# Gluon propagators near the phase transition in SU(2) gluodynamics

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- Motivation
- Details of simulations
- Gribov copies
- Extrapolation to the infinite-volume limit
- Check of critical behavior of $M_E$
- Critical behavior of the asymmetry

## **Motivation** to compute lattice gluon propagators

- Relation to confinement via Gribov-Zwanziger (Refined Gribov-Zwanziger) scenario.
- To guide computations within other approaches (Dyson-Schwinger equations) at small and intermediate momenta (scaling solution, decoupling solution).
- Method to compute the running coupling.
- At finite temperatures the interesting quantities to be computed are the gluon screening masses and dimension two gluon condensates – quantities related to the gluon propagators.

# Controversy

Cucchieri, Maas and Mendes, 2007
Fischer, Maas, and Muller, 2010
Maas, Pawlowski, von Smekal, and Spielmann, 2012
Dramatic changes in the electric (longitudinal) gluon propagator in the vicinity of $T_c$ (results for $N_t = 4, 6$). Screening mass changes as

$$m_e = m_0 + \theta(t)\, at^{\kappa_1} + \theta(-t)\, a(-t)^{\kappa_2}$$

Cucchieri and Mendes, 2011, 2012, 2014
No specific signature of deconfinement associated with $D_L(p)$. The data for electric propagator are subject to severe finite lattice spacing effects at $T$ around $T_c$. As a result, only lattices with $N_t \geq 8$ seem to be free from systematic errors. Results with smaller $N_t$ should be revised.

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Maas, Pawlowski, von Smekal, and Spielmann Phys.Rev. D85, 034037 (2012)

R. Aouane, V.G. Bornyakov, E.M. Ilgenfritz, V.K. Mitrjushkin, M. Muller-Preussker, Phys.Rev. D85 (2012) 034501 A. Cucchieri and T. Mendes, Acta Phys. Pol. B Proceedings Supplement V.7 (2014)

P. J. Silva, O. Oliveira, P. Bicudo, and N. Cardoso, Phys.Rev. D89, 074503 (2014)

Wilson action and standard definition of the lattice gauge vector potential $\mathcal{A}_{x+\hat{\mu}/2,\mu}$ (Mandula, 1987):

$$\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2i}\left(U_{x\mu} - U^{\dagger}_{x\mu}\right) \equiv A^{a}_{x+\hat{\mu}/2,\mu}\frac{\sigma_a}{2} \,. \qquad (1)$$

**Landau gauge** fixing condition is

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4}\left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) = 0 \,, \qquad (2)$$

which is equivalent to finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V}\sum_{x\mu}\frac{1}{2}\,\mathrm{Tr}\,U^{g}_{x\mu} \,, \qquad (3)$$

with respect to gauge transformations $g_x$ .

We adopt the strategy of finding gauge copies being as close as possible to the global maximum of the gauge fixing functional - so called **absolute Landau gauge**.

Three improvements in comparison with many other studies but usual in our (Berlin - ITEP - IHEP - Dubna) studies:

- efficient optimization algorithm - simulated annealing.
- many gauge copies per MC configuration with the choice of the one with maximal $F_U$ - *best copy*.
- $Z_2$ flips to use full gauge freedom (with pbc).
  Bogolubsky, Bornyakov, Burgio, Ilgenfritz, Muller-Preussker, V. K. Mitrjushkin, 2008

Additionally, we compute the propagator in the infinite volume limit, for the first time to my knowledge.

On the asymmetric lattice there are two tensor structures for the gluon propagator:

$$D^{ab}_{\mu\nu}(p) = \delta_{ab}\left(P^T_{\mu\nu}(p)D_T(p) + P^L_{\mu\nu}(p)D_L(p)\right) \,, \tag{4}$$

where $P^{T;L}_{\mu\nu}(p)$ - orthogonal transverse (longitudinal) projectors
Two scalar propagators - longitudinal $D_L(p)$ and transverse $D_T(p)$ - are given by

$$D_T(p) = \frac{1}{6}\sum_{a=1}^{3}\sum_{i=1}^{3} D^{aa}_{ii}(p) \,; \qquad D_L(p) = \frac{1}{3}\sum_{a=1}^{3} D^{aa}_{44}(p) \tag{5}$$

For $\vec{p} = 0$ propagators $D_T(0)$ and $D_L(0)$ are defined as follows

$$D_T(0) = \frac{1}{9}\sum_{a=1}^{3}\sum_{i=1}^{3} D^{aa}_{ii}(0)\,; \quad D_L(0) = \frac{1}{3}\sum_{a=1}^{3} D^{aa}_{00}(0). \tag{6}$$

