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Analysis of short-distance current correlators using OPE

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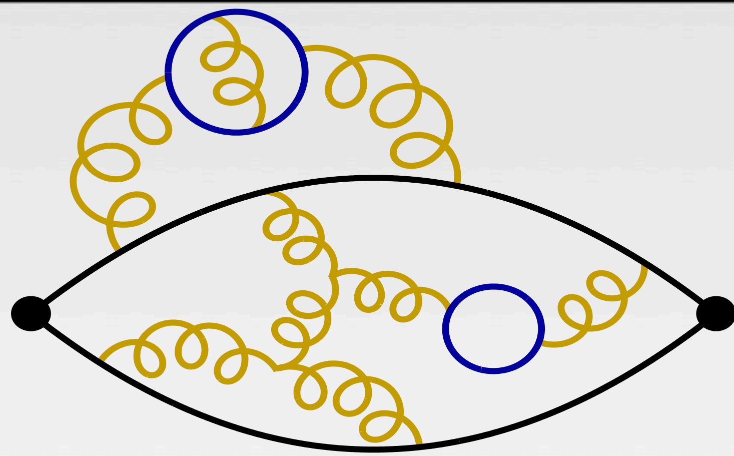
Current Correlators



High-energy

- Perturbative at short-distance

  (Continuum PT well-developed, 4-loop)

0.1fm

▲ OPE in the middle

Test & use
This region

1fm

- Non-perturbative at long-distance ($\gtrsim 1/\Lambda_{\rm QCD}$)

  ChPT

Low-energy

Operator Product Expansion (OPE)

$$\Pi(x) = \underbrace{\frac{c_0(\alpha_s)}{x^6}} + \frac{c_2 m_{\mathrm{q}}^2}{x^4} + \underbrace{\frac{c_{4,\bar{\mathrm{q}}\mathrm{q}} m_{\mathrm{q}} \langle \bar{\mathrm{q}}\mathrm{q} \rangle + c_{4,\mathrm{G}} \langle \mathrm{GG} \rangle + \cdots}{x^2}} + \cdots$$

Massless Perturbation (up to 4-loop)

tiny mass correction

First non-perturbative effect through condensates

- Test this theoretical formulation against the "experiment" on the lattice
- Use as an input for NPR

Two-point Functions

- Definitions

$$\Pi_{\mathrm{S}}(x) = \langle S(x)S(0)^\dagger \rangle, \qquad \Pi_{\mathrm{P}}(x) = \langle P(x)P(0)^\dagger \rangle,$$

$$\Pi_{\mathrm{V},\mu\nu}(x) = \langle V_\mu(x)V_\nu(0)^\dagger \rangle, \qquad \Pi_{\mathrm{A},\mu\nu}(x) = \langle A_\mu(x)A_\nu(0)^\dagger \rangle,$$

$$\Pi_{\mathrm{V/A}}(x) = \sum_\mu \Pi_{\mathrm{V/A},\mu\mu}(x)$$

- Non-singlet local operators are used

$$S(x) = \bar{\mathrm{u}}\mathrm{d}(x), \qquad P(x) = \bar{\mathrm{u}}\mathrm{i}\gamma_5\mathrm{d}(x),$$

$$V_\mu(x) = \bar{\mathrm{u}}\gamma_\mu\mathrm{d}(x), \qquad A_\mu(x) = \bar{\mathrm{u}}\gamma_\mu\gamma_5\mathrm{d}(x)$$

- Degeneracy
  - Massless perturbation

$$\Pi_{\mathrm{S}} = \Pi_{\mathrm{P}}, \quad \Pi_{\mathrm{V},\mu\nu} = \Pi_{\mathrm{A},\mu\nu}$$

  - Non-perturbative effects breaks these degeneracies

Outline

0. Set up
   - Ensembles: 2+1-flavor configs with Möbius domain-wall fermion

1. Test of OPE
   - Subtraction/Elimination of discretization effects at short distances
   - Convergence of perturbation theory
   - Effect of condensates

2. Renormalization
   - Discrimination of discretization effects and non-perturbative
   - Z_V, Z_A
   - Z_S, Z_P

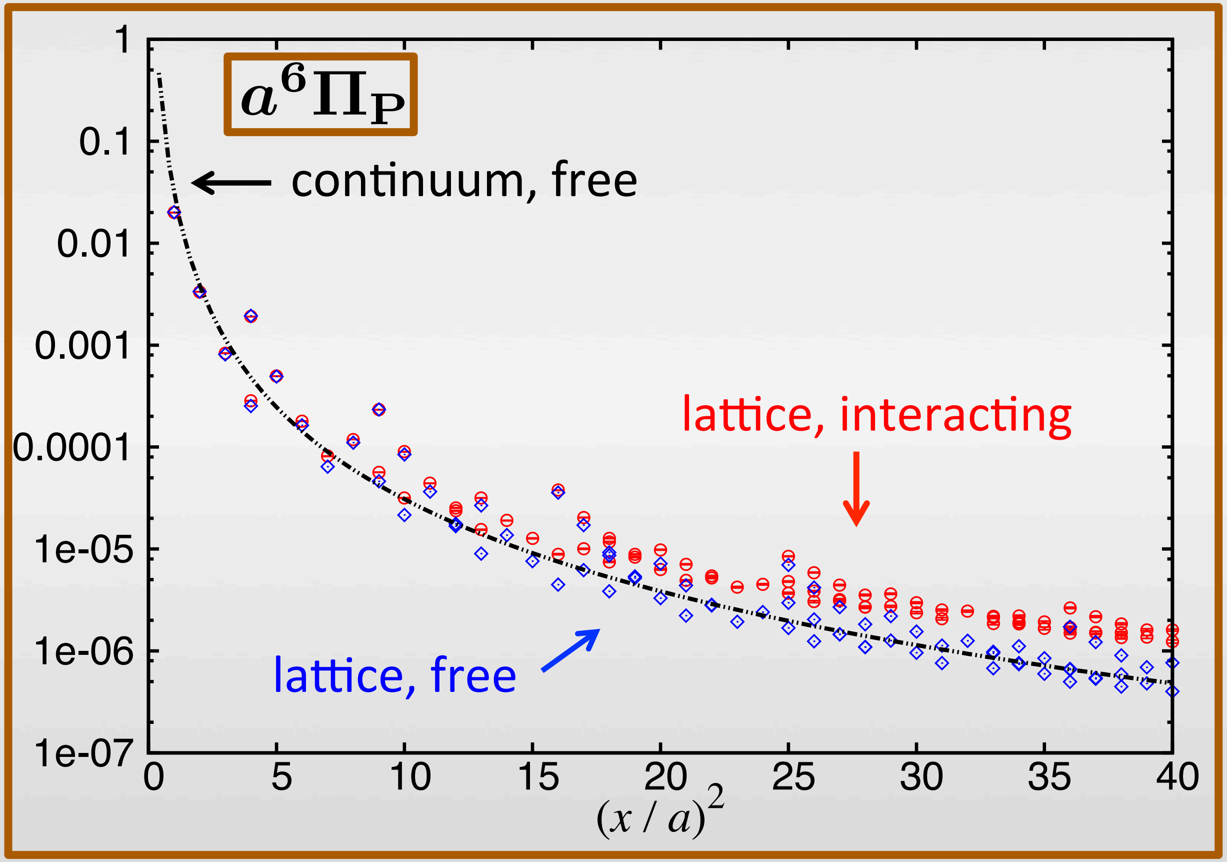
Lattice Setup

- 2+1 Möbius DW fermions w/ 3 times stout smeared links
- Symanzik improved gauge action
- 3 kinds of different cutoff w/ nearly same physical volume

a^{-1} [GeV]	Volume	am_s	am_{ud} (M_π[MeV])
2.453(4)	32^3x64x12	0.0300	0.0070 (300), 0.0120 (400), 0.0190 (500)
		0.0400	0.0035(230), 0.0070(300), 0.0120 (400), 0.0190 (500)
	48^3x96x12	0.0400	0.0035 (230)
3.610(9)	48^3x96x8	0.0180	0.0042 (300), 0.0080 (400), 0.0120 (500)
		0.0250	0.0042 (300), 0.0080 (400), 0.0120 (500)
4.496(9)	64^3x128x8	0.0150	0.0030 (300)

