Topological observables in many-flavour QCD

Ed Bennett
for the LatKMI Collaboration

Swansea University
Prifysgol Abertawe

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The LatKMI Collaboration

- KMI
  - Y. Aoki
  - T. Aoyama
  - T. Maskawa
  - K. Nagai
  - K. Yamawaki
- KEK
  - M. Kurachi
  - A. Shibata
- Lawrence Livermore National Laboratory
  - E. Rinaldi
- Aix*Marseille université CPT
  - K. Miura
- RIKEN
  - H. Ohki
- Swansea University Prifysgol Abertawe
  - E. Bennett
- 筑波大学 University of Tsukuba
  - T. Yamazaki

Outline

Motivation

- Investigation and classification gauge theories is an area of interest
- Topological observables can check ergodicity, as well as a variety of other uses
- Can we identify (near-)conformal gauge theories from their topology?

Topological charge

Topological charge density:

$$q(x) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left\{ F_{\mu\nu}(x) F_{\rho\sigma}(x) \right\}$$

Topological charge:

$$Q = \sum_x q(x)$$

UV fluctuations dominate over topology; remove with the gradient flow:

$$\dot{B}_\mu = D_\nu G_{\mu\nu}$$

$$B_\mu\big|_{t=0} = A_\mu$$

Can also use gradient flow to define scale t_0 as:

$$t^2 E(t)\big|_{t=t_0} = 0.3$$

$$\text{where } E = \frac{1}{4} \mathrm{tr}\, G_{\mu\nu} G_{\mu\nu}$$

Topological susceptibility and instanton size

Topological susceptibility:

$$\chi = \frac{\langle Q^2 \rangle}{V} \equiv \frac{\langle Q^2 \rangle - \langle Q \rangle^2}{V}$$

Instanton size:

$$q_{\mathsf{peak}} = \frac{6}{\pi^2 \rho^4}$$

Frozen topology and subvolumes

- Moving towards physical region (chiral, continuum limits) can trap simulation at one Q.
- $\Rightarrow$ Must verify sufficient ergodicity.
- Can we find χ for frozen ensembles with insufficient statistics to estimate $\langle Q^2 \rangle$?
- Yes: look instead at a finite subvolume V_s. Then:

$$Q_\mathrm{s} = \sum_{V_\mathrm{s}} q(x)$$

$$\chi = \frac{\langle Q_\mathrm{s}^2 \rangle - \frac{V_\mathrm{s}}{V} \langle Q \rangle^2}{V_\mathrm{s}}$$

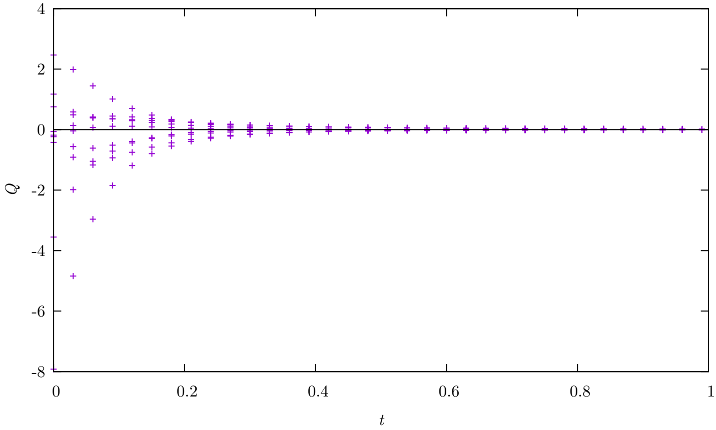
Topological behaviour near the conformal window

- Conformal theory with finite deforming mass behaves as confining with heavy fermions
- Thus topological observables will be as in pure gauge theory
- Deforming mass will alter scale of theory, so match with appropriate observables

Setup

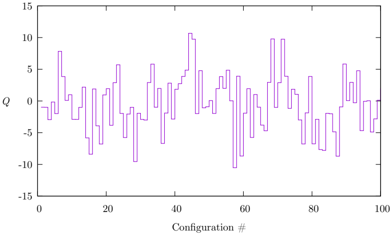
- Symanzik gauge action
- HISQ fermion action
- LatKMI configurations:
  - $N_f = 4$: $V = 30 \times 20^3$, $\beta = 3.7$
  - $N_f = 8$: $V = \frac{4}{3}L \times L^3$, $18 \le L \le 42$, $\beta = 3.8$
  - $N_f = 12$: $V = \frac{4}{3}L \times L^3$,$18 \le L \le 36$, $\beta = 3.7, 4.0$
  - $N_f = 16$: $V = 24^4, 48^4$, $\beta = 12.0$
  - Plus pure gauge: $V = 32 \times 24^3$, $4.0 \le \beta \le 5.0$

