(2+1)-flavor QCD Thermodynamics from the Gradient Flow

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collaboration with
H.Suzuki (Kyushu U.), Y.Taniguchi (Tsukuba U.), T.Umeda (Hiroshima U.)
based on FlowQCD and Whot QCD joint-collaboration

references:
Asakawa, Hatsuda, E.I., Kitazawa, Suzuki (FlowQCD coll.) Phys.Rev. D90 (2014) 1, 011501
work in progress

Energy-momentum tensor on the lattice

Basic Idea

Quantum field theory

(UV divergence)

perturbation with dim. reg.

+YM gradient flow

(general covariance OK!)

lattice reg.

+Wilson flow

(with a->0 limit)

At finite flow time, UV finite!

Luescher and Weisz, JHEP 1102, 051(2011)

Firstly, we obtain the relation between them perturbatively.

Assume that it applies to the nonperturbative regime.

``Suzuki method"

for quenched QCD

Suzuki, PTEP 2013, no8, 083B03

relation··· dim.=4 op on the lattice vs. renormalized EMT

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t),$$

coefficient··· given by renormalized coupling and coeff. of beta fn.

$$\alpha_U(t)(g;\mu) = \bar{g}^2\left\{1 + 2b_0\left[\ln(\sqrt{8t}\mu) + s_1\right]\bar{g}^2 + O(\bar{g}^4)\right\},$$

$$\alpha_E(t)(g;\mu) = \frac{1}{2b_0}\left\{1 + 2b_0 s_2 \bar{g}^2 + O(\bar{g}^4)\right\},$$

b_0 1-loop coeff. of beta fn.

MSbar scheme

$$s_1 = 0.03296\ldots$$
$$s_2 = 0.19783\ldots$$

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Suzuki, PTEP 2013, no8, 083B03

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b_0 1-loop coeff. of beta fn.

MSbar scheme

$s_1 =$ -0.0863575

$s_2 =$ 0.05578512

How to get EMT

for quenched QCD

Step 1

Generate gauge configuration at t=0 (usual process)

Step 2

Solve the Wilson flow eq. and generate the gauge configuration at flow time (t)

$$a \ll \sqrt{8t} \ll \Lambda_{QCD}^{-1} \text{ or } T^{-1}$$

Step 3

Measure two dim=4 ops. using flowed gauge configuration

$$U_{\mu\nu}(t,x), E(t,x)$$

Step 4

Take the continuum limit. Then take t->0 limit.
(Take care the feasible window of flow time)

$$T_{\mu\nu}^{R}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

One-point fn. of EMT in finite temperature QCD

Asakawa, Hatsuda, E.I., Kitazawa, Suzuki (FlowQCD coll.)
Phys.Rev. D90 (2014) 1, 011501