Discretization effect

- Subtraction of tree-level discretization effect

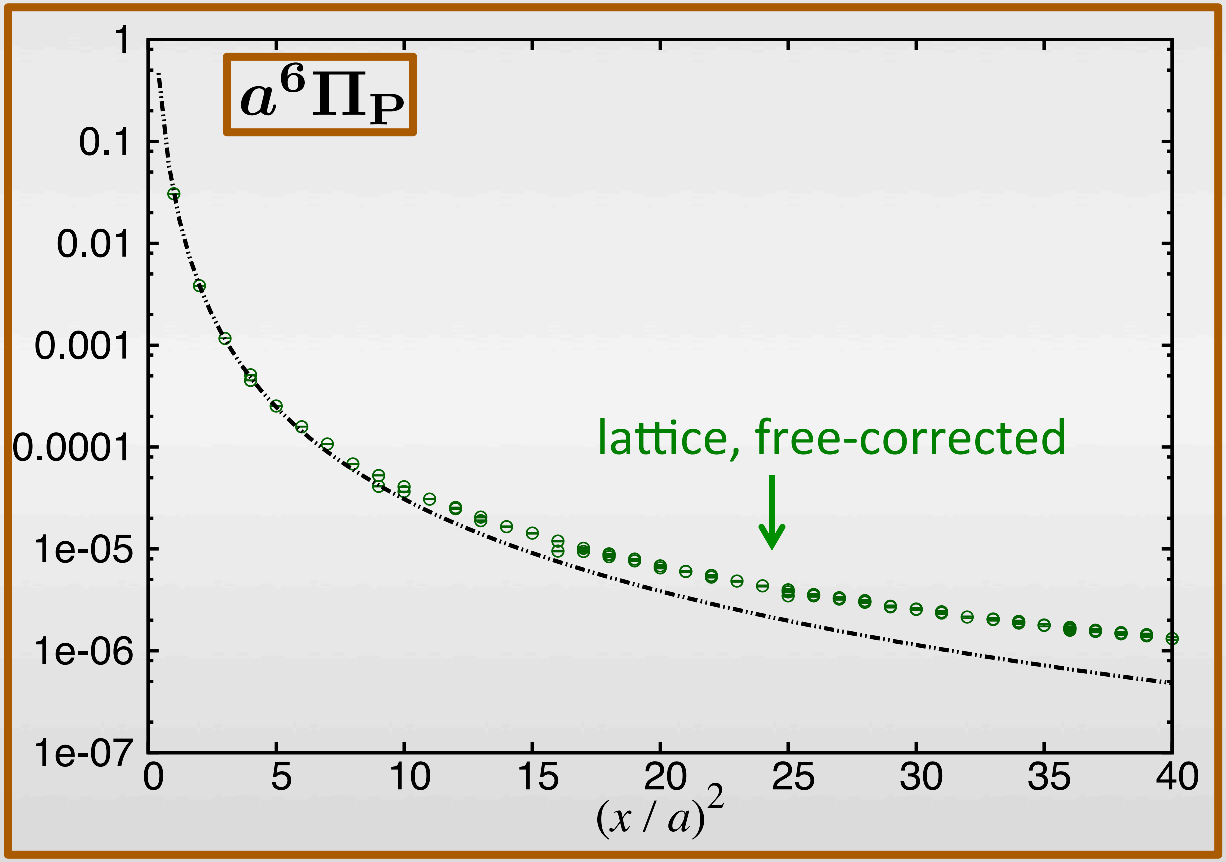


✦ Replace

$$\Pi_\Gamma^{lat}(x) \longrightarrow \Pi_\Gamma^{lat}(x) - \left(\Pi_\Gamma^{lat,free}(x) - \Pi_\Gamma^{cont,free}(x)\right)$$

Discretization effect

- Subtraction of tree-level discretization effect



✦ Replace

$$\Pi_\Gamma^{lat}(x) \longrightarrow \Pi_\Gamma^{lat}(x) - \left(\Pi_\Gamma^{lat,free}(x) - \Pi_\Gamma^{cont,free}(x)\right)$$

Continuum Perturbation

- Perturbative series

$$\Pi = c_0 + c_1 a_s(\mu) + c_2 a_s(\mu)^2 + \cdots$$

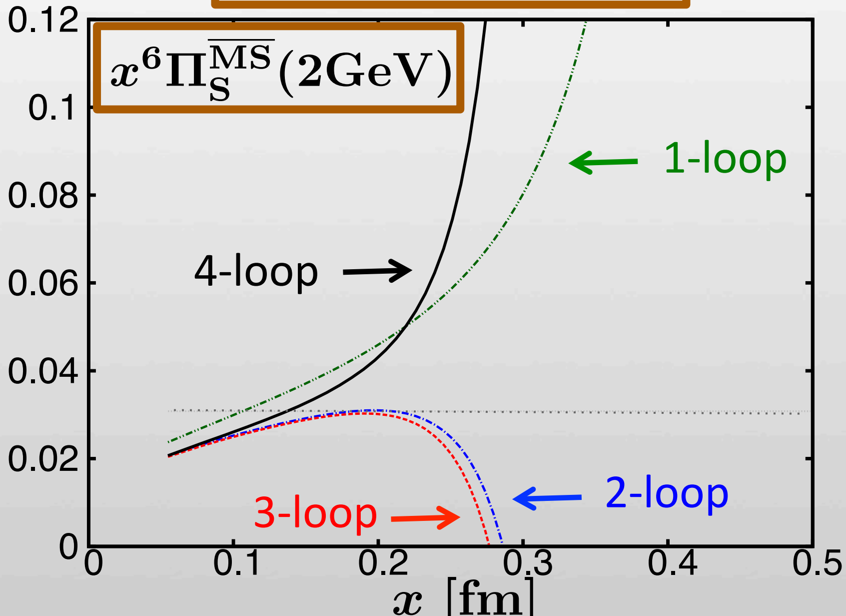
$$a_s(\mu) = a_s(\mu^*) + O(a_s(\mu^*)^2)$$