Simulations on $N_s^3 \times N_t$ lattices
$N_t = 8$ (lattice spacing is a bit smaller than 0.1 fm)
$N_s = 32, 40, 48, 56, 64, 78$;
for $\beta = 2.506, 2.510$ we also consider $N_s = 88$.

| $\beta = 4/g^2$ | 2.478 | 2.488 | 2.495 | 2.501 | 2.506 | 2.508 | 2.509 | 2.510 |
|---|---|---|---|---|---|---|---|---|
| $1/a$ (GeV) | 2.143 | 2.213 | 2.263 | 2.307 | 2.344 | 2.359 | 2.367 | 2.374 |
| $T/T_c$ | 0.901 | 0.931 | 0.952 | 0.970 | 0.986 | 0.992 | 0.996 | 0.999 |

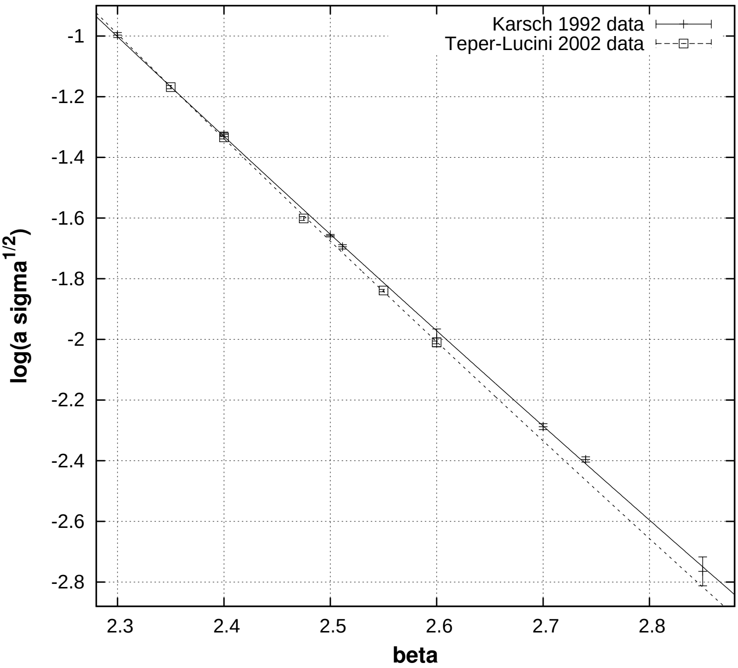
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\beta = 4/g^2$ | 2.511 | 2.512 | 2.513 | 2.515 | 2.518 | 2.521 | 2.527 |
| $1/a$ (GeV) | 2.382 | 2.389 | 2.397 | 2.412 | 2.436 | 2.459 | 2.507 |
| $T/T_c$ | 1.0019 | 1.0051 | 1.0083 | 1.0148 | 1.0246 | 1.0345 | 1.0545 |

## Scale fixing

$$\ln(\sigma a^2) = -\frac{4\pi^2}{b_0}\beta + \frac{2b_1}{b_0^2}\ \ln\left(\frac{4\pi^2}{b_0}\beta\right) + c' + \frac{4\pi^2}{b_0}\frac{d'}{\beta}$$

Results of the fit to the Karsch data gives $\chi^2/N_{dof} = 1.02,\ \ N_{dof} = 6$.
The combined fit using both Karsch and Teper-Lucini data gives

$$\chi^2/N_{dof} = 6.14,\ N_{dof} = 11$$



Karsch 1992 data
Teper-Lucini 2002 data
log(a sigma^1/2)
beta
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
2.3
2.4
2.5
2.6
2.7
2.8

$$T = \frac{\sqrt{\sigma}}{N_t} \exp\left( - \frac{3\pi^2}{11} f(\beta, c, d) \right),$$

$$f(\beta, c, d) = -\beta + \frac{17}{11\pi^2} \ln \beta + c + \frac{d}{\beta} \, .$$

