## Neutron-Antineutron Oscillation Matrix Elements with DW Fermions at the Physical Point

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## Motivation

- Baryon number violation ( $\Delta B=2$ )

One of Sakharov's conditions for baryogenesis

- Nuclear matter stability

Decay of nuclei through (nn)-annihilation

Probing BSM physics, $\Delta(B-L)$
Connection to lepton number violation and
seesaw neutrino mass mechanism?
[R.Mohapatra, R.Marshak (1980)]

Alternative to proton decay ( $\Delta B=1$ )
Which one (or both?) realized in nature?
neutron/antineutron oscillation through $\Delta B=1$ is suppressed

## Basics of $\mathrm{n} \leftrightarrow \overline{\mathrm{n}}$ Oscillations

$\mathcal{L}_{n \bar{n}}$ is a vacuum operator and preserves spin $\Rightarrow 2$-state system

$$
\begin{aligned}
& \mathcal{H}_{\mathrm{osc}}=\left(\begin{array}{cc}
M_{n}+\frac{1}{2} \Delta M & \delta m \\
\delta m & M_{n}-\frac{1}{2} \Delta M
\end{array}\right) \\
& \text { where } \delta m=\langle\bar{n}| \mathcal{L}_{n \bar{n}}|n\rangle
\end{aligned}
$$

$$
\Delta M=\text { induced by magnetic field or nuclear media }
$$

Oscillation probability:

$$
\left.P_{n \rightarrow \bar{n}}(t)=\left|\langle\bar{n}| e^{-i \mathcal{H}_{\text {osc }} t}\right| n\right\rangle\left.\right|^{2}=\left[\frac{(\delta m)^{2}}{(\Delta M / 2)^{2}+(\delta m)^{2}}\right] \sin ^{2}\left[\frac{1}{2} \Delta E t\right]
$$

Current bound $\tau_{n \bar{n}} \gtrsim 10^{8} s \quad \Longleftrightarrow \delta m \lesssim 6 \cdot 10^{-24} \mathrm{eV}$
Earth magnetic field $=0.5$ Gauss: $\quad \Delta M=2 \mu_{n} B_{\oplus} \approx 6 \cdot 10^{-12} \mathrm{eV}$

$$
\Delta E=\sqrt{(\Delta M)^{2}+(2 \delta m)^{2}} \approx \Delta M \gg \delta m
$$

Quasifree condition $(\Delta E \cdot t<1)$ for $\mathrm{t}=1 \mathrm{sec}$ :

$$
B<\left(2 \mu_{n} t\right)^{-1}=5 \mathrm{nT}=10^{-4} B_{\oplus}
$$

## Searches for $n \rightarrow \bar{n}$

Stability of a nucleus w.r.t ( $n n$ ) annihilation
${ }^{56} \mathrm{Fe}$ [Soudan 2] $T_{d}\left({ }^{56} \mathrm{Fe}\right)>0.72 \cdot 10^{32} \mathrm{yr} \longrightarrow \tau_{n \bar{n}}>1.4 \cdot 10^{8} \mathrm{~s}$
${ }^{16} \mathrm{O}$ [Super-K] $T_{d}\left({ }^{16} \mathrm{O}\right)>1.77 \cdot 10^{32} \mathrm{yr} \longrightarrow \tau_{n \bar{n}}>3.3 \cdot 10^{8} \mathrm{~s}$
${ }^{2} \mathrm{H}$ [SNO] $\quad T_{d}\left({ }^{2} H\right)>0.54 \cdot 10^{32} \mathrm{yr} \longrightarrow \tau_{n \bar{n}}>1.96 \cdot 10^{8} \mathrm{~s}$