Ref: Brodsky et al, 1983

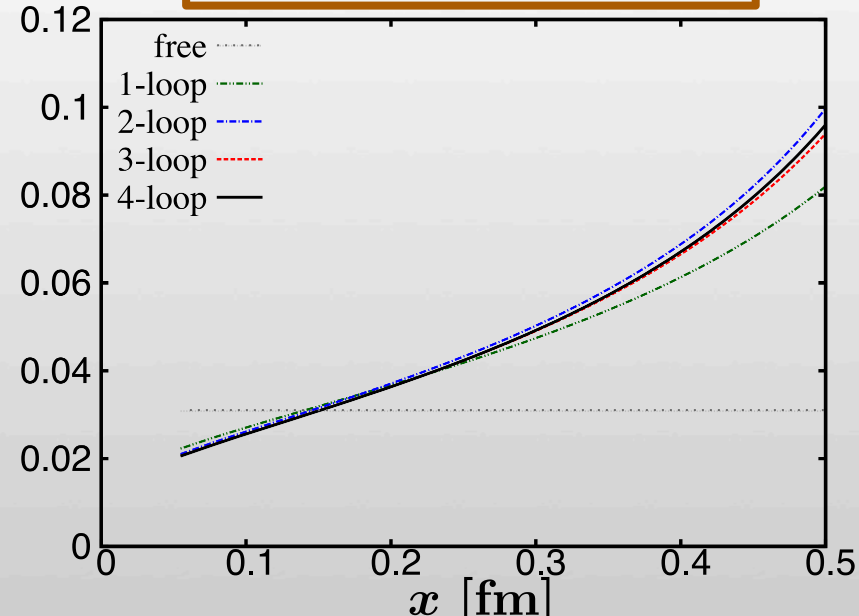
$$= c_0 + c_1^* a_s(\mu^*) + c_2^* a_s(\mu^*)^2 + \cdots$$ ← good convergence if c_i^* : small

- μ^* : arbitrary

Coupling scale $\mu = x^{-1}$

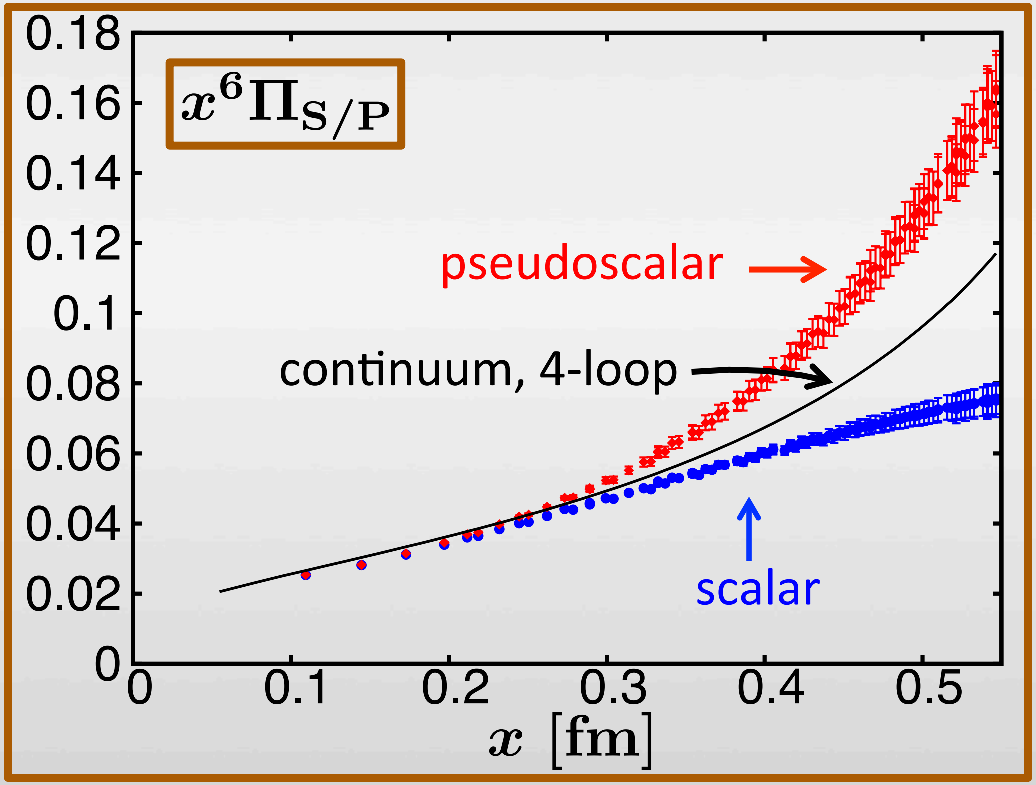


Optimal coupling scale μ^*



Lattice V.S. Continuum

- Comparison
  - ✦ Lattice : a^{-1} = 3.61 GeV, $M\pi$ = 300 MeV, RC is multiplied
  - ✦ Continuum : $\overline{\text{MS}}$ 2GeV, 4-loop



- Non-perturbative effect → Gap between 2 channels

⇒ Check consistency with OPE

OPE — (Axial-)Vector Channel

- Momentum space

$$\begin{aligned}\Pi_{\Gamma,\mu\nu}(q) &= \int \mathrm{d}x\ \mathrm{e}^{-\mathrm{i}qx}\ \Pi_{\Gamma,\mu\nu}(x) \\ &= (\delta_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_\Gamma^{(1)}(q^2) - q_\mu q_\nu \Pi_\Gamma^{(0)}(q^2) \\ &= \underbrace{(\delta_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_\Gamma^{(1+0)}(q^2)}_{\text{Conserved}} - \underbrace{\delta_{\mu\nu}q^2\Pi_\Gamma^{(0)}(q^2)}_{\text{Non-conserved}}\end{aligned}$$

$$\Pi_\Gamma^{(1+0)}(q^2) \equiv \Pi_\Gamma^{(1)}(q^2) + \Pi_\Gamma^{(0)}(q^2)$$

- Non-conserved part

$$q_\mu \Pi_{\Gamma,\mu\nu}(q) = q_\nu q^2 \Pi_\Gamma^{(0)}(q^2)$$

$$\Pi_\mathrm{V}^{(0)}(q^2) = 0$$

$$\Pi_\mathrm{A}^{(0)}(q^2) = \frac{4m_\mathrm{q}\langle\bar{\mathrm{q}}\mathrm{q}\rangle}{q^4} + O(m_\mathrm{q}^2)$$