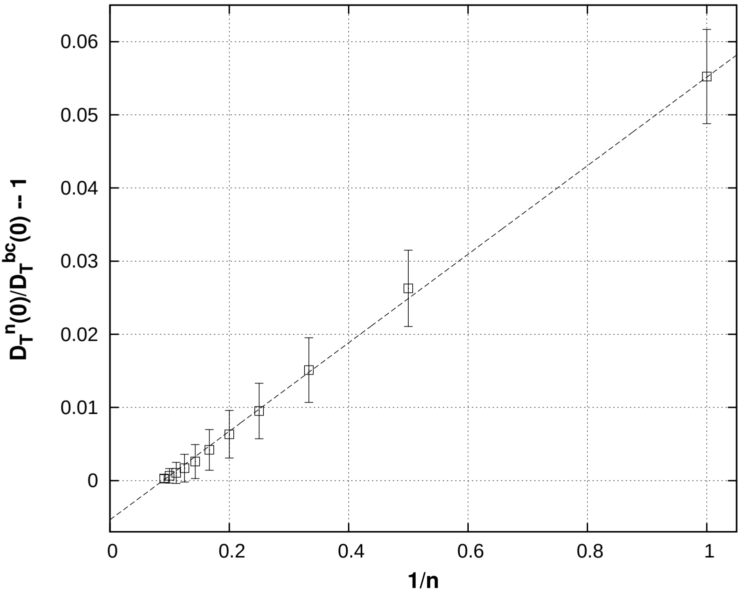
$$c = 0.110(21), \quad d = 1.58(5)$$

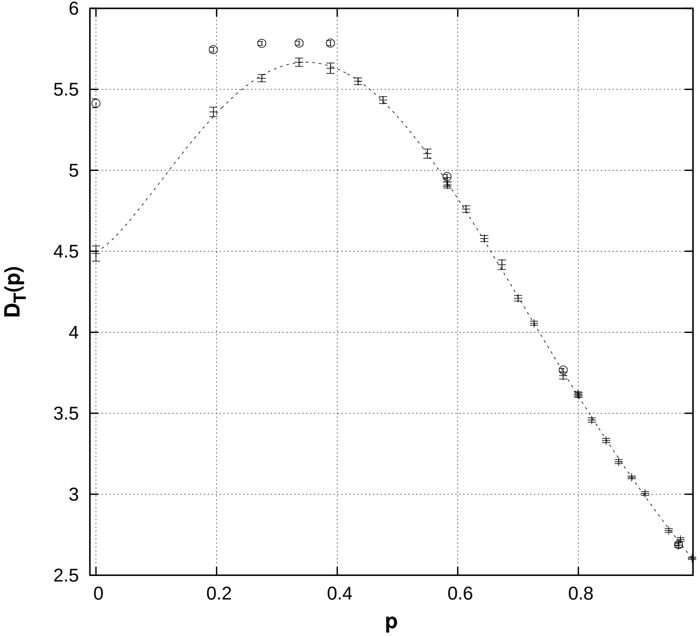
Karsch *et al.* 1992: $\beta_c = 2.5115(40)$, $N_t = 8$
Mitrjushkin *et al.* 1999: $\beta_c = 2.5105(10)$,
Velytsky 2010: $\beta_c = 2.5104(2)$.

| $\beta$ | $\xi = T/T_c$ | error |
|---|---|---|
| 2.478 | 0.901328 | 0.000669 |
| 2.488 | 0.930747 | 0.000476 |
| 2.495 | 0.951881 | 0.000334 |
| 2.506 | 0.986020 | 0.000098 |
| 2.509 | 0.995532 | 0.000032 |
| 2.511 | 1.001921 | 0.000014 |
| 2.512 | 1.005131 | 0.000036 |

Table: We consider $\beta_c = 2.5104$ exactly.

# The effect of **Gribov copies**





6
5.5
5
4.5
4
3.5
3
2.5
D_T(p)
0
0.2
0.4
0.6
0.8
p

$$\Delta_{T,L}(p) = \frac{D^{fc}_{T,L}(p) - D^{bc}_{T,L}(p)}{D^{bc}_{T,L}(p)} \ , \tag{7}$$

We present $\Delta_{T,L}(p)$ as a sum

$$\Delta_{T,L}(p) = \mathcal{Z}_{T,L}(p) + \mathcal{Y}_{T,L}(p) \tag{8}$$

where

$$\mathcal{Z}_{T,L}(p) = \frac{D^{fc}_{T,L}(p) - D^{bc}_{T,L}(p, n_c = 1)}{D^{bc}_{T,L}(p)} \ , \tag{9}$$

$$\mathcal{Y}_{T,L}(p) = \frac{D^{bc}_{T,L}(p, 1) - D^{bc}_{T,L}(p, n_c)}{D^{bc}_{T,L}(p)} \ , \tag{10}$$

