MASS ANOMALOUS DIMENSION OF SU(2) WITH $N_F = 8$ USING THE SPECTRAL DENSITY METHOD

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BACKGROUND

Infrared-conformal gauge theories have been considered as models for physics beyond the Standard Model. In these models the anomalous dimension of the fermion operator $\bar{\psi}\psi$, γ_m, plays an important role. The scaling exponent of the spectral density of the massless Dirac operator is a function of the mass anomalous dimension, and thus it can be extracted by studying the behaviour of the eigenvalue distribution of the Dirac operator.

The mass anomalous dimension can be obtained by using the Schrödinger functional mass step scaling function method [1], and our results based on previous work using the method are shown in Fig. 1. Using the method we obtained results in agreement with the perturbative prediction at small couplings g^2_{GF}, but see deviation from the curve at higher coupling.

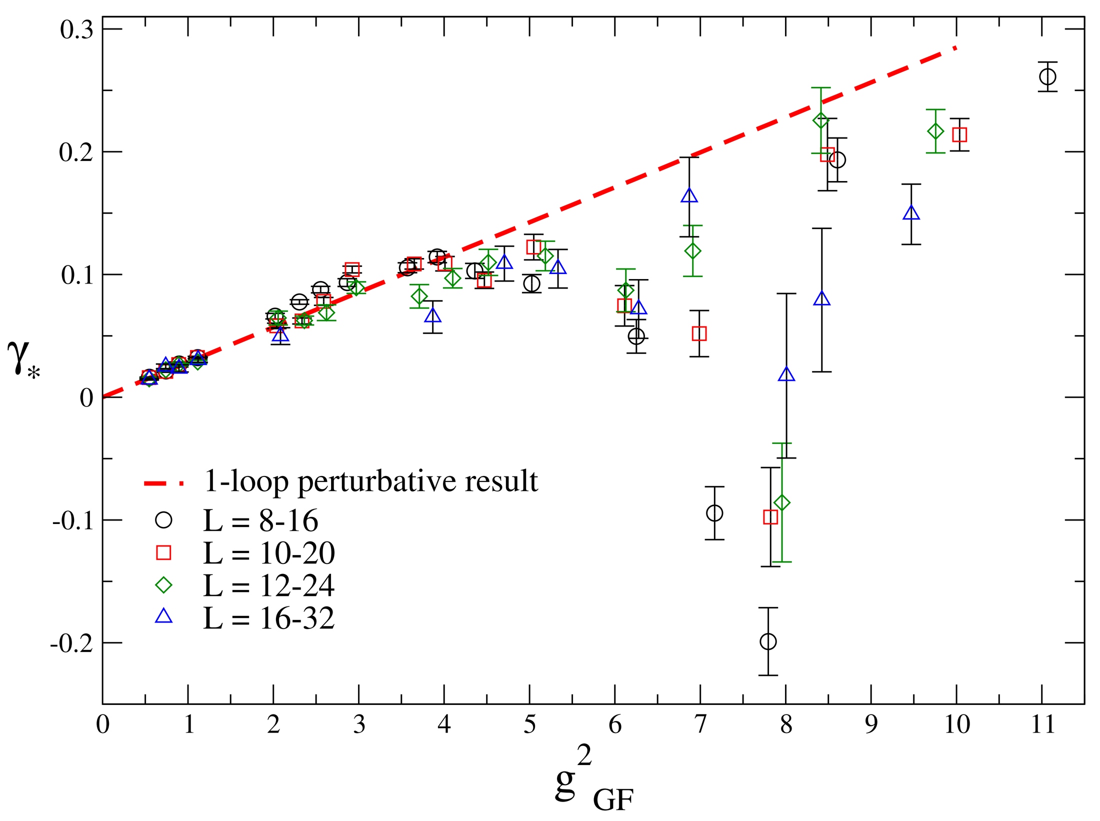

Figure 1: The mass anomalous dimension as a function of the gradient flow coupling constant obtained using the mass step scaling function method.