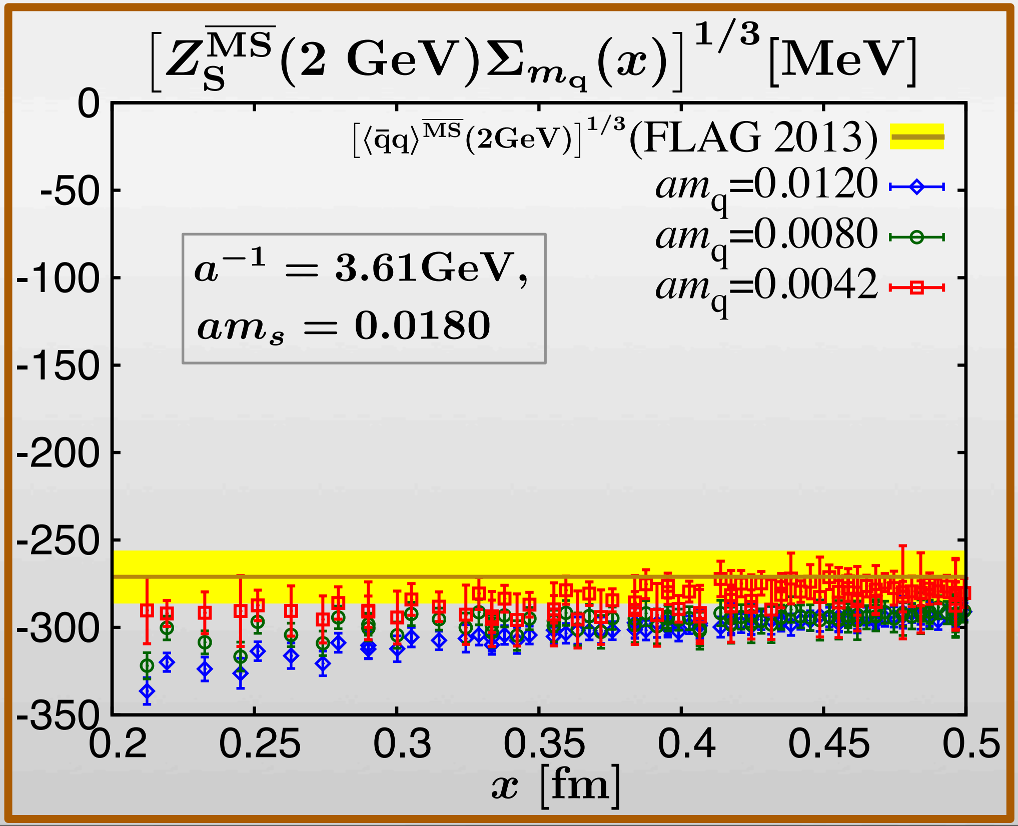
OPE — (Axial-)Vector Channel

- Non-conserved part in X-space

$$\boxed{\Sigma_{m_q}(x) \equiv -\frac{\pi^2}{2m_q} x^2 x_\nu \partial_\mu \Pi_{A-V,\mu\nu}(x) = \langle \bar{q}q \rangle + O(m_q)\cdot O(x^{-2})}$$

$$\Pi_{A-V,\mu\nu}(x) \equiv \Pi_{A,\mu\nu}(x) - \Pi_{V,\mu\nu}(x)$$

- Very close to FLAG's value of $<\bar{q}q> = [-271(15)\ \mathrm{MeV}]^3$
- Vector & Axial-vector : consistent with OPE
- How about scalar & pseudo-scalar?



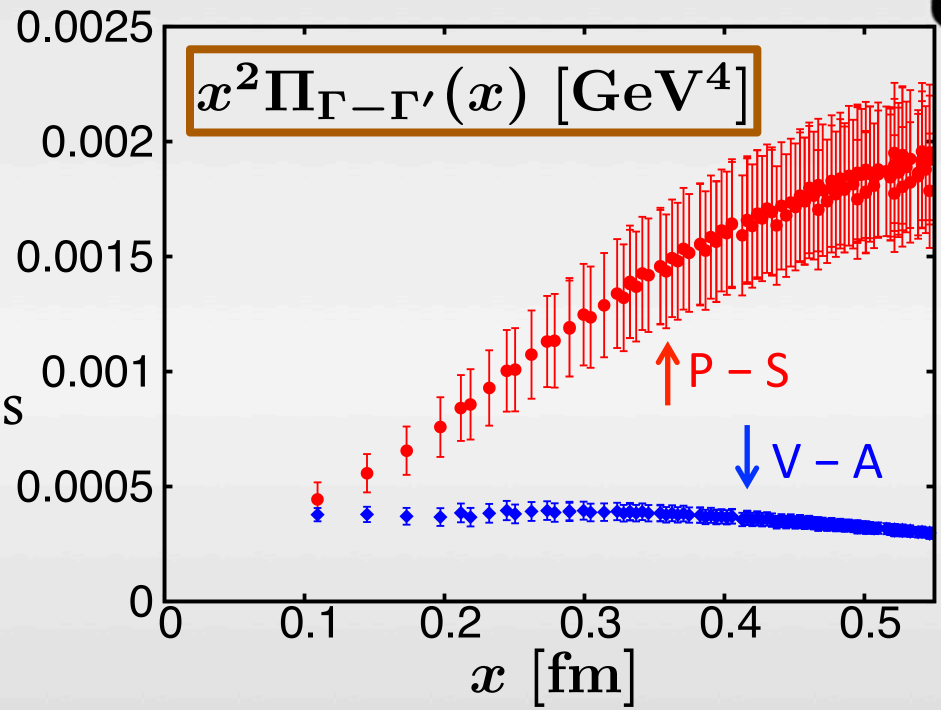
OPE — (Pseudo-)Scalar Channel

- Gap : $\Pi_{\Gamma-\Gamma'}(x) \equiv \Pi_{\Gamma}(x) - \Pi_{\Gamma'}(x)$

  - OPE : $\Pi_{P-S} \sim 0.5\,\Pi_{V-A}$
  - Lattice : $\Pi_{P-S} \gg \Pi_{V-A}$

  $P-S$: too large

  - Known on quenched lattices
    Ref: Chu et al, 1993;
    Faccioli, DeGrand, 2003
  - Not understood
  - Maybe related to $U(1)_A$ anomaly
  - Detailed study underway



Renormalization in X-space

- Renormalization to $\overline{\mathrm{MS}}$ scheme

$$\mathcal{O}^{lat}(1/a) \to \mathcal{O}^{\overline{\mathrm{MS}}}(\mu) = Z^{\overline{\mathrm{MS}}/lat}(1/a \to \mu)\mathcal{O}^{lat}(1/a)$$

- Renormalization condition in X-space method
  (Martinelli et al '97, Giménez et al 2004, Cichy et al 2012)

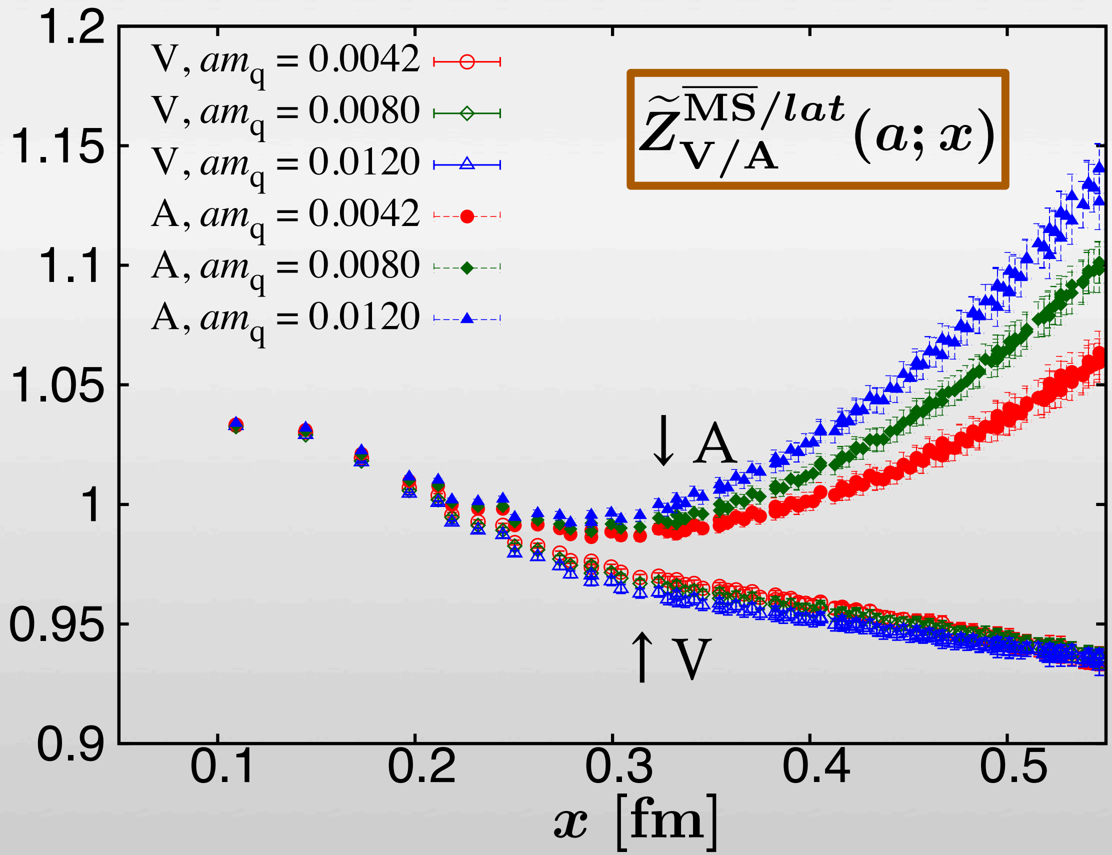
$$Z_\Gamma^{\overline{\mathrm{MS}}/lat}(\mu)^2 \Pi_\Gamma^{lat}(x) = \Pi_\Gamma^{\overline{\mathrm{MS}}}(\mu;x) \Rightarrow \boxed{Z_\Gamma^{\overline{\mathrm{MS}}/lat}(\mu) = \sqrt{\frac{\Pi_\Gamma^{\overline{\mathrm{MS}}}(\mu;x)}{\Pi_\Gamma^{lat}(x)}}}$$