## Sensitivity is limited by atmospheric neutrinos



Quasifree neutrons ( $\Delta E t \ll 1$ ) in vacuum:
ILL Grenoble high-flux reactor, 1990 [M.Baldo-Ceolin et al, 1994)]


$$
\tau_{n \bar{n}}>0.86 \cdot 10^{8} \mathrm{sec}
$$

 matter stability bound $\approx 10^{35} \mathrm{yr}$

## Neutron $\leftrightarrow$ Antineutron Operators

Effective 6-quark operators From Beyond (the Standard Model) :
interaction with a massive Majorana lepton, unified theories, etc
[T.K.Kuo, S.T.Love, PRL45:93 (1980)]
[R.N.Mohapatra, R.E.Marshak, PRL44:1316 (1980)]

$$
\begin{array}{ll}
\mathcal{H}_{n \bar{n}}=\left(\begin{array}{cc}
E+V & \delta m \\
\delta m & E-V
\end{array}\right) & \tau_{n \bar{n}}=(2 \delta m)^{-1} \\
\mathcal{L}_{\text {eff }}=\sum_{i}\left[c_{i} \mathcal{O}_{i}^{6 q}+\text { h.c. }\right] & \left.\delta m=-\langle\bar{n}| \int d^{4} x \mathcal{L}_{\text {eff }}|n\rangle=-\sum_{i} c_{i}\left\langle\langle\bar{n}| \mathcal{O}_{i}^{6 \mathrm{q}} \mid n\right\rangle\right\rangle
\end{array}
$$

Dimension-9 point-like operators suppressed by $\left(M_{X}\right)^{-5}$
What would be the scale for new physics behind $n \leftrightarrow n^{-}$?
Current limit on $\tau_{n-n}$ requires $M_{x} \approx$ few $\cdot 10^{2} \mathrm{TeV}$
Sensitivity of matter to BN -violating terms is determined by nuclear scale physics and non-perturbative QCD


## Neutron $\leftrightarrow$ Antineutron Matrix Elements

Operators: pseudoscalar singlets w.r.t $S U(3)_{\mathrm{c}} \otimes U(1)_{\mathrm{em}}\left[\otimes S U(2)_{L}\right]$
$\mathcal{O}_{1 \chi_{1}\left\{\chi_{2} \chi_{3}\right\}}=T_{i j k l m n}^{s}\left[u_{\chi_{1}}^{i T} \mathcal{C} u_{\chi_{1}}^{j}\right]\left[{ }_{\chi_{2}}^{k T} \mathcal{C} d_{\chi_{2}}^{l}\right]\left[d_{\chi_{3}}^{m T} \mathcal{C} d_{\chi_{3}}^{n}\right]$
$\chi_{1,2,3}=R, L$

$\mathcal{O}_{2\left\{\chi_{1} \chi_{2}\right\} \chi_{3}}=T_{i j k l m n}^{s}\left[u_{\chi_{1}}^{i T} \mathcal{C} d_{\chi_{1}}^{j}\right]\left[u_{\chi_{2}}^{k T} \mathcal{C} d_{\chi_{2}}^{l}\right]\left[d_{\chi_{3}}^{m T} \mathcal{C} d_{\chi_{3}}^{n}\right]$
$\mathcal{O}_{3\left\{\chi_{1} \chi_{2}\right\} \chi_{3}}=T_{i j k l m n}^{a}\left[u_{\chi_{1}}^{i T} \mathcal{C} d_{\chi_{1}}^{j}\right]\left[u_{\chi_{2}}^{k T} \mathcal{C} d_{\chi_{2}}^{l}\right]\left[d_{\chi_{3}}^{m T} \mathcal{C} d_{\chi_{3}}^{n}\right]$