Simulation setup

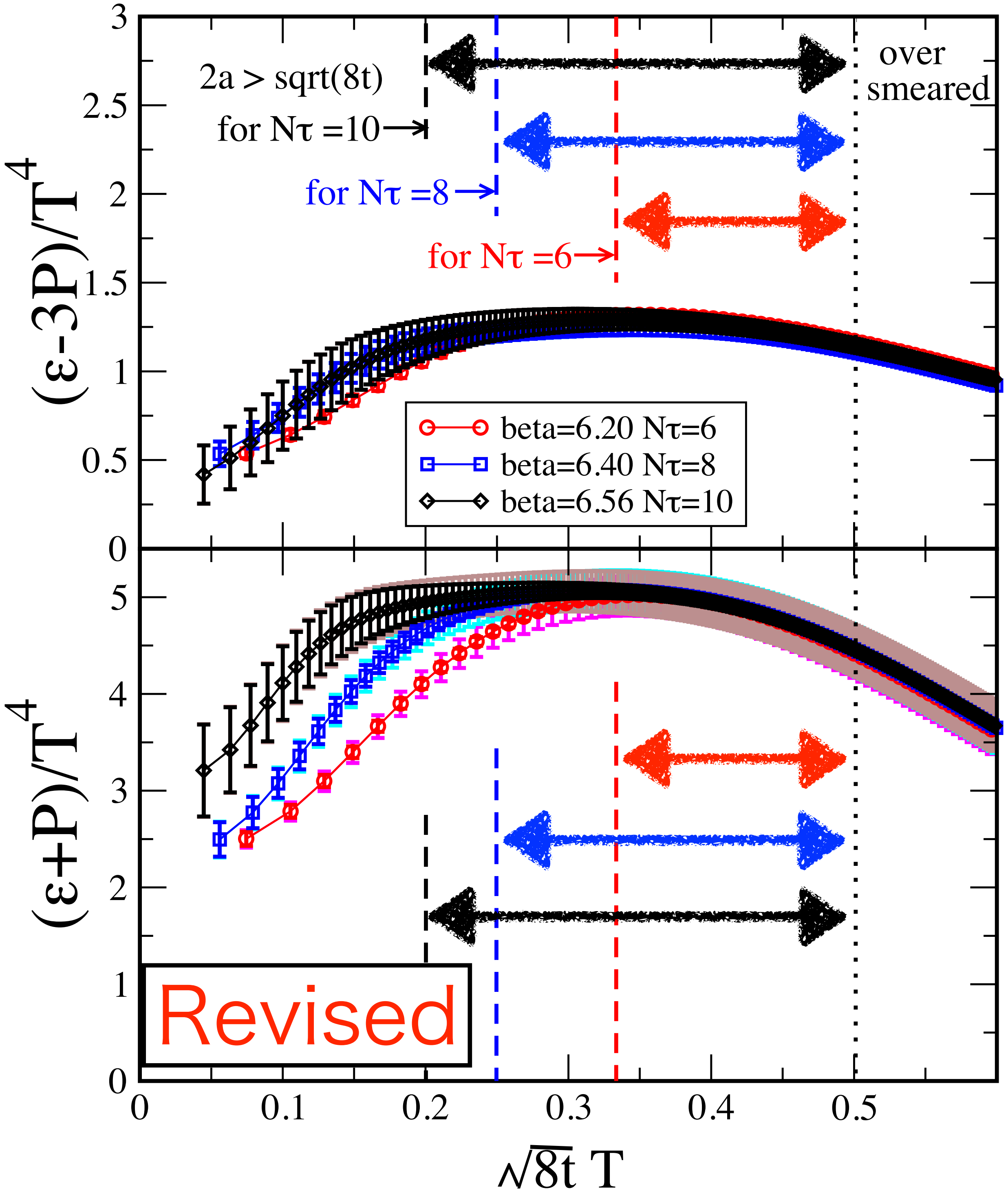
- Wilson plaquette gauge action
- lattice size (Ns=32, Nt=6,8,10,32)
- # of confs. is 100 - 300
- simulation parameters

N_τ	6	8	10	T/T_c
β	6.20	6.40	6.56	1.65
	6.02	6.20	6.36	1.24
	5.89	6.06	6.20	0.99

Temperature is determined by
Boyd et. al. NPB469,419 (1996)

Parametrization is given by
alpha collaboration NPB538,669 (1999)

flow time dependence
(T=1.65Tc)