- Advantage
  - Renormalization by gauge invariant quantities
  - We know $\Pi_\Gamma^{\overline{\mathrm{MS}}}(\mu;x)$ to 4-loop level (Chetyrkin, Maier, 2011)

- Window problem : we need to extract Z_Γ from $\underset{①}{a \ll} x \underset{②}{\ll} \Lambda_{\mathrm{QCD}}^{-1}$
  in order to avoid ① discretization effect,
  ② non-perturbative effect

Renormalization of Vector Current

- $Z_{\mathbf{V}}^{\overline{\mathrm{MS}}} = Z_{\mathbf{A}}^{\overline{\mathrm{MS}}}$: scale independent
- x-dependence of $\sqrt{\dfrac{\Pi_{\mathbf{V}}^{\overline{\mathrm{MS}}}(x)}{\Pi_{\mathbf{V/A}}^{lat}(x)}} \equiv \widetilde{Z}_{\mathbf{V/A}}^{\overline{\mathrm{MS}}/lat}(a;x)$: not a constant



Renormalization of Vector Current

- OPE

Too small $\Rightarrow$ neglected

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{\mathrm{V/A}}(a;x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(a) + \cancel{c_2^{\mathrm{V/A}} m_{\mathrm{q}}^2 x^2}$$

$$+(c^{\mathrm{V/A}}_{4,\bar{\mathrm{q}}\mathrm{q}} m_{\mathrm{q}} \langle \bar{\mathrm{q}}\mathrm{q} \rangle + c^{\mathrm{V/A}}_{4,\mathrm{G}} \langle \mathrm{GG} \rangle + \cancel{c^{\mathrm{V/A}}_{4,m} m_{\mathrm{q}}^4}) x^4 + \cdots$$

$$\underline{c^{\mathrm{V}}_{4,\bar{\mathrm{q}}\mathrm{q}} / c^{\mathrm{A}}_{4,\bar{\mathrm{q}}\mathrm{q}} = -3/5}$$

$$\Rightarrow \widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(5\mathrm{V}+3\mathrm{A})/8}(a;x) \equiv \sqrt{\frac{\Pi^{\overline{\mathrm{MS}}}_{\mathrm{V}}(x)}{\left(5\Pi^{lat}_{\mathrm{V}}(x) + 3\Pi^{lat}_{\mathrm{A}}(x)\right)/8}}$$

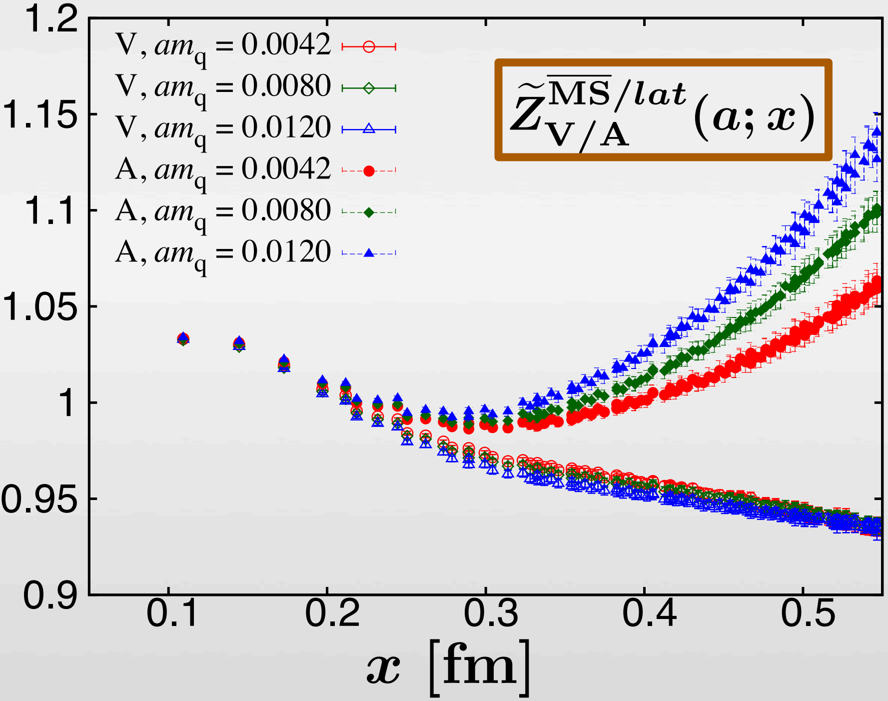
$$= Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(a) + 0 \cdot m_{\mathrm{q}} \langle \bar{\mathrm{q}}\mathrm{q} \rangle x^4 + c_{4,\mathrm{G}} \langle \mathrm{GG} \rangle x^4 + \cdots$$

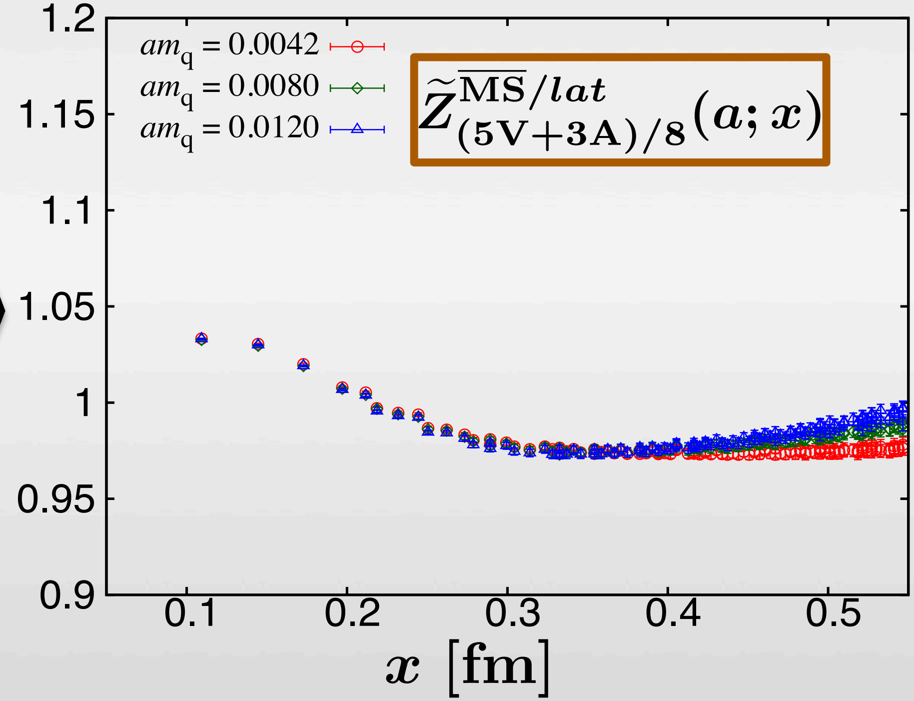
- Mass dependence in short-range can be eliminated