SETUP

The theory which we are studying is SU(2) with $N_f = 8$ fermions in the fundamental representation. We use HEX smeared, clover improved Wilson fermions with Schrödinger functional boundary conditions, and we have tuned the PCAC quark mass to zero. We calculate the mode number per unit volume

$$\nu(\Lambda) = 2\int_0^{\sqrt{\Lambda^2-m^2}} \rho(\lambda)d\lambda, \qquad (1)$$

where $\rho(\lambda)$ is the spectral density of the Dirac operator, by using

$$\nu(\Lambda) = \lim_{V\to\infty} \frac{1}{V}\langle \mathrm{tr}\, \mathbb{P}(\Lambda)\rangle \,. \qquad (2)$$

The operator $\mathbb{P}(\Lambda)$ projects from the full eigenspace of $M = m^2 - \not{D}^2$ to the eigenspace of eigenvalues lower than Λ^2, and the trace is calculated stochastically [2]. We use from 8 to 16 configurations for the calculation for each value of the gauge coupling and lattice size.

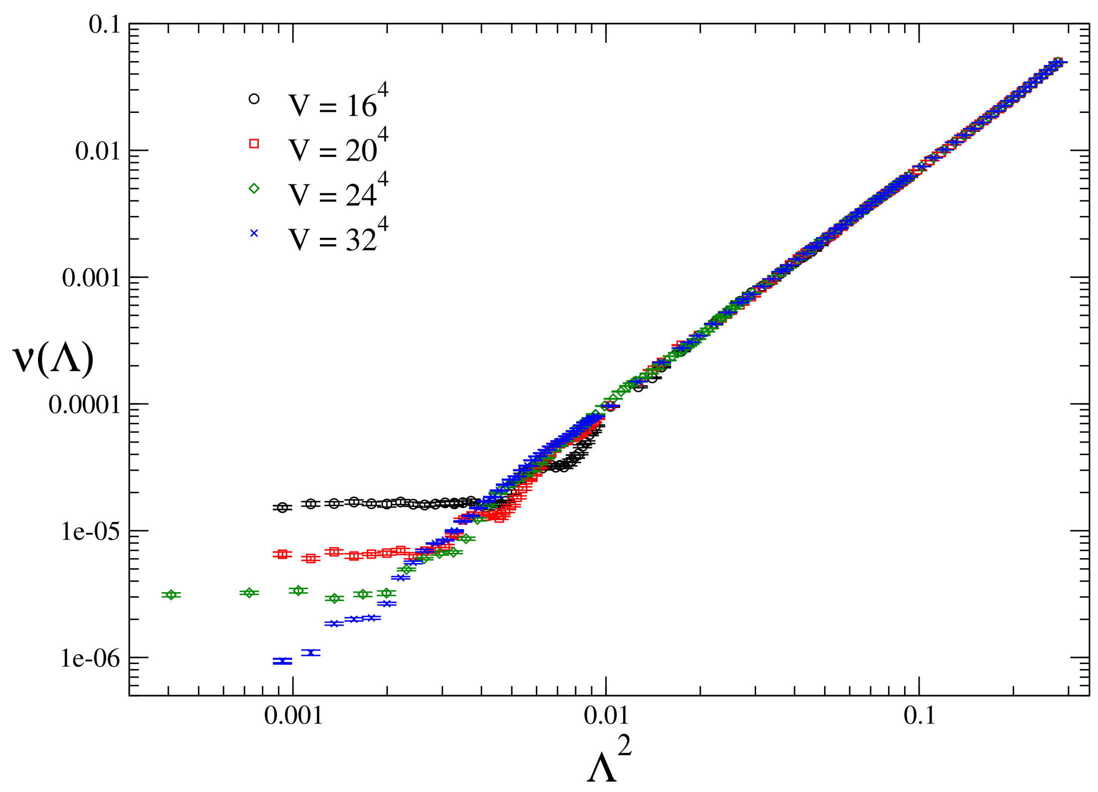

Figure 2: The mode number calculated for different different lattice volumes with a $g^2_{GF} = 0.90$ coupling.