| Lattice | $n_c$ | $\mathcal{Z}_L(0)$ | $\mathcal{Y}_L(0)$ | $\mathcal{Z}_T(0)$ | $\mathcal{Y}_T(0)$ |
|---|---|---|---|---|---|
| $32^3, 2.506$ | 4 | -0.127(13) | -0.009(7) | 0.469(8) | 0.014(4) |
| $40^3, 2.511$ | 3 | -0.115(20) | 0.006(19) | 0.419(11) | 0.020(9) |
| $48^3, 2, 501$ | 3 | -0.018(19) | -0.039(30) | 0.361(13) | 0.061(17) |
| $56^3, 2.501$ | 12 | -0.059(12) | -0.040(29) | 0.332(8) | 0.121(14) |
| $64^3, 2.501$ | 2 | -0.061(18) | -0.010(35) | 0.311(13) | 0.071(19) |
| $78^3, 2.511$ | 3 | -0.026(15) | 0.054(47) | 0.214(11) | 0.192(25) |
| $88^3, 2.506$ | 2 | -0.034(19) | 0.065(41) | 0.187(11) | 0.109(24) |

Table: The flip effect $\mathcal{Z}$ and the copy effect $\mathcal{Y}$ for the longitudinal and transverse propagators at $p = 0$ for various lattices.

Normalization conditions for the longitudinal propagator

- (MOM): $$D^{MOM}(p)|_{p^2=\mu^2} = \frac{1}{\hat{p}^2}, \qquad \hat{p}_\mu = \frac{2}{a}\sin\frac{p_\mu a}{2}.$$
- (NAT): $$\frac{1}{D^{NAT}(p)} = m_e^2 + p^2 + \underline{O}(p^4).$$

$$D^{NAT}(p) = Z_{NAT} D^{bare}(p), \qquad D^{MOM}(p) = Z_{MOM} D^{bare}(p)$$

we consider three masses,

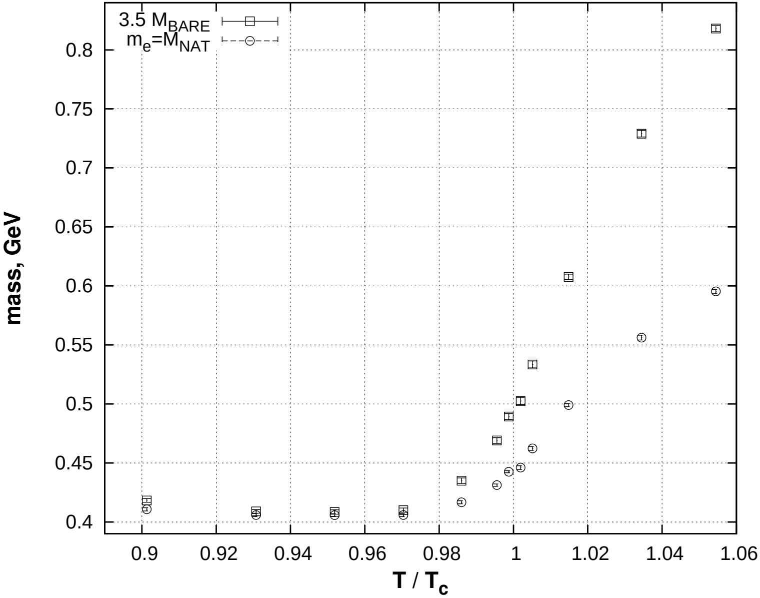
$$M_{bare} = \frac{1}{\sqrt{D^{bare}(0)}}, \tag{11}$$

Fischer, Maas and Muller, 2010

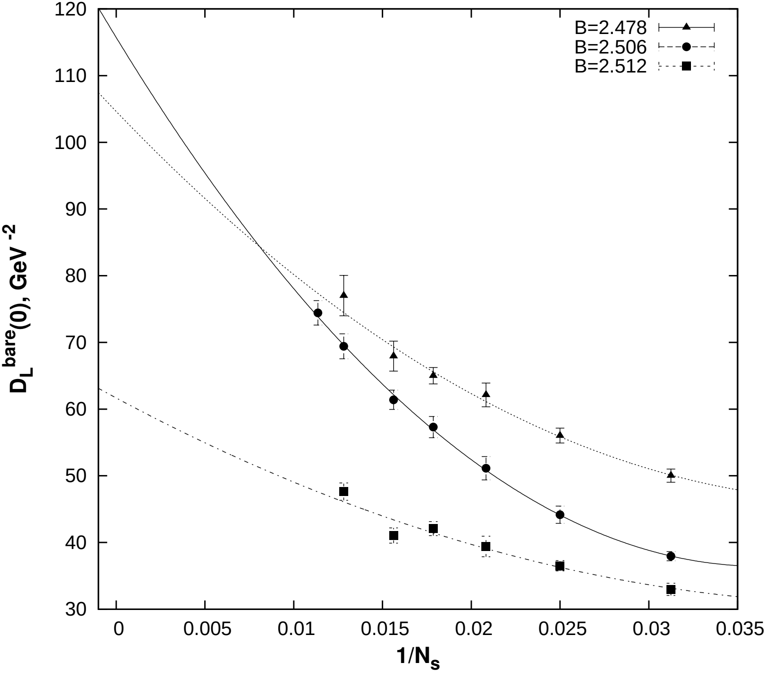
$$M_{NAT} \equiv m_e = \frac{1}{\sqrt{D^{NAT}(0)}} = \frac{M_{bare}}{\sqrt{Z_{NAT}}}, \tag{12}$$