Renormalization of Vector Current

- New combination

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(5\mathrm{V}+3\mathrm{A})/8}(a;x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(a) + c_{4,\mathrm{G}}\langle \mathrm{GG}\rangle x^4 + \cdots$$





- Mass dependence become much smaller

Renormalization of Vector Current

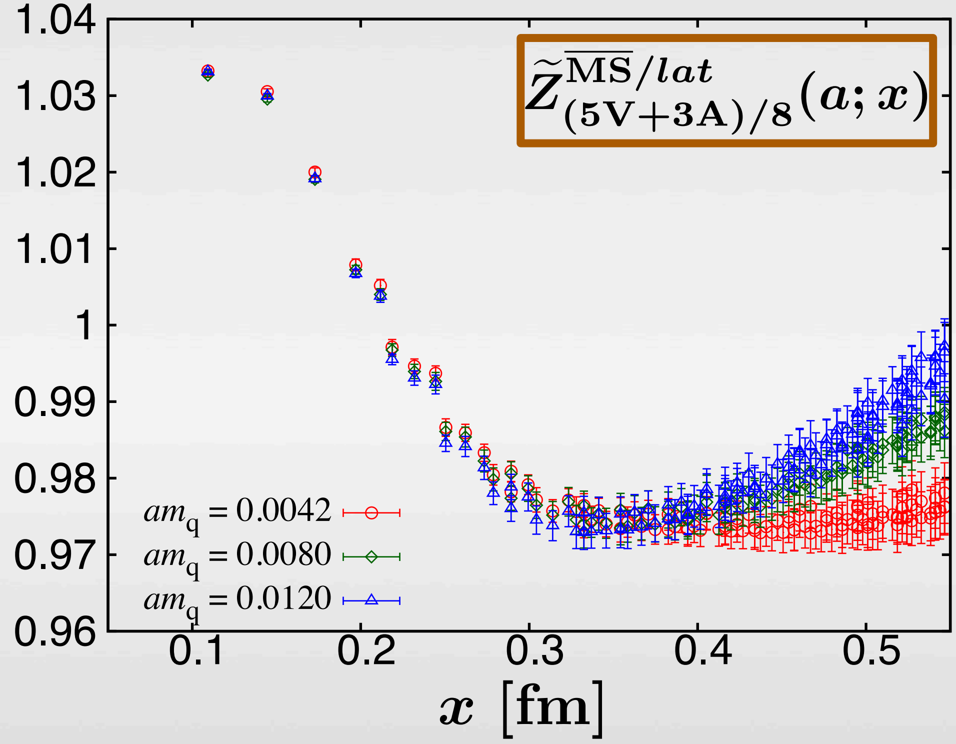
- Up to x^4, data are independent of mass

$\Rightarrow$ O(x^6) : necessary

at $x > 0.4$ fm

$$c_{6,\bar{q}q^2}\langle\bar{q}\bar{q}qq\rangle x^6 + c_{6,mG}m_q^2\langle GG\rangle x^6 + \cdots$$

Zoomed version



Renormalization of Vector Current

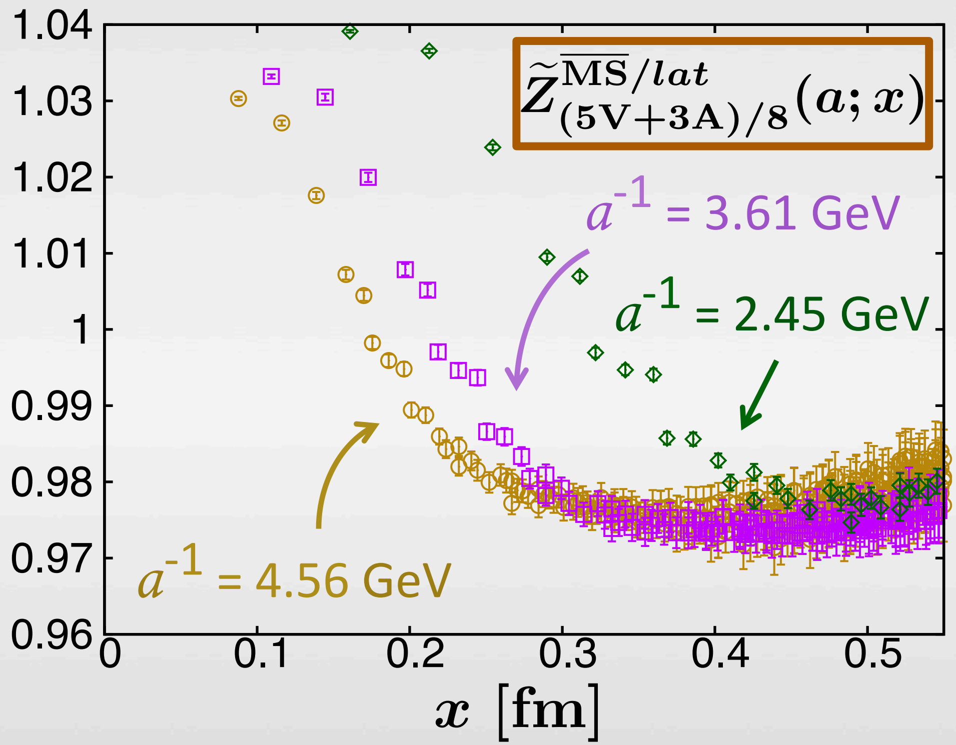
- Up to x^4, data are independent of mass

$\Rightarrow$ O(x^6) : necessary at $x > 0.4$ fm

$$c_{6,\bar{q}q^2}\langle\bar{q}\bar{q}qq\rangle x^6 + c_{6,mG} m_q^2 \langle GG\rangle x^6 + \cdots$$

- Increasing behavior in short range depends on a

Result for each a at $M_\pi = 300$ MeV



$\rightarrow$ Discretization effect $(a/x)^2$, $(am_q)^2$, $(a/x)^4$, $\cdots$

Significant

Renormalization of Vector Current

- Fit function:

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(5\mathrm{V}+3\mathrm{A})/8}(a;x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(a) + c_{-2}(a/x)^2 + c_4 x^4 + (c_6 + c'_6 m_{\mathrm{q}}^2)x^6$$

