) \approx m_{\pi} + m_N$$

- we observe it, but with large errorbars.

. No hope for the S11 resonance with this kind of correlators.



The measure branching ratio (PDG2014) ,

$$Br(\tau \to \pi^- \pi^0 \nu_\tau) = 25.52(9)\%$$

$$Br(\tau \to (\pi^- \pi^0)(non-\rho)\nu_\tau = 0.30(32)\%$$

So we estimate the branching ratio ,

$$Br(\tau \to \rho \nu_\tau) = 25.22(33)\%$$

For the neutral rho meson **(PDG2014),**

$$\Gamma\left(\rho^0 \to e^+ e^-\right) = \frac{4\pi}{3} \alpha_{QED}^2 \overline{Q}_\rho^2 \frac{f_\rho^2}{m_\rho^0} = 7.04(6) keV$$

Using $\overline{Q}_\rho^2 = 1/2$ from the valence approximation, $\left|\rho^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|u\bar{u}\right\rangle + \left|d\bar{d}\right\rangle\right)$

$$f_{\rho^0} = 221(1) MeV$$