Nf=2+1+1 renormalisation of four-quark operators

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Motivations

The story so far

- The Rome-Southampton method with momentum sources has proven to be very efficient
- We have presented last year a $N_f = 2 + 1 + 1$ strategy to treat the charm threshold
- ... and preliminary results for B_K

What more can we do?

- All systematics were not fully under control
- We need generalisation to SUSY B_K and $K \to \pi\pi$

Operators and chiral sectors

sector	dim	B_K	SUSY B_K	$K \to \pi\pi$ $I = 2$	$K \to \pi\pi$ $I = 0$
$(27, 1)$	1	✓	✓	✓	✓
$(8, 8)$	2		✓	✓	✓
$(8, 1)$	4				✓
$(6, 6)$	2		✓		

With our chiral action, we can consider each chiral sector separately.

$$Q^{(27,1)} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma^\mu(1-\gamma_5)d] \quad (1)$$
$$Q_1^{(8,8)} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma^\mu(1+\gamma_5)d] \quad (2)$$
$$Q_2^{(8,8)} = [\bar{s}(1-\gamma_5)d]\,[\bar{s}(1+\gamma_5)d] \quad (3)$$
$$Q_1^{(6,6)} = [\bar{s}\sigma_{\mu\nu}(1-\gamma_5)d]\,[\bar{s}\sigma^{\mu\nu}(1-\gamma_5)d] \quad (4)$$
$$Q_2^{(6,6)} = [\bar{s}(1-\gamma_5)d]\,[\bar{s}(1-\gamma_5)d] \quad (5)$$

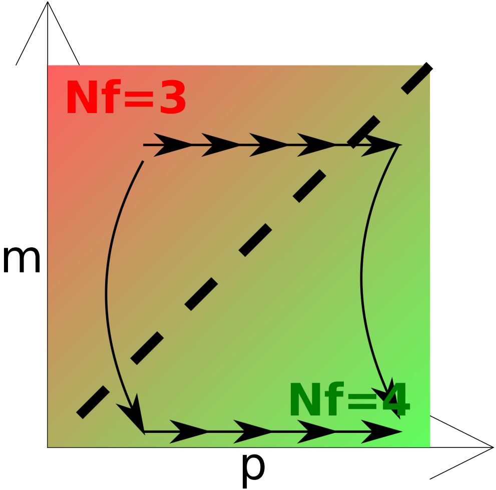
Step-scaling

If one chooses a scaling trajectory and compute at constant physical masses the continuum limit

$$\sigma(p, p_0 \mid m) = \lim_{a \to 0} \frac{Z(p \mid m)}{Z(p_0 \mid m)}, \tag{6}$$

this is universal, i.e. only dependent on the continuum RGEs from one scale to another.
Does not depend on your scaling trajectory, your lattice action, the $O(4)$ orbit of momenta, and so on.
Therefore those ratios are convenient building-blocks that we can combine with as many ratios as we want into a telescopic product.

Charm threshold



Ensembles

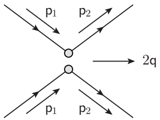
$N_f = 2+1$ Ensembles

Matrix elements have been computed on a wide set of (M)DWF ensembles, including two ensembles at the physical quark masses, and lattice spacing going up to 3 GeV.

$N_f = 2+1+1$ Ensembles

β	$L^3 \times T \times L_5$	m_l	m_c	a^{-1}
5.70	$32^3 \times 64 \times 12$	0.0047	0.243, 0.1, 0.0186	3.0 GeV
5.70	$32^3 \times 64 \times 12$	0.002	0.243	3.0 GeV
5.77	$32^3 \times 64 \times 12$	0.0044	0.213	3.6 GeV
5.84	$32^3 \times 64 \times 12$	0.0041	0.183, 0.0146	4.3 GeV
5.84	$32^3 \times 64 \times 12$	0.002	0.183	4.3 GeV

RI-SMOM scheme



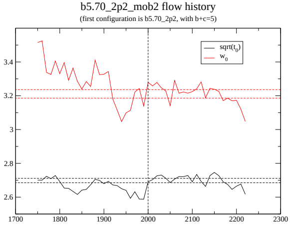
Kinematics

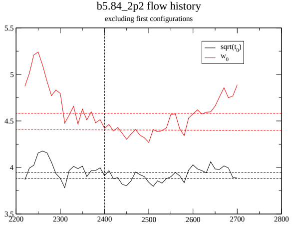
- Non-exceptional schemes avoid π pole
- $p_1^2 = p_2^2 = (p_1 - p_2)^2$
- no $\sum p_i$ combination cancels out
- many orientations satisfy this condition but cont. limit is universal