each dark color shows statistical error

each light color includes systematic error

feasible flow time

longer than lattice cutoff

avoid an over-smeared regime

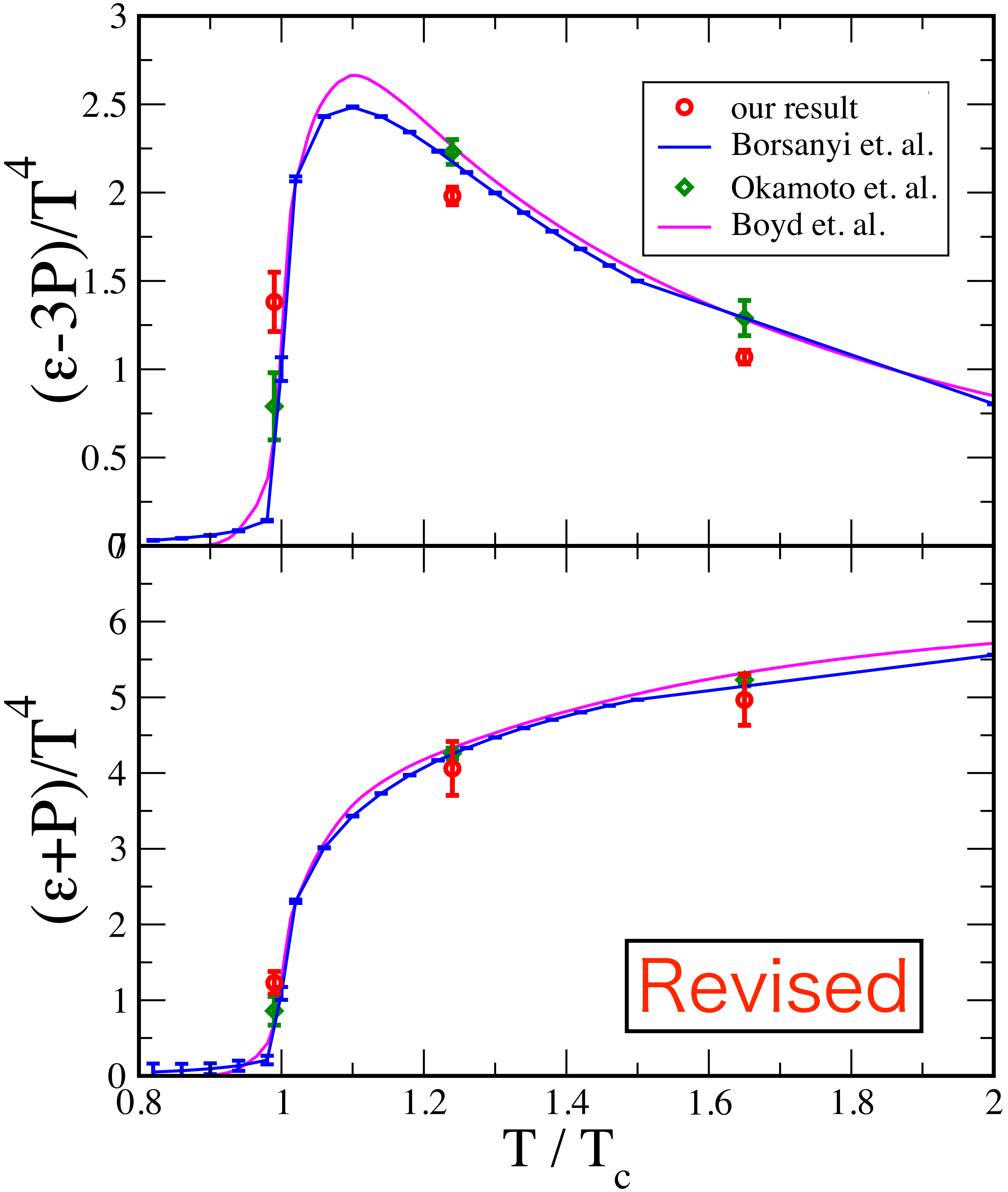
$$2a < \sqrt{8t} < N_\tau a/2$$

- show a plateau
  (small higher dimensional op.)
  Practically, no need t-> 0 limit

**finer lattice simulation shows a slope
M.Kitazawa's talk on Wed.

- systematic error coming from scale setting is dominated in entropy density

Comparison with the results given by integration method



Boyd et. al.
NPB469,419 (1996)

Okamoto et. al. (CP-PACS)
PRD60, 094510 (1999)

Borsanyi et. al.
JHEP 1207, 056 (2012)

(2+1)flavor QCD

How to get EMT

for quenched QCD

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$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

How to get EMT

for full QCD

Step 1

Generate gauge configuration at t=0 (usual process)

With fermion

Step 2

Solve the Wilson flow eq. and generate the gauge configuration at flow time (t)

$$a \ll \sqrt{8t} \ll \Lambda_{QCD}^{-1} \text{ or } T^{-1}$$

With fermion flow

M.Luescher, JHEP 04 (2013) 123

Step 3

Measure two dim=4 ops. using flowed gauge configuration

$$U_{\mu\nu}(t,x), E(t,x)$$

Add operators with fermion

H.Makino and H.Suzuki, PTEP 2014 (2014) 6, 063B02

Step 4

Take the continuum limit. Then take t->0 limit.

(Take care the feasible flow time window)

Fermion flow

M.Luescher, JHEP 04 (2013) 123

t

Gauge flow

$$\partial_t V_t = Z(V_t) V_t,$$

Runge-Kutta step

$$W_3 = \exp\{\tfrac{3}{4}Z_2 - \tfrac{8}{9}Z_1 + \tfrac{17}{36}Z_0\}W_2,$$

$$W_2 = \exp\{\tfrac{8}{9}Z_1 - \tfrac{17}{36}Z_0\}W_1,$$

$$W_1 = \exp\{\tfrac{1}{4}Z_0\}W_0,$$

$$W_0 = V_t,$$

initial cond. $V|_{t=0} = U$

Fermion (adjoint) flow

$$\partial_t \chi_t = \Delta(V_t)\chi_t,$$

initial cond. $\xi_t^{\epsilon}(x) = \eta(x),$

Runge-Kutta step

$$\lambda_3 = \xi_{s+\epsilon}^{\epsilon},$$

$$\lambda_2 = \tfrac{3}{4}\Delta_2\lambda_3,$$

$$\lambda_1 = \lambda_3 + \tfrac{8}{9}\Delta_1\lambda_2,$$

$$\lambda_0 = \lambda_1 + \lambda_2 + \tfrac{1}{4}\Delta_0(\lambda_1 - \tfrac{8}{9}\lambda_2)$$