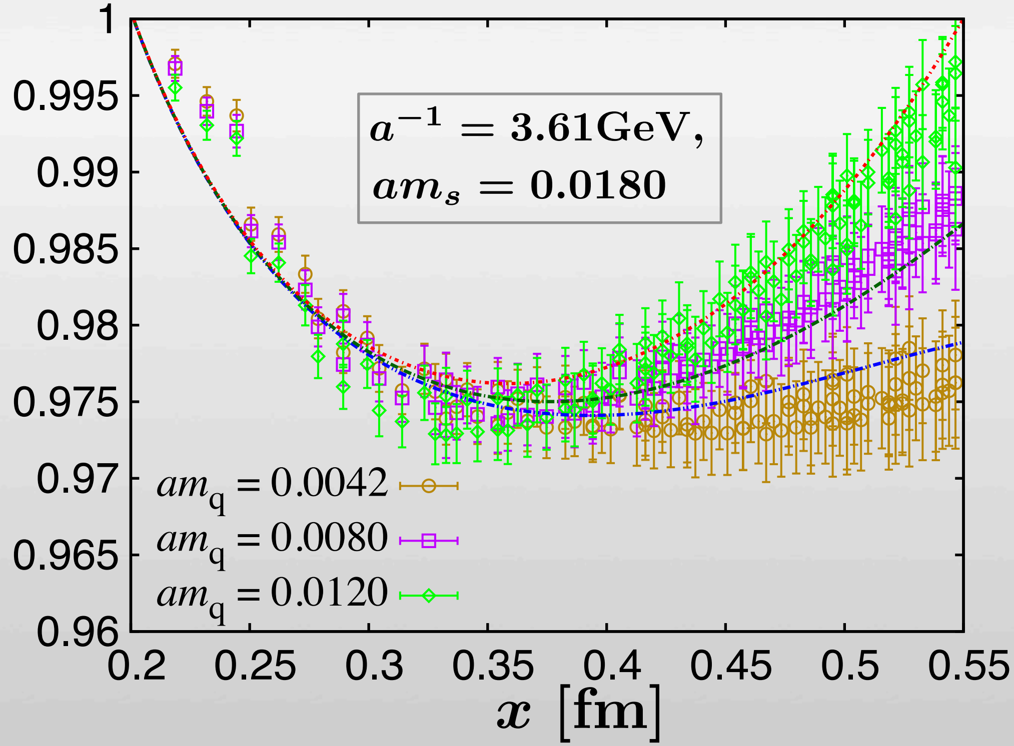
- χ^2 fitting simultaneously among each ensembles

- Preliminary results

$$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(\beta = 4.17) = 0.951(4),$$

$$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(\beta = 4.35) = 0.956(3),$$

$$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(\beta = 4.47) = 0.961(3)$$



Renormalization of Scalar Density

- Good combination of Π_S & Π_P

Naïve average : $\mathbf{\Pi_{(S+P)/2} = \frac{1}{2}(\Pi_S + \Pi_P)}$ ← $U(1)_A$-singlet

- Subtracting remaining non-perturbative effect

$$\boldsymbol{\Pi_{(S+P)/2} = C_{pert}x^{-6} + \left(C_{4,\bar{q}q}m_q\langle\bar{q}q\rangle + C_{4,G}\langle GG\rangle\right)x^{-2} + \cdots}$$

Reduce number of fit parameters

using

$$\boldsymbol{\Pi_{V-A} = -16C_{4,\bar{q}q}m_q\langle\bar{q}q\rangle x^{-2} + \cdots}$$

✦ $\mathbf{\Pi_{(S+P)/2+(V-A)/16}}$: independent of $m_q \langle\bar{q}q\rangle\, x^{-2}$

Renormalization of Scalar Density

- Fit function:

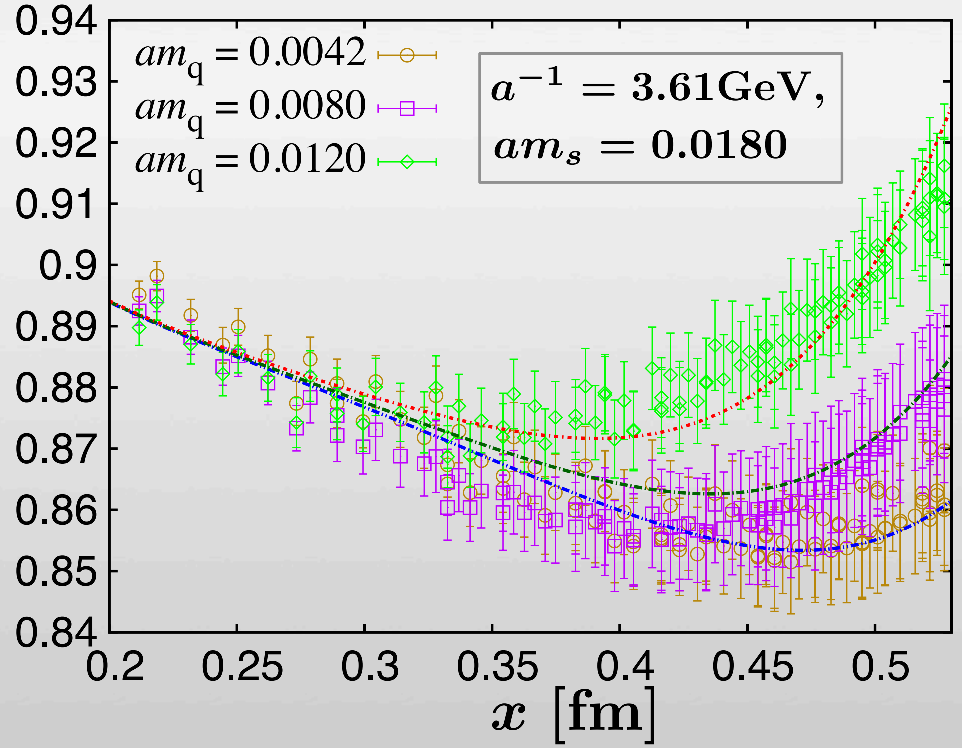
$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(\mathrm{S+P})/2+(\mathrm{V-A})/16}(2\ \mathrm{GeV}; a; x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{S}}(2\ \mathrm{GeV}; a)$$
$$+c_{-2}(a/x)^2 + c_4 x^4 + (c_6 + c_6' m_{\mathrm{q}}^2)x^6$$

- Preliminary results for $Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{S}}(2\ \mathrm{GeV}; a)$

$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{S}}(\beta = 4.17) = 0.975(9),$

$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{S}}(\beta = 4.35) = 0.881(7),$

$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{S}}(\beta = 4.47) = 0.842(5)$