Bornyakov, Mitrjushkin, 2011;
Silva, Oliveira, Bicudo, and Cardoso, 2014

$$M_{MOM} = \frac{1}{\sqrt{D^{MOM}(0)}} = \frac{M_{bare}}{\sqrt{Z_{MOM}}}. \tag{13}$$




Mass at $N_s = 64$; lattice size $b = 5.0 \div 5.8 fm$



B=2.478
B=2.506
B=2.512
$D_L^{bare}(0)$, GeV$^{-2}$
$1/N_s$
120
110
100
90
80
70
60
50
40
30
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035

## Extrapolation to the infinite-volume limit

$$\frac{1}{D(p;N_s)} \simeq c_{00} + c_{20}p^2 + c_{40}p^4 + c_{60}p^6 + c_{01}\frac{1}{N_s} + c_{21}\frac{p^2}{N_s} + c_{41}\frac{p^4}{N_s}$$

$$\frac{1}{D(p;N_s)} \simeq c_{00} + c_{20}p^2 + c_{40}p^4 + c_{60}p^6 + c_{01}\frac{1}{N_s} + c_{21}\frac{p^2}{N_s} + c_{41}\frac{p^4}{N_s} + c_{22}\frac{p^2}{N_s^2} + c_{42}\frac{p^4}{N_s^2}$$