operators with fermion

H.Makino and H.Suzuki, PTEP 2014 (2014) 6, 063B02

$$\{T_{\mu\nu}\}_R(x) = c_1(t)\left[\tilde{\mathcal{O}}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right]$$
$$+ c_2(t)\left[\tilde{\mathcal{O}}_{2\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right\rangle\right]$$

added

$$+ c_3(t)\left[\tilde{\mathcal{O}}_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}_{4\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}_{4\mu\nu}(t,x)\right\rangle\right]$$
$$+ c_4(t)\left[\tilde{\mathcal{O}}_{4\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}_{4\mu\nu}(t,x)\right\rangle\right]$$
$$+ c_5(t)\left[\tilde{\mathcal{O}}_{5\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}_{5\mu\nu}(t,x)\right\rangle\right] + O(t),$$

$$\mathcal{O}_{3\mu\nu}(x) \equiv \bar{\psi}(x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\psi(x),$$
$$\mathcal{O}_{4\mu\nu}(x) \equiv \delta_{\mu\nu}\bar{\psi}(x)\overleftrightarrow{\not{D}}\psi(x),$$
$$\mathcal{O}_{5\mu\nu}(x) \equiv \delta_{\mu\nu}m_0\bar{\psi}(x)\psi(x).$$

essentially summarize

$$t^r_{\mu\nu}(t) \equiv \frac{1}{N_\Gamma}\sum_x \left\langle \bar{\chi}_r(t,x)\gamma_\mu\left(D_\nu - \overleftarrow{D}_\nu\right)\chi_r(t,x)\right\rangle,$$
$$s^r(t) \equiv \frac{1}{N_\Gamma}\sum_x \left\langle \bar{\chi}_r(t,x)\chi_r(t,x)\right\rangle,$$

Lattice setup

- Iwasaki gauge action + improved Wilson fermion
- lattice size (Ns=32, Nt=8)
- m_ps/m_v=0.6337(38) for u,d quarks
- m_ps/m_v=0.7377(28) for s quark
- each configuration is separated by 100 MC trj.

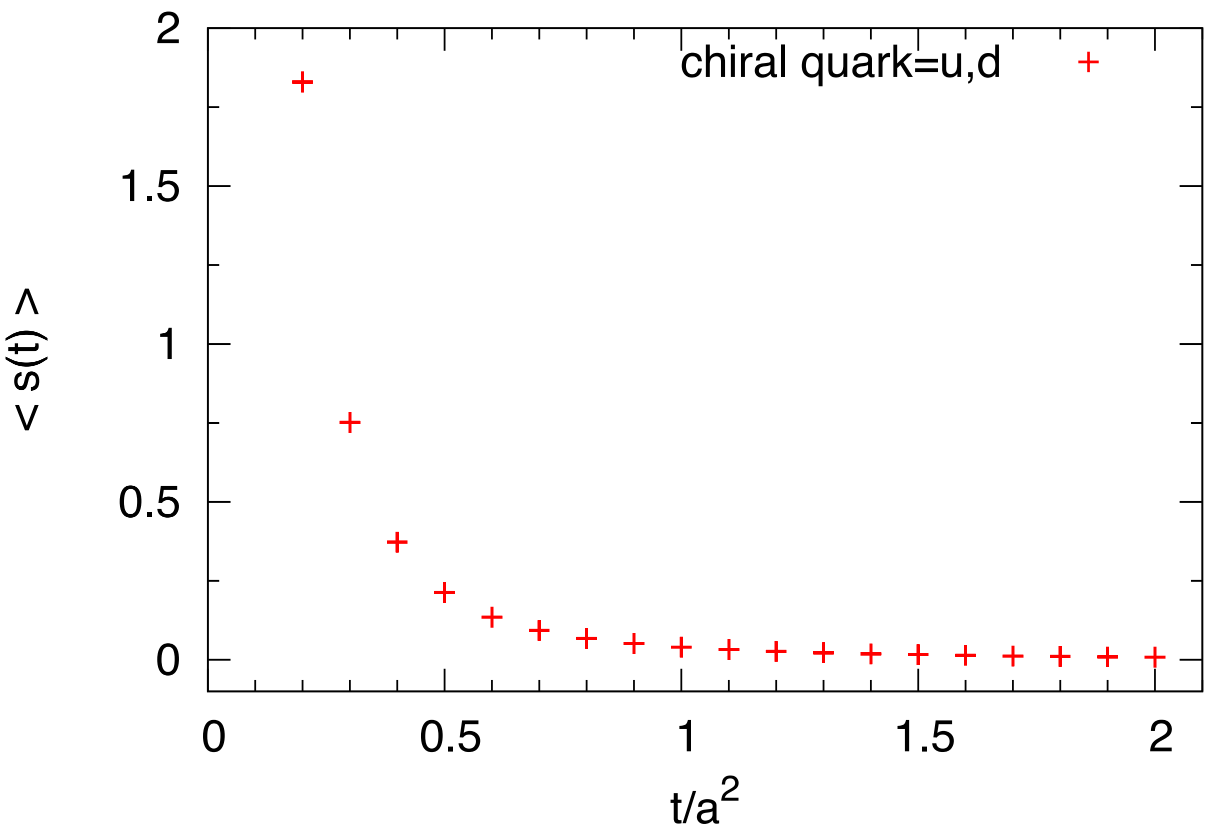
Parametrization is given by
T.Umeda et.al. for WHOT-QCD coll., Phys.Rev.D85,094508(2012)