## Computed using MIT bag model

[T.Kuo, S.Love, PRL45:93 (1980)]
[S.Rao, R.Shrock, PLB116:238 (1982)]
Chiral $\operatorname{SU}(2)_{L, R}$ multiplet classification:
$\left.\begin{array}{l|l|c|c}{\left[(R R R)_{\mathbf{3}}\right]} & \mathcal{O}_{R(R R)}^{1}+4 \mathcal{O}_{(R R) R}^{2} & \mathbf{3}_{R} \otimes \mathbf{0}_{L} & \left(\alpha_{S} / 4 \pi\right)(-12) \\ \hline\left[(R R R)_{\mathbf{1}}\right] & \mathcal{O}_{(R R) R}^{( }-\mathcal{O}_{R(R R)}^{1} \equiv 3 \mathcal{O}_{(R R) R}^{3} & \mathbf{1}_{R} \otimes \mathbf{0}_{L} & \left(\alpha_{S} / 4 \pi\right)(-2) \\ {\left[R_{\mathbf{1}}(L L)_{\mathbf{2}}\right]} & \mathcal{O}_{(L L) R}^{2}-\mathcal{O}_{L(L R)}^{1} \equiv 3 \mathcal{O}_{(L L) R}^{3} & \mathbf{1}_{R} \otimes \mathbf{0}_{L} & 0 \\ {\left[(R R)_{\mathbf{1}} L_{\mathbf{0}}\right]} & 3 \mathcal{O}_{(L R) R}^{3} & \mathbf{1}_{R} \otimes \mathbf{0}_{L} & \left(\alpha_{S} / 4 \pi\right)(+2)\end{array}\right\}$ sU(2)L×U(1) -symmetric

Chiral symmetry is essential for simple renormalization

## Lattice Calculation

$$
\begin{gathered}
\left\langle N_{\uparrow}^{(+)}\left(t_{2}\right) \mathcal{O}^{6 \mathrm{q}}(0) N_{\downarrow}^{(-)}\left(-t_{1}\right)\right\rangle \underset{t_{1}, t_{2}, t_{1}+t_{2} \rightarrow \infty}{\sim} e^{-M_{n}\left(t_{2}+t_{1}\right)}\left\langle n_{\uparrow}\right| \mathcal{O}^{6 \mathrm{q}}\left|\bar{n}_{\uparrow}\right\rangle \\
\hline
\end{gathered}
$$


No quark-disconnected contractions!
Single propagator $\longrightarrow \quad \forall t_{1}, t_{2}$
Initial calculation with anisotropic Wilson in [M.Buchoff, C.Schroeder, J.Wasem, arXiv:1207.3832 (LATTICE2012)]

$$
\begin{aligned}
\left.\langle n| \mathcal{O}|\bar{n}\rangle\right|_{\text {lat }}= & \langle n| \mathcal{O}|\bar{n}\rangle \\
& +O\left(e^{-\Delta E_{\text {exc }} t_{1}}, e^{-\Delta E_{\text {exc }} t_{2}}, e^{-\Delta E_{\text {exc }}\left(t_{1}+t_{2}\right)}\right)
\end{aligned}
$$



Complete set of correlators for sophisticated exc.state analysis:

- Exponential fits
- Variational (GPoF)


## Preliminary Results

Physical pions $m_{\pi}=140 \mathrm{MeV}$[RBC and UKQCD collaborations, arXiv:1411.7017]
$\uparrow$ lattice $48^{3} \times 96=5.5^{3} \times 10.9 \mathrm{fm}$
$\uparrow$ lattice spacing $a=0.123 \mathrm{fm}, a^{-1}=(1.730(4)) \mathrm{GeV} ; \delta\left(a^{-6}\right) \approx 1.4 \%$

- chiral (Möbius Domain Wall Fermions)
$\checkmark \quad 28 \times 81$ samples (AMA)


## PRELIMINARY ANALYSIS:

$\downarrow$ simplified analysis of exc.states

$$
\langle n| \mathcal{O}|\bar{n}\rangle \sim \frac{C_{n \mathcal{O} \bar{n}}\left(t_{2}, 0,-t_{1}\right)}{\sqrt{C_{n n}\left(t_{2}, 0\right) C_{\bar{n} \bar{n}}\left(0,-t_{1}\right)}}
$$