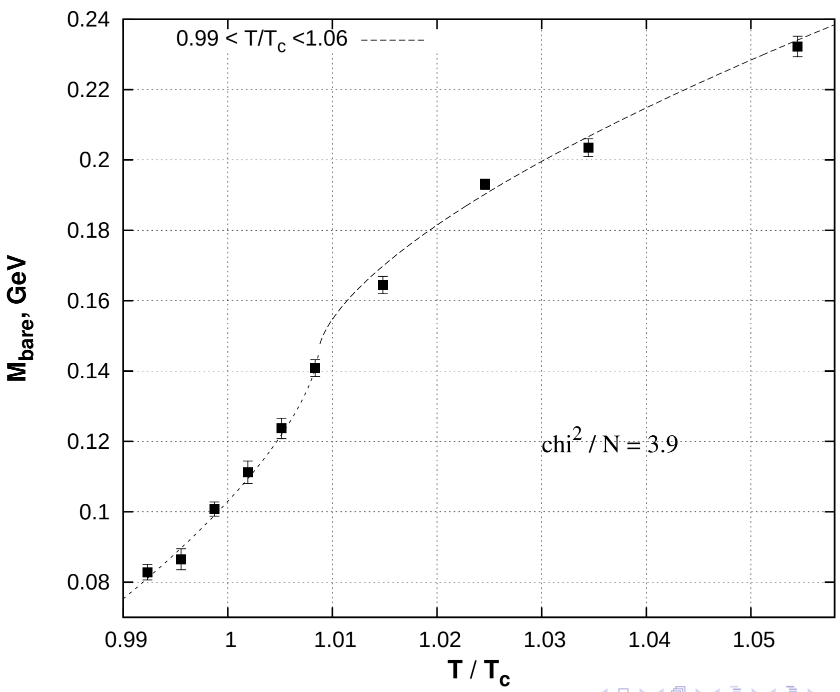
## RESULTS

We use the fit function

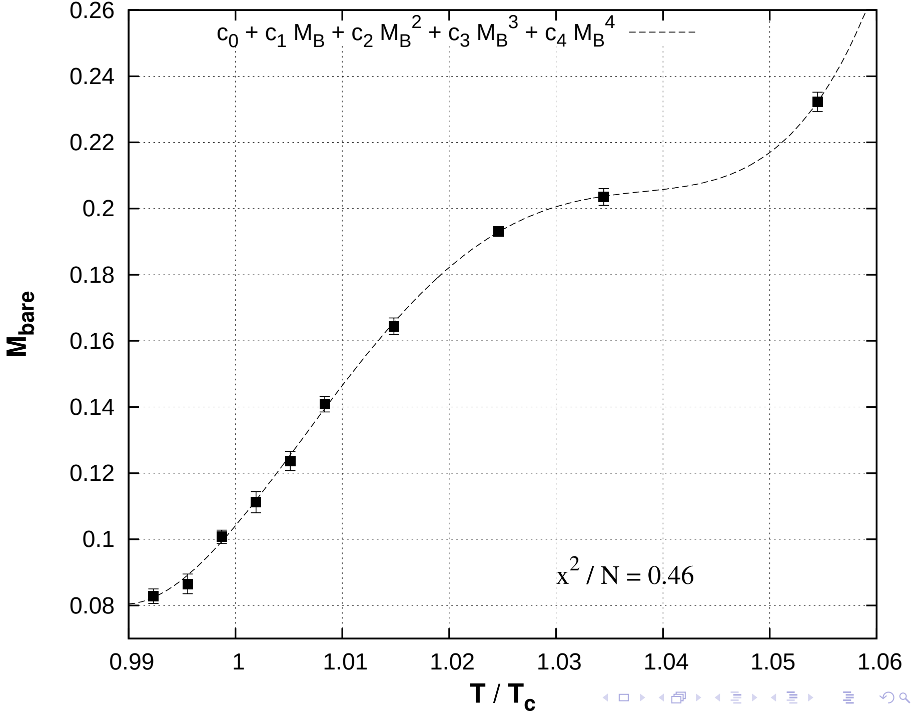
- at $T/T_c > d$ $\quad M_{BARE}(T) = m_{BARE} + c(T/T_c - d)^b$
- at $T/T_c < d$ $\quad M_{BARE}(T) = m_{BARE} - q(d - T/T_c)^b$

Parameters obtained in the fit:

- $m_{BARE} = 0.146(4)$ GeV;
- $d = 1.0087(4)$;
- $b = 0.65(4)$;
- $c = 0.65(8)$ GeV;
- $q = 0.94(18)$ GeV;



0.99 < T/T$_c$ <1.06
chi$^2$ / N = 3.9
M$_{bare}$, GeV
T / T$_c$
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.99
1
1.01
1.02
1.03
1.04
1.05



$c_0 + c_1 M_B + c_2 M_B^2 + c_3 M_B^3 + c_4 M_B^4$
$x^2 / N = 0.46$
$M_{bare}$
$T / T_c$
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.99
1
1.01
1.02
1.03
1.04
1.05
1.06

- critical behavior:

$$\frac{\chi^2}{N_{d.o.f}} = 3.90$$

versus

- smooth behavior (polynomial fit function)

$$\frac{\chi^2}{N_{d.o.f}} = 0.46$$

We see no critical behavior of $M_{bare}(T)$, only crossover.

# Asymmetry

Electric-magnetic asymmetry ( Chernodub, Ilgenfritz, 2008)

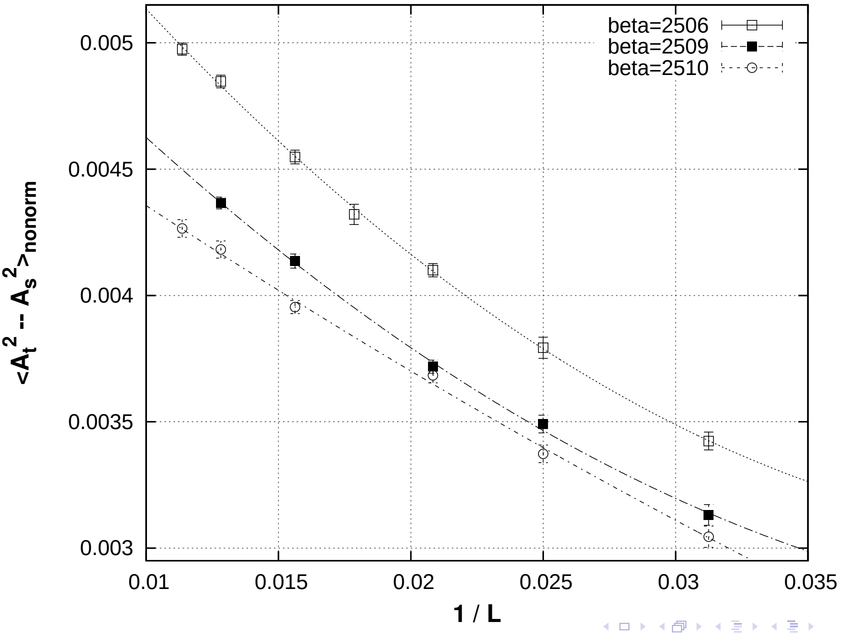
$$\mathcal{A} = \frac{g^2}{V\ T_c^2} \int dx\ \left(A_4^a(x) A_4^a(x) - \frac{1}{3} A_i^a(x) A_i^a(x)\right)\ .$$