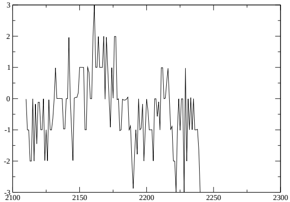
Renormalisation condition

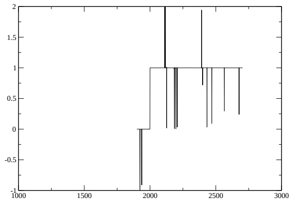
- $Z \, \mathrm{Tr}\left[P_{ijkl} G_{ijkl}\right] = \mathrm{Tr}\left[P_{ijkl} G_{ijkl}\right]|_{\mathrm{tree}}$
- $P_{ijkl} = \gamma_i \delta_{ij} \gamma_k \delta_{kl}$ or $P_{ijkl} = \not{q}_{ij} \not{q}_{kl}$
  different schemes allow us to evaluate the truncation error
- Very versatile method, with many knobs to turn
- Cheap once we have configs, excellent signal/noise

Wilson flow and topological charge







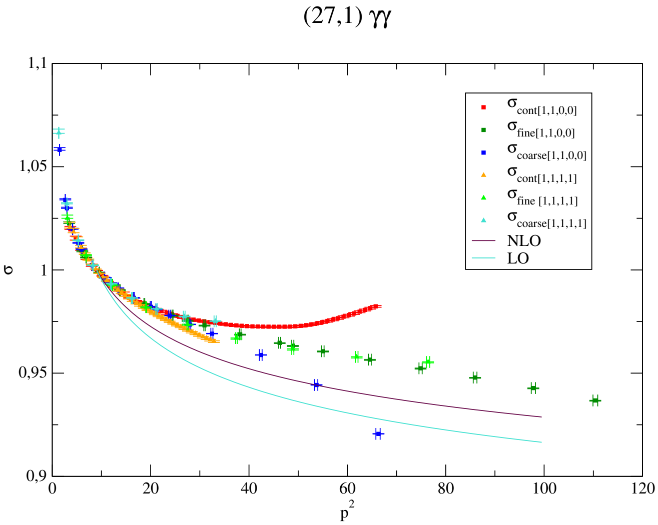


Continuum limit (intro)

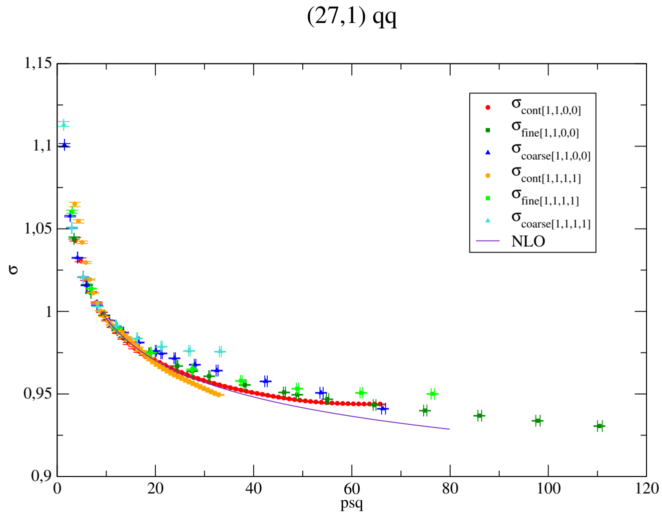
The following results are produced through this process:

1. Compute step-scaling from 3 GeV for each ensemble (and orientation)
2. Interpolate them to a common set of p^2 values
3. For each interpolated p^2 and each orientation, extrapolate independently in a^2
4. The difference between two orientations is an estimate of the systematics from higher order discretisation terms