## Effective Mass: Gauging Excited States



## Lattice Matrix Elements







scaled $\times 10^{6}$, kinematics factors not divided out Separation T=10 : ~10\% stat.errorbars, consistent with T=12

## Renormalization : RI-(S)MOM on a lattice

$$
\left(G_{I}\right)_{\alpha \beta \gamma \delta \epsilon \eta}^{i j k l m n}\left(x, p_{1} \ldots p_{6}\right)=\left\langle\mathcal{O}_{I} \bar{d}_{\eta}^{n}\left(p_{6}\right) \bar{d}_{\epsilon}^{m}\left(p_{5}\right) \bar{d}_{\delta}^{l}\left(p_{4}\right) \bar{d}_{\gamma}^{k}\left(p_{3}\right) \bar{u}_{\beta}^{j}\left(p_{2}\right) \bar{u}_{\alpha}^{i}\left(p_{1}\right)\right\rangle
$$



$$
\begin{aligned}
& p_{1}=p_{3}=p_{5}=p \\
& p_{2}=p_{4}=p_{6}=-p
\end{aligned}
$$

Ext.momenta assigned to match the 2-loop pert.QCD calculation [M.Buchoff, M.Wagman, arXiv:1506.00647]

Contractions : loop over $\left(N_{s} N_{c}\right)^{6}$, for each $\left(N_{s} N_{c}\right)^{6}$ elements, every site (with vol.sources)

- use Fouirer-transf. propagators $\mathrm{x}_{0} \longrightarrow \mathrm{p}_{\mathrm{i}}$ for all $p_{i}$ (reuse RHQBBar data, 20*81 samples)
- fermion symmetry: antisymmetrize propagators ( $u_{p} \otimes u_{-p}$ ), ( $d_{p} \otimes d_{-p}$ ) before contractions

Use 4d diagonal $p=(k, k, k, k),(a \cdot k)<\pi / 2$ to minimize discretization errors at higher scale

## Restoring Chiral Isomultiplets



## "Scale-independent" Ren.factors



Perturbative 1-loop running from
W.Caswell et al PLB122:373 (1983)]

Take variances between $2-4 \mathrm{GeV}$ and $4-6 \mathrm{GeV}$ fits as estimates of syst.errors

## Preliminary Results in MSbar(2GeV)

## DO NOT QUOTE PRELIMINARY DO NOT QUOTE PRELIMINARY DO NOT QU

|  | $Z(\mathrm{lat} \rightarrow \overline{M S})$ | $\mathcal{O}^{\overline{M S}(2 \mathrm{GeV})}$ | Bag "A" | $\frac{\mathrm{LQCD}}{\text { Bag "A" }}$ | Bag "B" | $\frac{\mathrm{LQCD}}{\text { Bag "B" }}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $\left[(R R R)_{\mathbf{3}}\right]$ | $0.62(12)$ | 0 | 0 | - | 0 | - |
| $\left[(R R R)_{\mathbf{1}}\right]$ | $0.454(33)$ | $45.4(5.6)$ | 8.190 | 5.5 | 6.660 | 6.8 |
| $\left[R(R L)_{\mathbf{0}}\right]$ | $0.435(26)$ | $44.0(4.1)$ | 7.230 | 6.1 | 6.090 | 7.2 |
| $\left[(R R)_{\mathbf{1}} L_{\mathbf{0}}\right]$ | $0.396(31)$ | $-66.6(7.7)$ | -9.540 | 7.0 | -8.160 | 8.1 |
| $\left[(R R)_{\mathbf{2}} L_{\mathbf{1}}\right]^{(1)}$ | $0.537(52)$ | $-2.12(26)$ | 1.260 | -1.7 | -0.666 | 3.2 |
| $\left[(R R)_{\mathbf{2}} L_{\mathbf{1}}\right]^{(2)}$ | $0.537(52)$ | $0.531(64)$ | -0.314 | -1.7 | 0.167 | 3.2 |
| $\left[(R R)_{\mathbf{2}} L_{\mathbf{1}}\right]^{(3)}$ | $0.537(52)$ | $-1.06(13)$ | 0.630 | -1.7 | -0.330 | 3.2 |