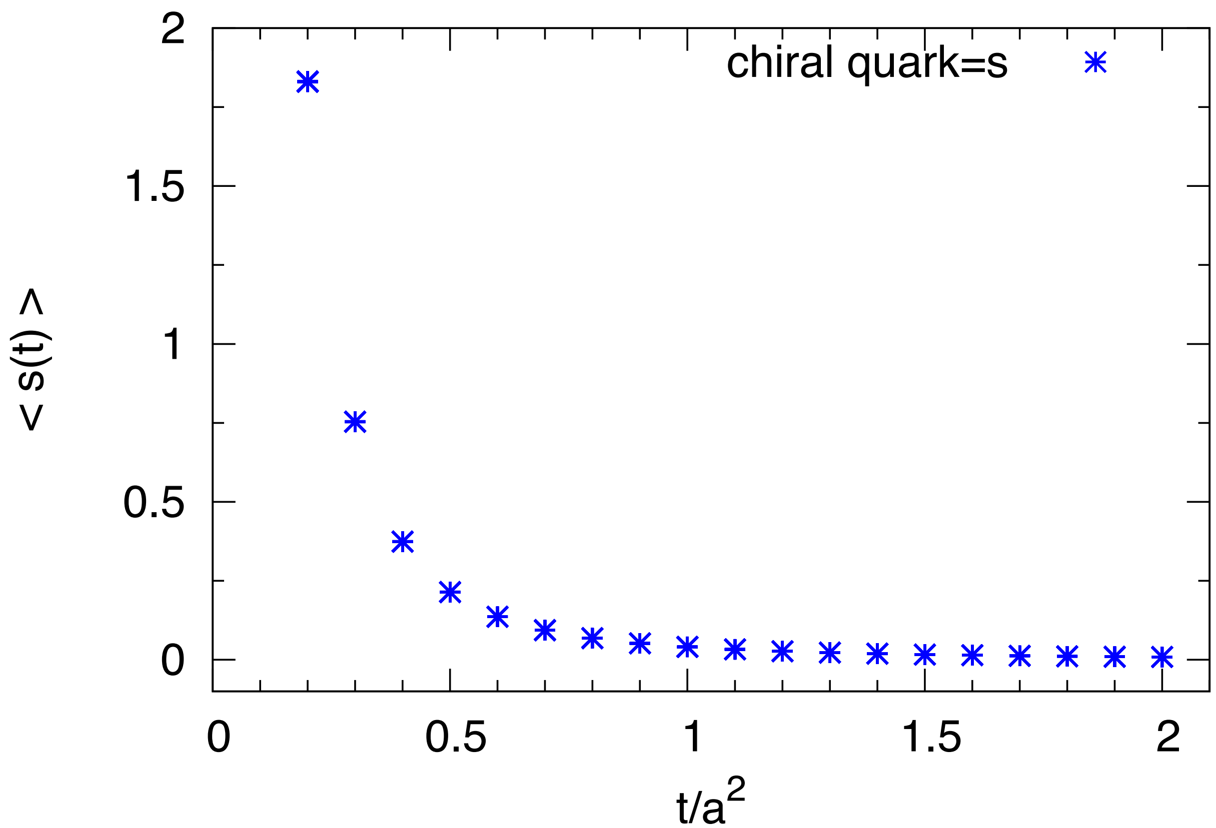
preliminary results

T=280MeV

Nt=8, beta=1.9728, c_sw=1.66922, kappa_ud=0.136147, kappa_s=0.135417

$$s^r(t) \equiv \frac{1}{N_\Gamma}\sum_x \langle \bar{\chi}_r(t,x)\chi_r(t,x)\rangle$$