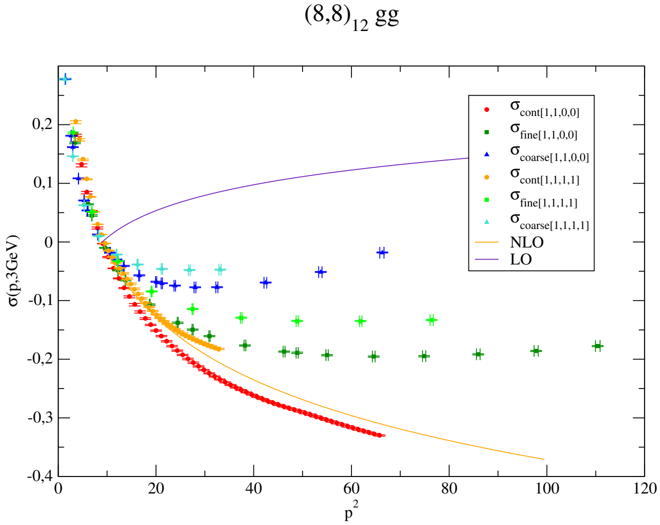
Continuum limit I



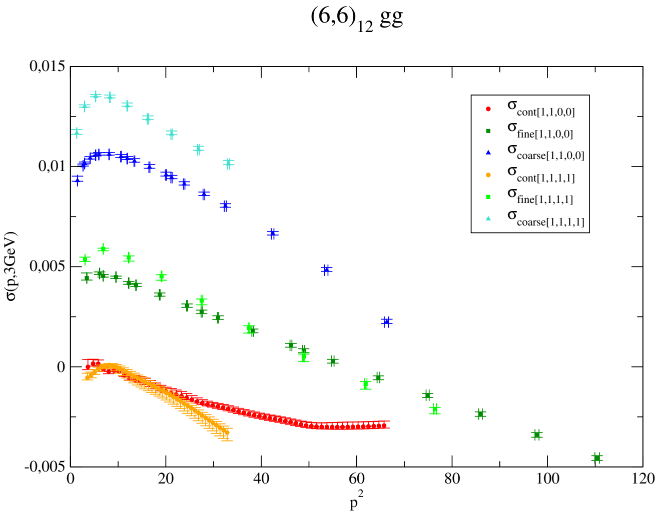
Continuum limit II



Continuum limit III



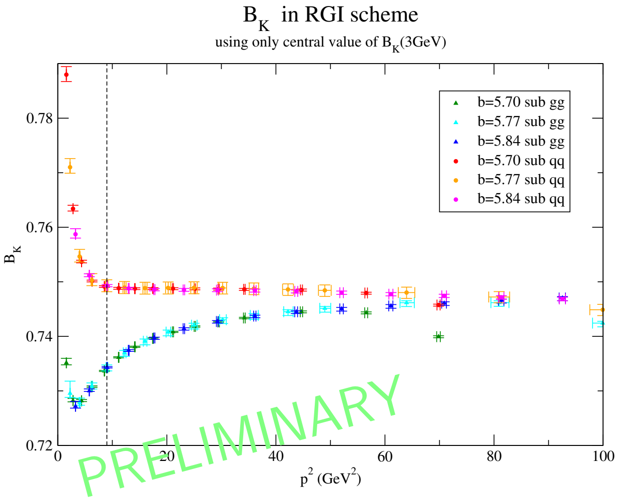
Continuum limit IV



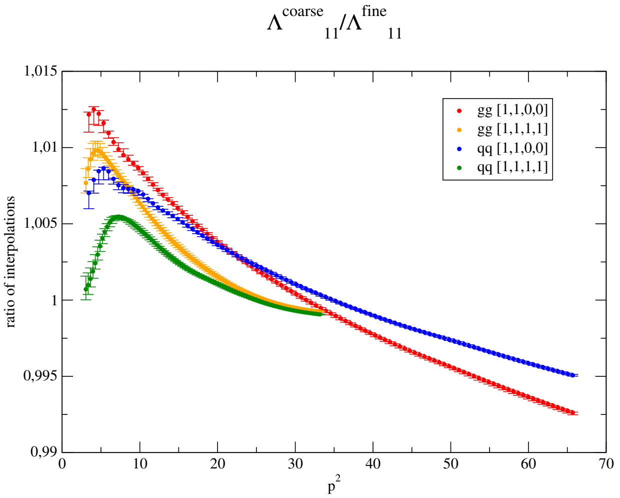
Method

- Because most of the running comes from low-order PT, discretisation effects might be well-described by PT
- However we do not need any explicit perturbative formula
- At high-p^2, Λ_{QCD} becomes irrelevant, dimensional analysis suggest discr is mostly $(ap)^n$-dependent.
  More precisely: the boosted coupling constant only grows logarithmically (see hep-lat/1412.0834)
- When combined with this all-order theoretical argument, the p^2-dependence gives additional information
- Of course it doesn't replace a third ensemble, but the latter would be much more expensive than this project