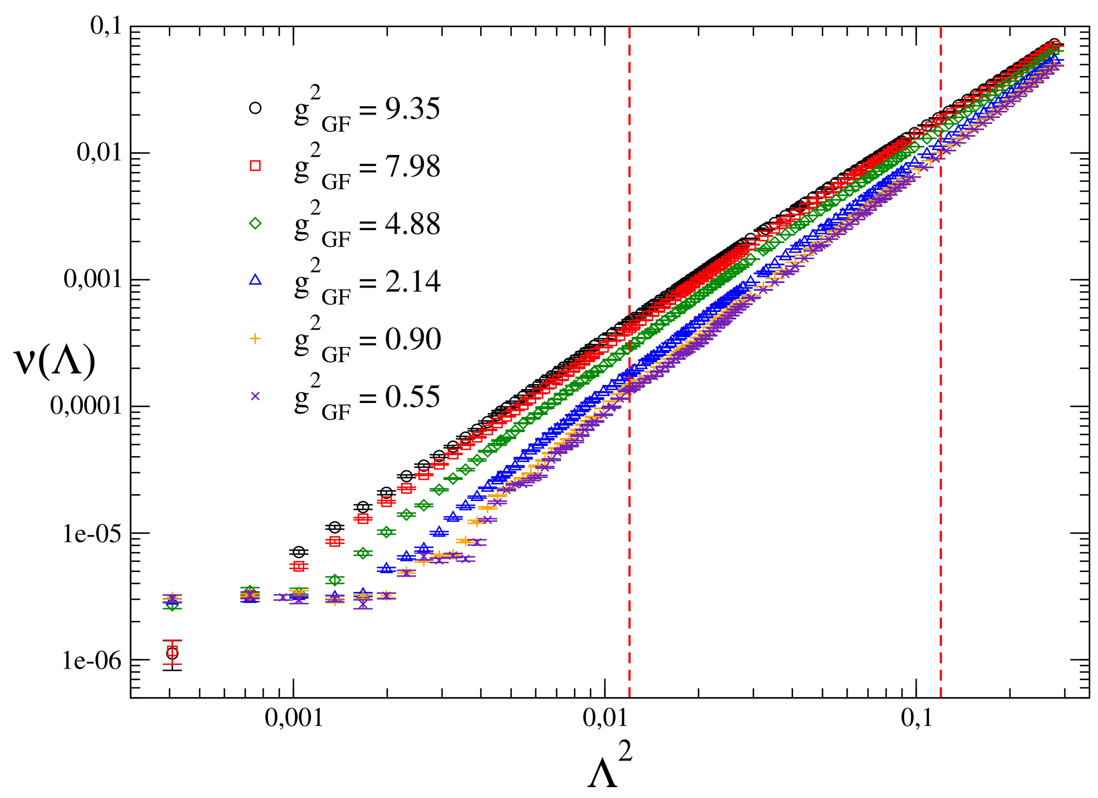

Figure 3: The mode number calculated for different couplings on a $V = 24^4$ lattice. The dashed lines correspond roughly to the fit range.

POWER LAW FIT

The spectral density in the vicinity of the IR-fixed point has approximately a form of a power law [3,4]

$$\nu(\Lambda) \simeq \nu_0 + A\left[\Lambda^2 - m^2\right]^{\frac{2}{1+\gamma_*}}, \qquad (3)$$

where ν_0 and A are an additive and a multiplicative constant respectively, m is the quark mass and γ_* is the mass anomalous dimension. All four parameters are used for fitting. The range of eigenvalues where this form holds is not known *a priori*, and needs to be determined by trial and error. This range was typically found to be between $\Lambda^2 \simeq 0.01 \ldots 0.11$.