on a lattice is rearranged to

$$\langle \mathcal{A} \rangle\ = 4 N_t^2 \xi^2 \langle \Delta u^2 \rangle,$$

$$\Delta u^2 = \frac{1}{N_s^3 N_t} \sum_{x,t} \sum_{a=1}^{3} \left( u_4^a(t,x) u_4^a(t,x) - \frac{1}{3}\ \sum_{i=1}^{3} u_i^a(t,x) u_i^a(t,x) \right)\ ;$$

$$U_{t,x;\mu} = u_\mu^0(t,x) + i u_\mu^a(t,x) \sigma^a$$


beta=2506
beta=2509
beta=2510
0.005
0.0045
0.004
0.0035
0.003
0.01
0.015
0.02
0.025
0.03
0.035
1 / L
$<A_t^2 -- A_s^2>_{nonorm}$

To determine the infinite-volume limit of $\mathcal{A}$, we compare two fit functions:

$$L: \qquad f_L(N_s) = c_0 + \frac{c_1}{N_s} \tag{14}$$

$$Q: \qquad f_Q(N_s) = d_0 + \frac{d_1}{N_s} + \frac{d_2}{N_s^2} \tag{15}$$

- the Fisher criterion gives clear-cut evidence for the fit function Q.

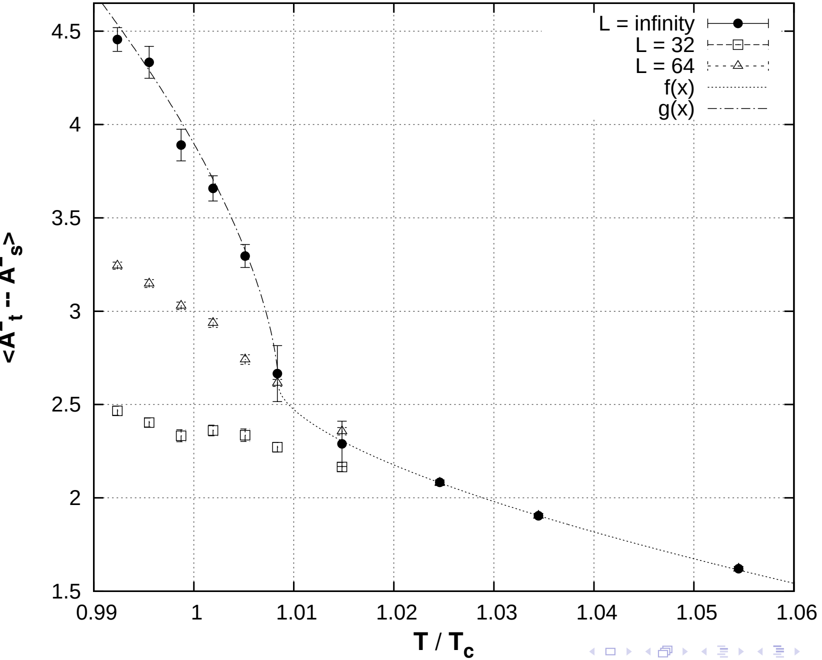
- at $\xi > d$  $\mathcal{A}(\xi) = a + c(\xi - d)^b$
- at $\xi < d$  $\mathcal{A}(\xi) = a - q(d - \xi)^b$

Parameters obtained in the fit:

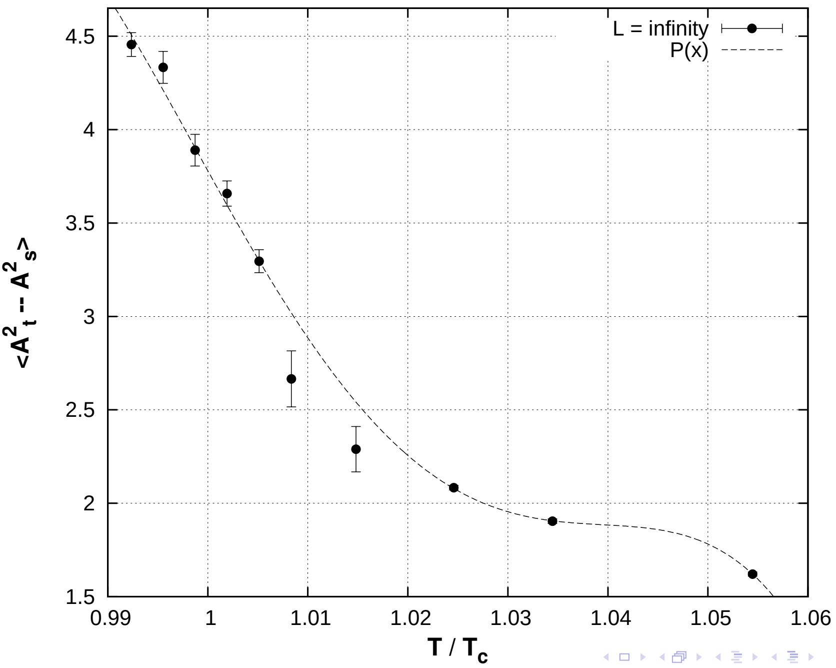
- $a = 2.59(9)$, $d = 1.0085(7)$;
- $b = 0.62(5)$; $c = 6.59(58)$, $q = 25.14(6.04)$
- critical behavior: $\frac{\chi^2}{N_{d.o.f}} = 1.00$

versus

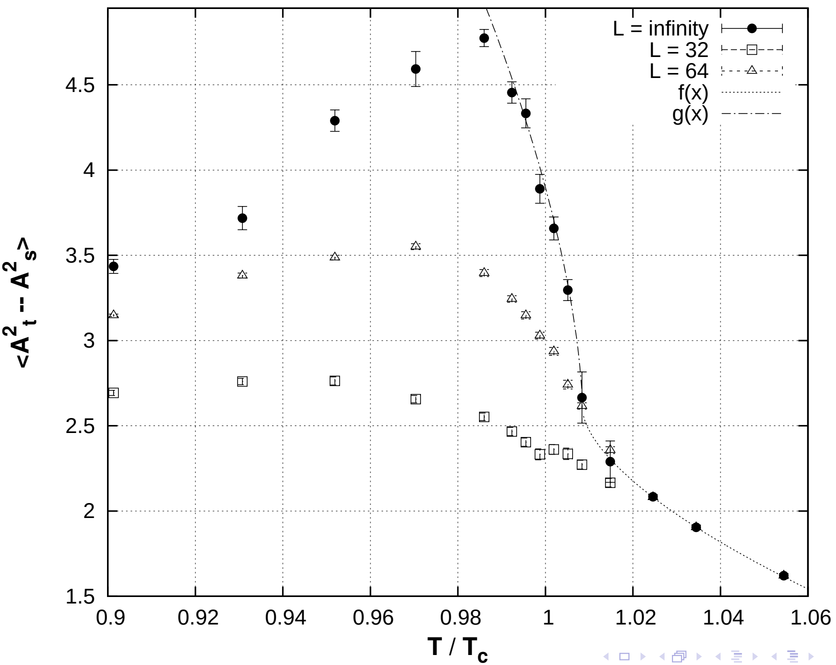
- smooth behavior (polynomial fit function)
  $\frac{\chi^2}{N_{d.o.f}} = 2.72$



L = infinity
L = 32
L = 64
f(x)
g(x)
$<A^2_t - A^2_s>$
T / $T_c$
4.5
4
3.5
3
2.5
2
1.5
0.99
1
1.01
1.02
1.03
1.04
1.05
1.06



L = infinity
P(x)
4.5
4
3.5
3
2.5
2
1.5
0.99
1
1.01
1.02
1.03
1.04
1.05
1.06
<A2t -- A2s>
T / Tc



L = infinity
L = 32
L = 64
f(x)
g(x)
4.5
4
3.5
3
2.5
2
1.5
0.9
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
T / $T_c$
$<A_t^2 - A_s^2>$

# Conclusions

- The effects of Gribov copies for $D_T(p)$ are significant at $p < 0.5 GeV$, the effects of flip sectors for $D_{44}(p)$ do not vanish at small lattices.
- The **infinite volume limit** for both $D_L(p)$ and $\mathcal{A}$ is evaluated for the first time in the vicinity of the phase transition, $0.93T_c < T < 1.008T_c$ .
- We reduce errors in $D_L(p, \infty)$ by fitting $D_L(p, N_s)$ as a function of two variables and performing regression analysis.
- We see that smooth behavior of the electric screening mass is more compatible with our data **However,** a more thorough extrapolation to the infinite volume is needed (larger statistics etc.).
- In the infinite-volume limit we find a critical behavior of $\mathcal{A}$; the respective critical temperature and index coincide, with those coming from the analogous fit to $m_e(T)$.
- The maximum of $\mathcal{A}$ occurs at $T = 0.979(5)T_c$.