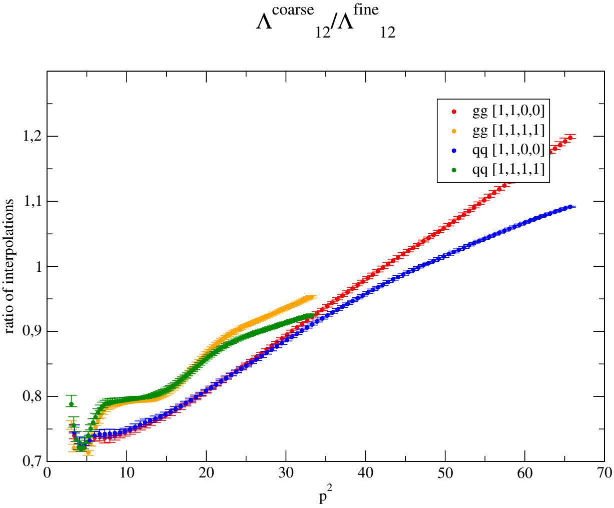
B_K


B_K in RGI scheme
using only central value of B_K(3GeV)
b=5.70 sub gg
b=5.77 sub gg
b=5.84 sub gg
b=5.70 sub qq
b=5.77 sub qq
b=5.84 sub qq
B_K
0.72
0.74
0.76
0.78
0
20
40
60
80
100
p^2 (GeV^2)
PRELIMINARY

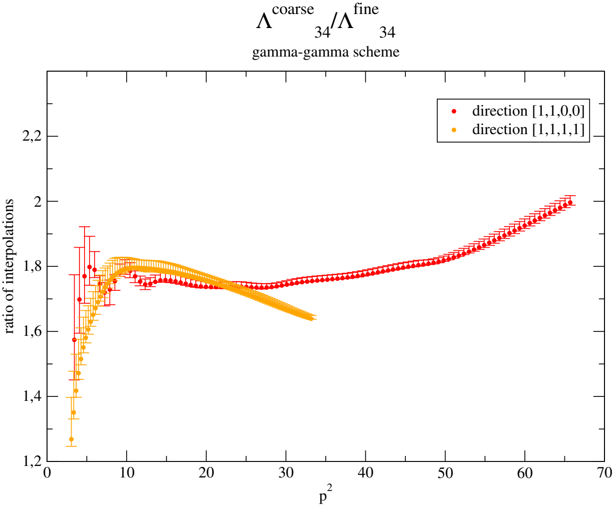
Perspectives for other sectors I



Perspectives for other sectors II



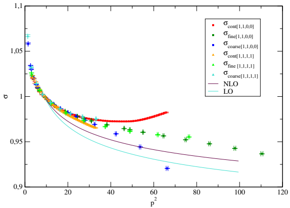
Perspectives for other sectors III



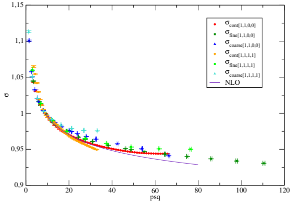
- We have presented preliminary results for the 5×5 SUSY B_K step-scaling
- A tiny "$Nf = 3/Nf = 4$" has not been included here
- With $O(a^2)$ extrapolations the discretisation errors are huge
- A third ensemble would be most useful
- Generating new ensembles, with stats for the low-energy scale-setting, is painful
- It is important to extract as much info as possible from ensembles we already have
- discr. effects in SUSY B_K have a very smooth p^2-dependence, this looks like a relatively operator-independent observation.

Thanks for your attention!