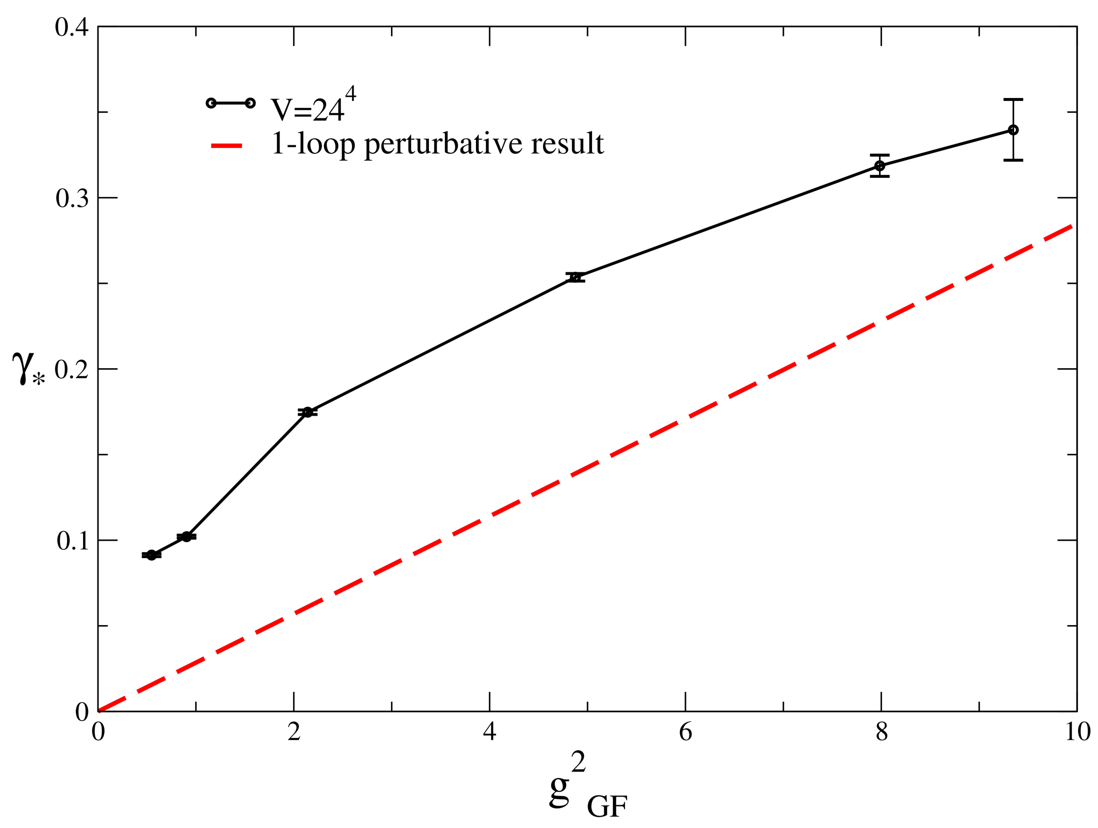

Figure 4: γ_* obtained by fitting Eq. 3 to the data in Fig. 3. The figure here is based on preliminary data, and the error bars are purely statistical and do not accurately reflect how well the fit works.

ENERGY SCALE DEPENDENCE OF γ_M

To examine the energy scale dependence of the mass anomalous dimension, we follow [5] and perform a linear fit of the data to

$$\log[\nu(\Lambda)] = \left(\frac{4}{1+\gamma_m(\Lambda)}\right)\log[\Lambda] + \text{ a constant} \qquad (4)$$

and obtain $\gamma_m(\Lambda)$ by differentiation with respect to $\log[\Lambda]$.