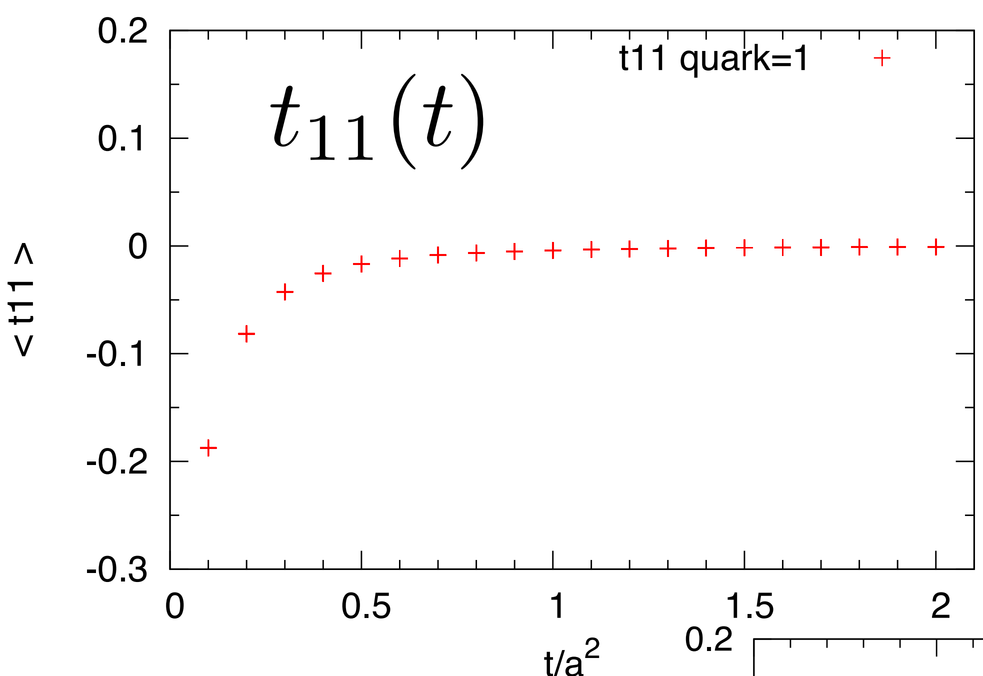
signal is very clear

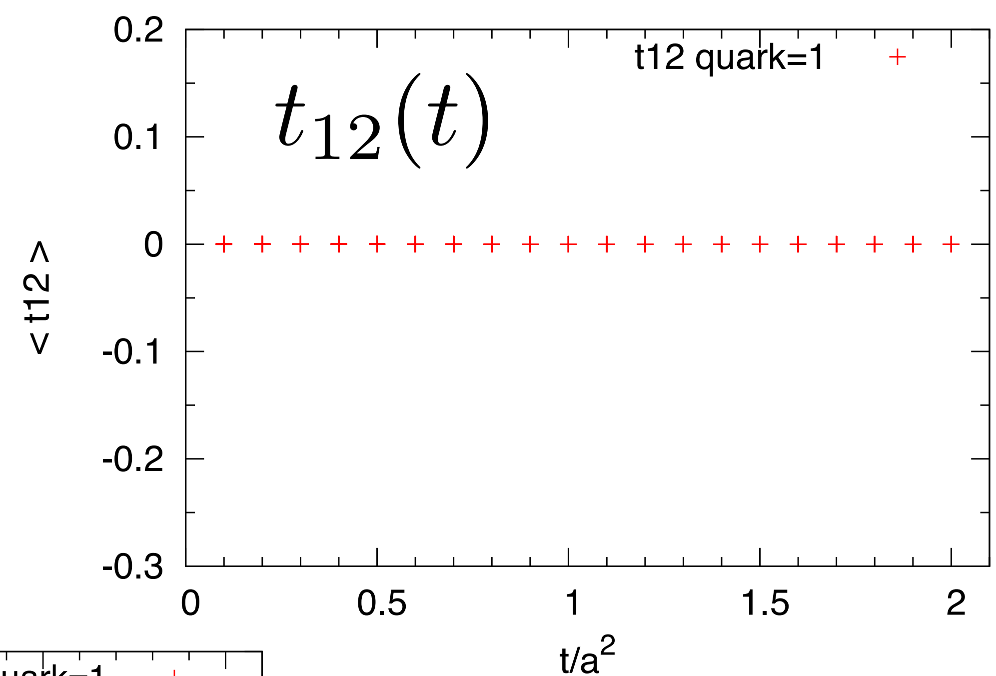
preliminary results

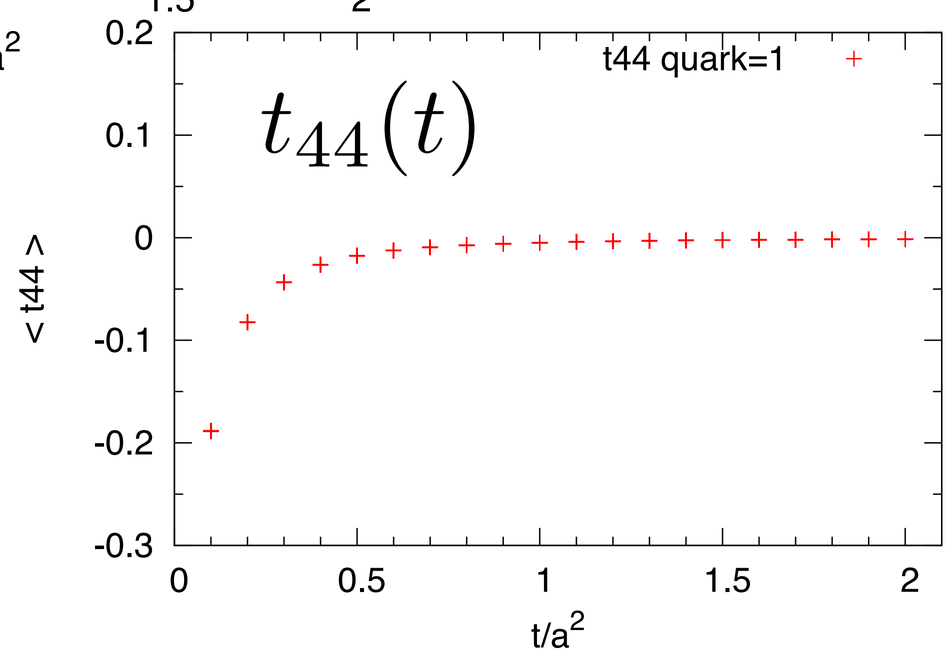
T=280MeV

Nt=8, beta=1.9728, c_sw=1.66922, kappa_ud=0.136147, kappa_s=0.135417

$$t^r_{\mu\nu}(t) \equiv \frac{1}{N_\Gamma}\sum_x \left\langle \bar{\chi}_r(t,x)\gamma_\mu \left(D_\nu - \overleftarrow{D}_\nu\right)\chi_r(t,x)\right\rangle$$