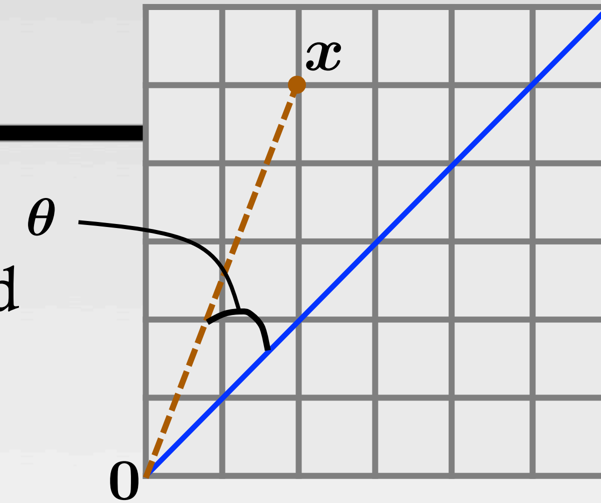


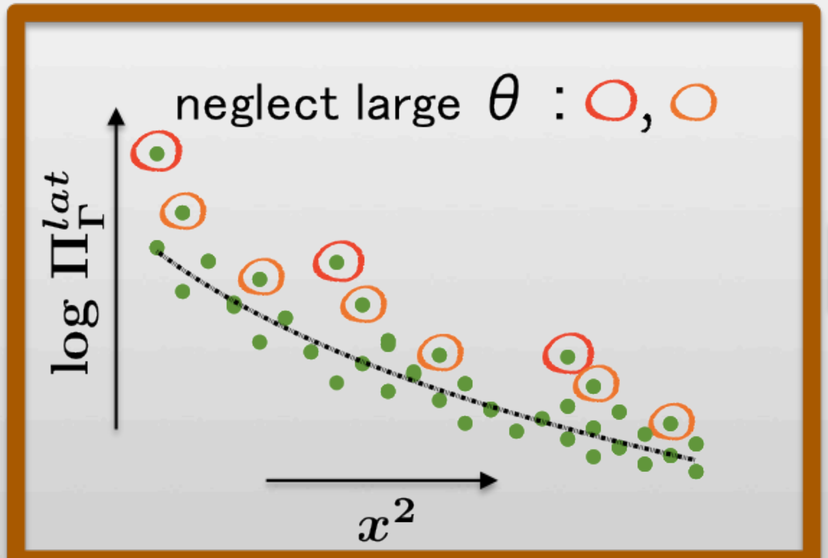
Summary

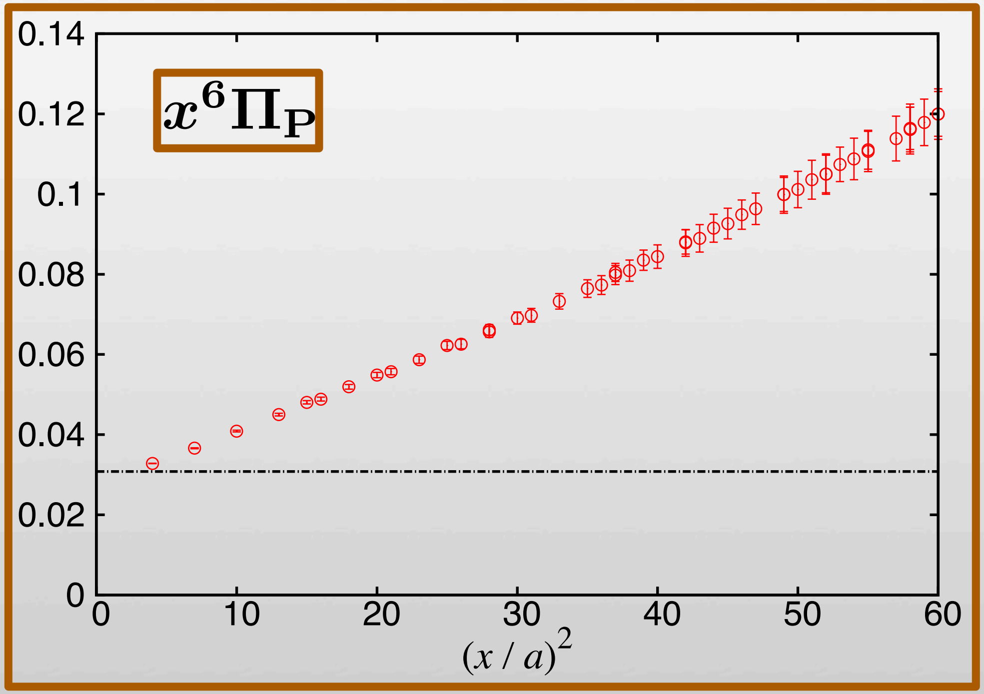
- We investigate two-point correlation functions for scalar and pseudoscalar densities, vector and axial-vector currents.
- Convergence of perturbative correlators is well improved by changing scale of coupling.
- Results of vector and axial-vector currents have good agreement with OPE.
- Chiral condensate is extracted from non-conserved part of the correlator of axial-vector current.
- Using the context of OPE, renormalization constants are determined with good precision ($\lesssim 1\ \%$).

Diagonal Cut

- θ : angle between x & (1,1,1,1)
- Correlators at large θ are more distorted
- Free correlators at small θ are closer to those in continuum theory
- Data with $\theta < 30°$ are used







Renormalization of Vector Current

- Fit function:

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(5\mathrm{V}+3\mathrm{A})/8}(a;x) = Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(a) + c_{-2}(a/x)^2 + c_4 x^4 + (c_6 + c'_6 m_{\mathrm{q}}^2)x^6$$

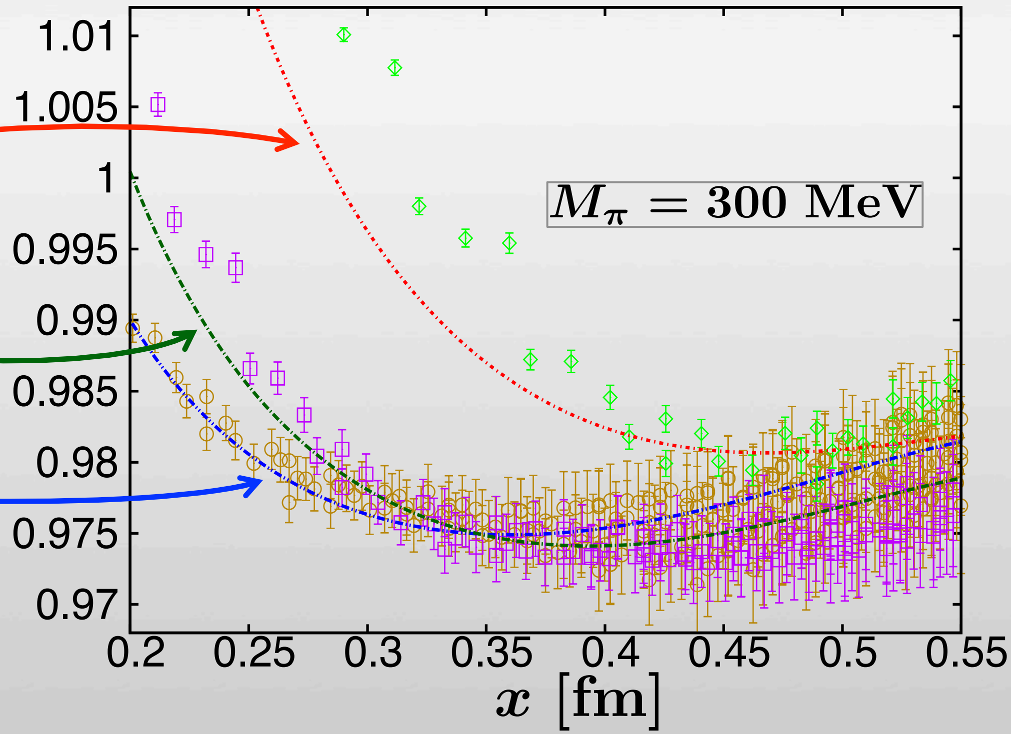
- Perform chi 2 fitting simultaneously for all ensembles

- Preliminary results

$$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(2.45\ \mathrm{GeV}) = 0.951(4),$$

$$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(3.61\ \mathrm{GeV}) = 0.956(3),$$

$$Z^{\overline{\mathrm{MS}}/lat}_{\mathrm{V}}(4.50\ \mathrm{GeV}) = 0.961(3)$$



Renormalization of Scalar Density

- Fit function:

$$\widetilde{Z}^{\overline{\mathrm{MS}}/lat}_{(\mathrm{S+P})/2+(\mathrm{V-A})/16}(2\ \mathrm{GeV}; a; x) = Z_{\mathrm{S}}^{\overline{\mathrm{MS}}/lat}(2\ \mathrm{GeV}; a) + c_{-2}(a/x)^2 + c_4 x^4 + (c_6 + c_6' m_{\mathrm{q}}^2)x^6$$

- Preliminary results for $Z_{\mathrm{S}}^{\overline{\mathrm{MS}}/lat}(2\ \mathrm{GeV}; a)$

$$Z_{\mathrm{S}}^{\overline{\mathrm{MS}}/lat}(a^{-1} = 2.45\ \mathrm{GeV}) = 0.975(9),$$

$$Z_{\mathrm{S}}^{\overline{\mathrm{MS}}/lat}(a^{-1} = 3.61\ \mathrm{GeV}) = 0.881(7),$$

$$Z_{\mathrm{S}}^{\overline{\mathrm{MS}}/lat}(a^{-1} = 4.50\ \mathrm{GeV}) = 0.842(5),$$