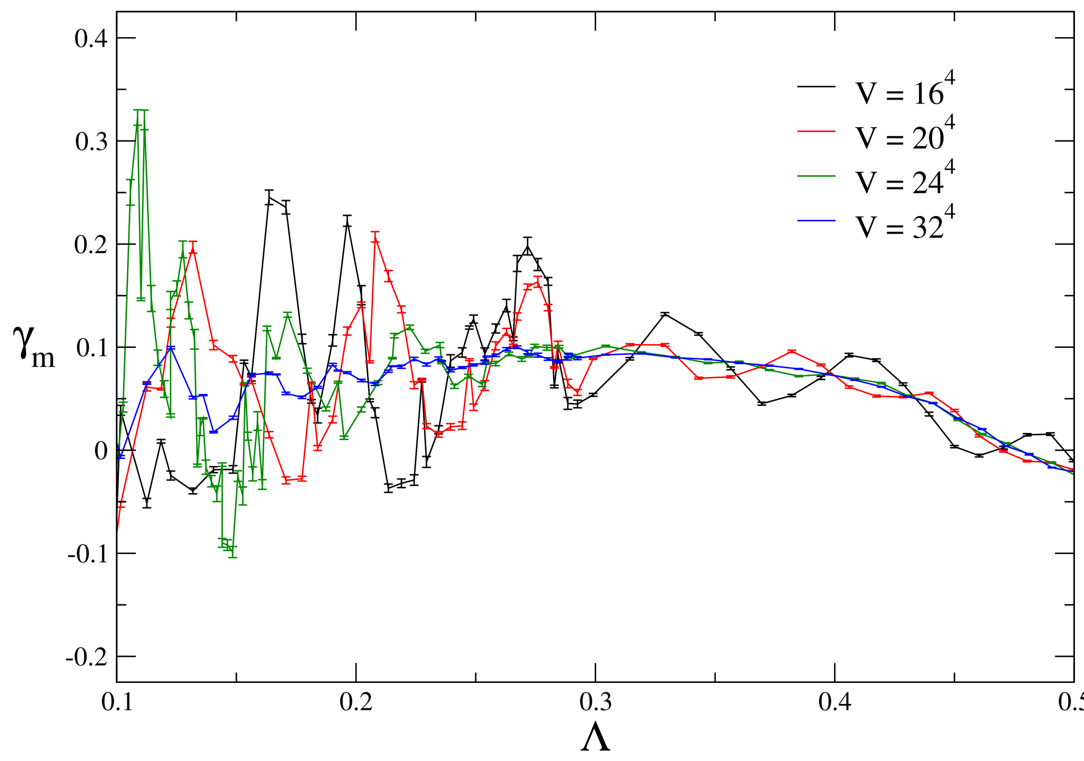

Figure 5: The mass anomalous dimension as a function of the energy scale for different lattice volumes at $g^2_{GF} = 0.90$.

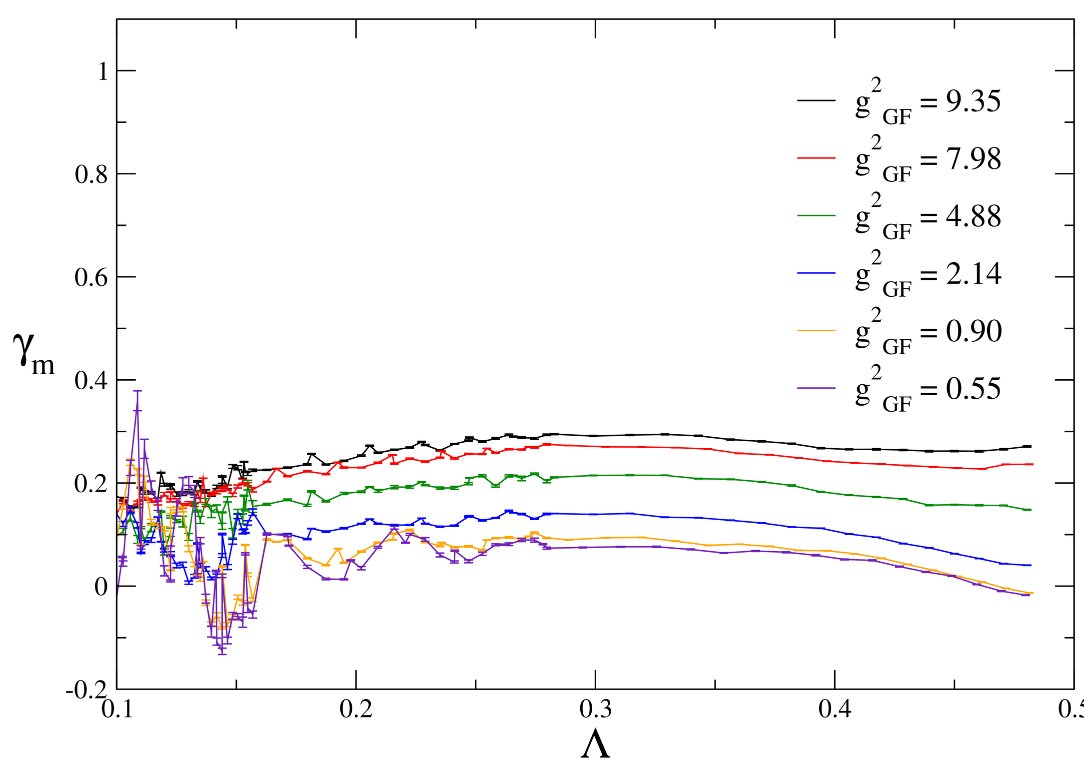

Figure 6: The mass anomalous dimension as a function of the energy scale for different couplings g^2_{GF} at volume $V = 24^4$.

CONCLUSIONS

In Fig. 4 we see a deviation from the perturbative result across all values of the coupling. The lower values of the coupling suffered from unstable fitting, and as such can not be trusted fully. As Fig. 2 shows, the volume dependence of the mode number is most apparent in the IR region where the power law fit should be done, suggesting further studies with larger lattices. Whether introducing sub-leading terms to Eq. 3 will correct the deviation or not is under investigation.

In a similar fashion, as Fig. 5 shows, the volume dependence has an effect on the range of validity for the differentiation method. Using a larger lattice would allow us to get better signal deeper in the IR region.

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