- matrix elements : $\mathrm{T}=10$ plateau average
- renormalization: only syst.errors, estimated from variation over entire range
- MIT Bag model results from [S.Rao, R.Shrock, PLB116:238 (1982)]


## Summary \& Outlook

Clear lattice signal for non-zero $\langle n| \mathcal{O}^{6 \mathrm{q}}|\bar{n}\rangle$ even with modest statistics
Physical $m_{\pi}=140 \mathrm{MeV}$ pion mass lattices with chiral symmetry
Comparison with the MIT Bag Model
Current stat\&sys. errors already not exceed $\sim 15 \%$
Reduction of model dependence of n-nbar oscillations phenomenology

## Outlook

- Study and improve control of syst.errors in renormalization

Alternative momentum arrangement?
Step-scaling?
Two-loop perturbative running and matching?

- Improve analysis to extract ground state M.E.

Easy to analyze excited state effects with chosen schemeStudy discretization effects with another (finer) lattice spacing

## BACKUP

## Neutron Oscillations and Baryogenesis

Baryosynthesis requires $B$ - or $L-$ violation :

- leptotenesis: $\Delta L$ above $T_{E W}$, transformed to $\Delta B$ by sphalerons
- (if exist) $n-\bar{n}$ oscillations can wash away $\Delta B$ during EW transition
- then, $n \leftrightarrow \bar{n}$ must explain post-sphaleron baryogenesis below $T_{E W}$ upper limit on $\tau_{n-n}<5 \cdot 10^{10} \sec$ (proton-decay already excluded)

Interplay of $T_{E W}$ and $\Delta B$ scales


## Neutron Oscillation and Neutrino See-Saw

( $) n$ - $\bar{n}$ oscillation $\Delta B=2$ breaks $(B-L)$ : beyond-SM physics
Similarly, $\Delta L=2$ from Majorana neutrino masses in the see-saw mechanism: is there connection between the two?
[Mohapatra, Marshak, PRL44:1316 (1980)]
Quark-lepton unification theory

$$
S U(2)_{R} \otimes S U(2)_{L} \otimes\left[U(1)_{(B-L)} \otimes S U(3)_{C}\right]_{\left(\text {remnants of } S U\left(4^{\prime}\right)\right)}
$$

with Majorana $\nu_{R}$ predicts

$$
\begin{aligned}
& \Delta L=2 \\
& \Delta B=2
\end{aligned}
$$

Spontaneous breaking

$S U(2)_{R} \otimes U(1)_{(B-L)} \rightarrow U(1)_{Y} \quad\left\langle\Delta_{R, 44}\right\rangle=v \neq 0$
with Higgs field

## Searches for $n \rightarrow \bar{n}$ : Proposed Improvements

## [Phillips et al, arXiv:1410.1100]

(1) Free-neutron oscillation (similar to ILL):

Maximize Prob $\sim N_{n}{ }^{*}\left(t_{\text {tree }}\right)^{2}$
$\downarrow$ Neutrons from spallation sources:

e.g. European Spallation source: x12 neutron flux
$\checkmark$ Elliptic mirror for slow neutrons (reflect $\sim 70 \%$ of $v_{\perp} \leqslant 40 \mathrm{~m} / \mathrm{s}$ neutrons)
$\checkmark$ Better mag.field screening ( $B<1 n T$ ) and longer flight time


Expected to increase sens. $x \mathbf{1 0}^{2}-10^{3} \mathrm{LLL}, \tau_{n-n} \geqslant 10^{9}-10^{10} \mathrm{~s}$, matter stability bound $\approx 10^{35} \mathrm{yr}$Other proposed experiments:

- stored ultra-cold neutrons ( $4-5 \mathrm{~m} / \mathrm{s}$ )
- vertical cold neutron beams