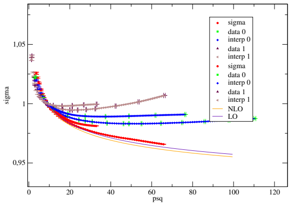
(27,1) γγ



(27,1) qq



results/StepCont_GGdir1_1_1.agr



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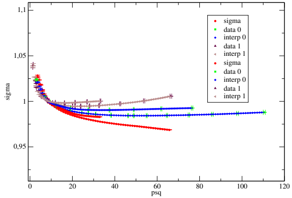
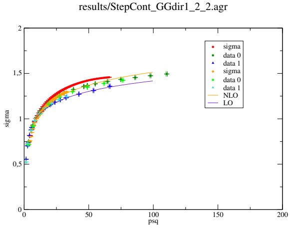
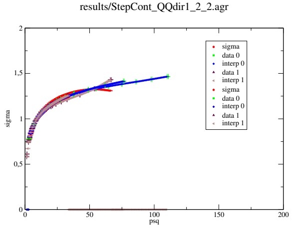
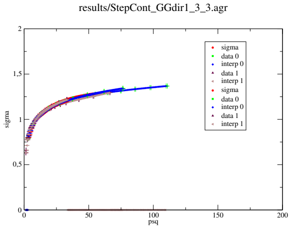



results/StepCont_GGdir1_2_2.agr
sigma
data 0
data 1
sigma
data 0
data 1
NLO
LO
sigma
psq
0
0,5
1
1,5
2
0
50
100
150
200


results/StepCont_QQdir1_2_2.agr
sigma
data 0
interp 0
data 1
interp 1
sigma
data 0
interp 0
data 1
interp 1
sigma
psq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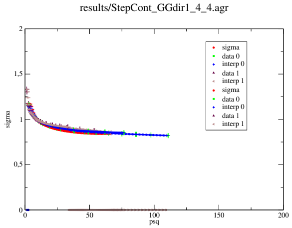
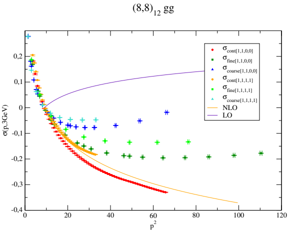
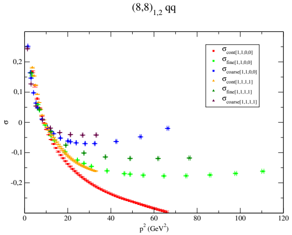
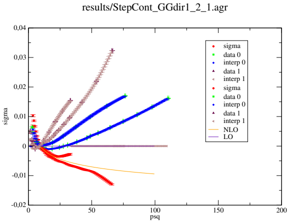
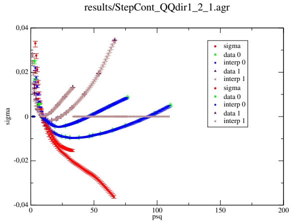
sigma
data 0
interp 0
data 1
interp 1
sigma
data 0
interp 0
data 1
interp 1
sigma
psq

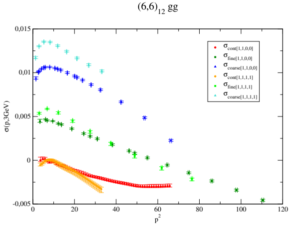

results/StepCont_GGdir1_4_4.agr
sigma
data 0
interp 0
data 1
interp 1
sigma
data 0
interp 0
data 1
interp 1
sigma
psq

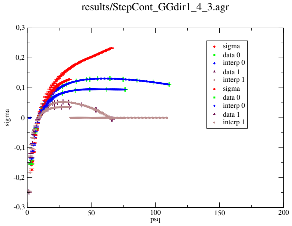

(8,8)12 gg
σ(p,3GeV)
p2
σcont[1,1,0,0]
σfine[1,1,0,0]
σcoarse[1,1,0,0]
σcont[1,1,1,1]
σfine[1,1,1,1]
σcoarse[1,1,1,1]
NLO
LO


(8,8)1,2 qq
σ
p2 (GeV2)
σcont[1,1,0,0]
σfine[1,1,0,0]
σcoarse[1,1,0,0]
σcont[1,1,1,1]
σfine[1,1,1,1]
σcoarse[1,1,1,1]


results/StepCont_GGdir1_2_1.agr
sigma
psq
sigma
data 0
interp 0
data 1
interp 1
NLO
LO


results/StepCont_QQdir1_2_1.agr
sigma
psq
sigma
data 0
interp 0
data 1
interp 1


(6,6)$_{12}$ gg
σ(p,3GeV)
p^2
σcont[1,1,0,0]
σfine[1,1,0,0]
σcoarse[1,1,0,0]
σcont[1,1,1,1]
σfine[1,1,1,1]
σcoarse[1,1,1,1]


results/StepCont_GGdir1_4_3.agr
sigma
psq
sigma
data 0
interp 0
data 1
interp 1
sigma
data 0
interp 0
data 1
interp 1