conclusion

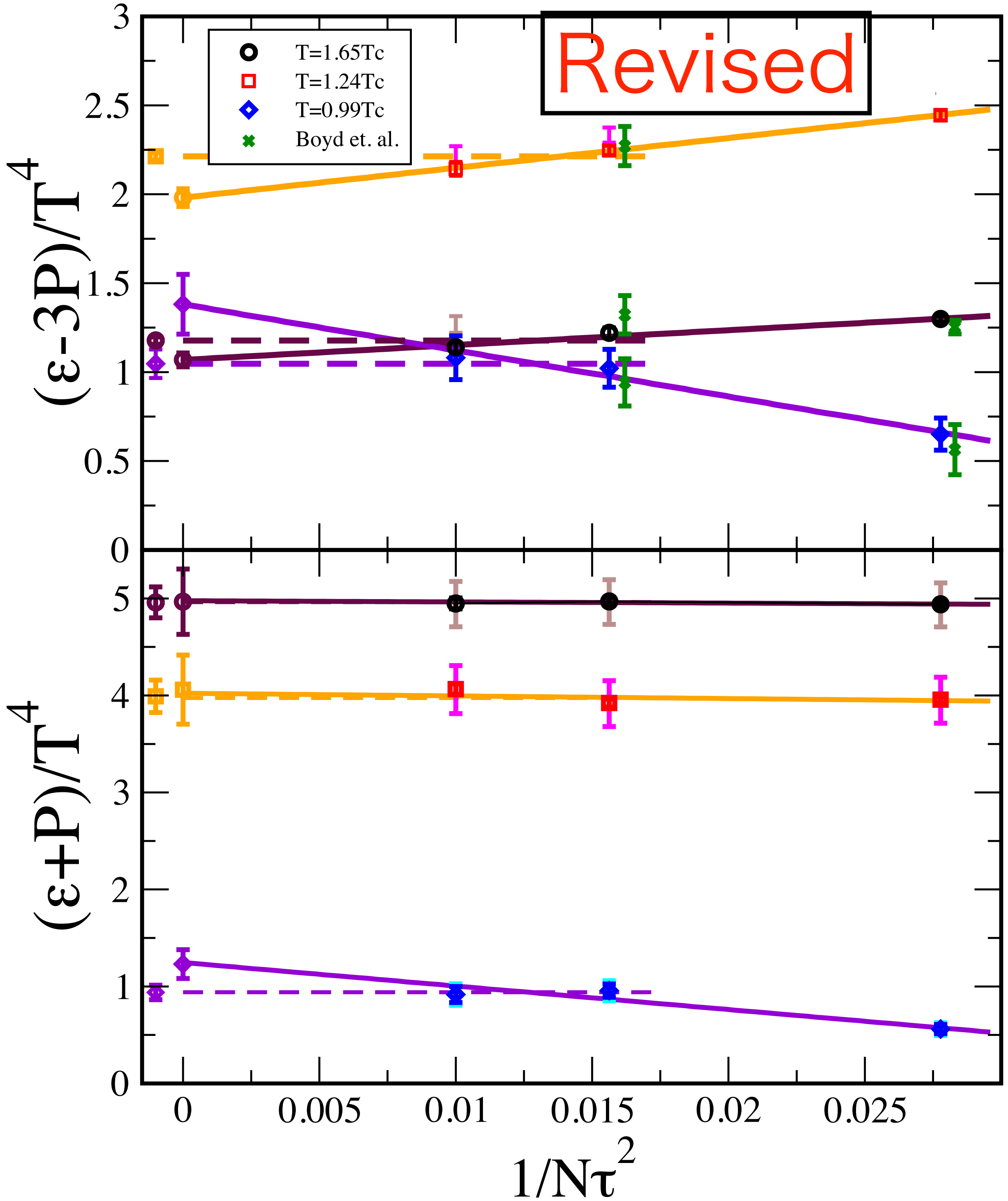
- ✦ Novel method to obtain EMT using the lattice simulation
- ✦ quenched results show that the small flow time expansion is promising
- ✦ clear statistical signal, small systematic error
- ✦ full QCD simulation is also doable!!

future direction

- ✦ combine 5 ops. for full QCD results (need Z-factor for fermion op.)
- ✦ precise parametrization is necessary
- ✦ two-point function of EMT (shear and bulk viscosity, heat capacity)
- ✦ application to conformal field theory (central charge, dilation physics)

backup

Continuum extrapolation



$$\sqrt{8t}T = 0.4$$

- raw data in Boyd et.al. (green)
- linear extrap. of three data points
- systematic error is estimated by constant fit of two data points
- data in different flow time give the consistent results

What is YM gradient flow？

Properties and application

Luescher, (Lattice2013)

- Novel definition of topological charge
- Scale setting (t0, w0)
- Novel renormalized coupling
- UV finiteness (No need wave fn. renormalization for bosonic composite op.)
- Novel calculation of chiral condensate

UV finiteness of the gradient flow

Flow eq. (continuum)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) \quad \text{initial condition} \quad B_\mu(t=0,x) = A_\mu(x)$$

leading order solution in perturbation

$$B_\mu(t,x) = \int d^D y K_t(x-y) A_\mu(y)$$

$$K_t(z) = \int \frac{d^D p}{(2\pi)^D} e^{ipz} e^{-tp^2}$$

$p^2 > 1/t$ modes are suppressed by a smooth UVcutoff

$|x| < \sqrt{8t}$ is smeared

All order finiteness is proven using the extended space-time method

Luescher and Weisz, JHEP 1102, 051(2011)