$N_\mathsf{f} = 16$

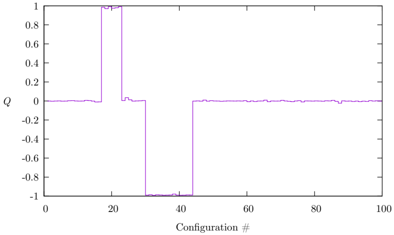


- Topology is strongly suppressed; $q(x)$ is zero
- Volume is too small
- Ignored in subsequent analysis

$N_\mathsf{f} = 0$



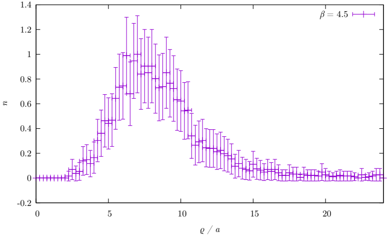
$\beta = 4.5$



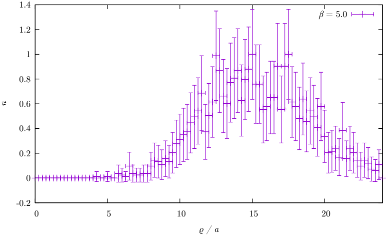
$\beta = 5.0$

- Moving towards continuum limit freezes topology
- Subvolume method used at larger β

$N_\mathsf{f} = 0$ size distribution



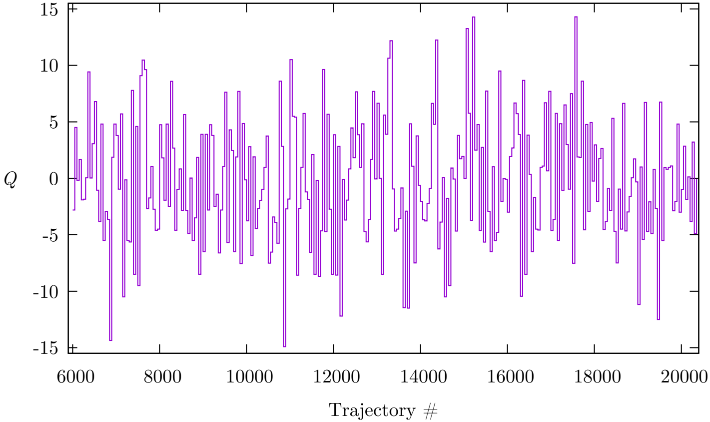
$\beta = 4.5$



$\beta = 5.0$

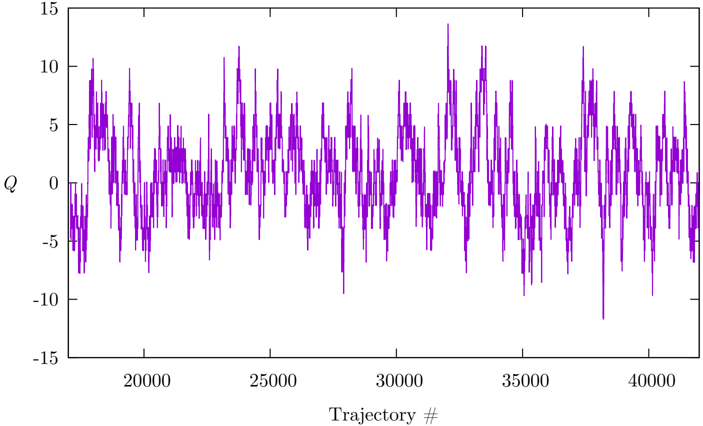
- $\beta = 5.0$ is overly volume constrained
- Ignored from subsequent analysis

$N_f = 4$



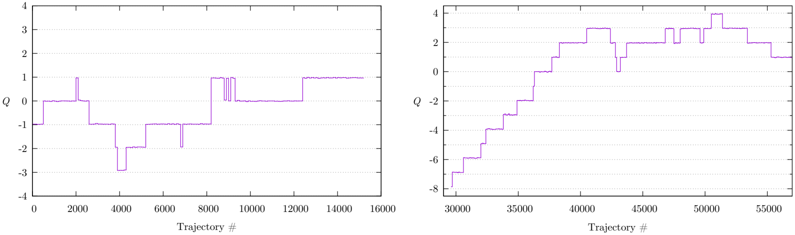
- Good ergodicity

$N_f = 8$



- Slightly more autocorrelation than $N_f = 4$, but good ergodicity still

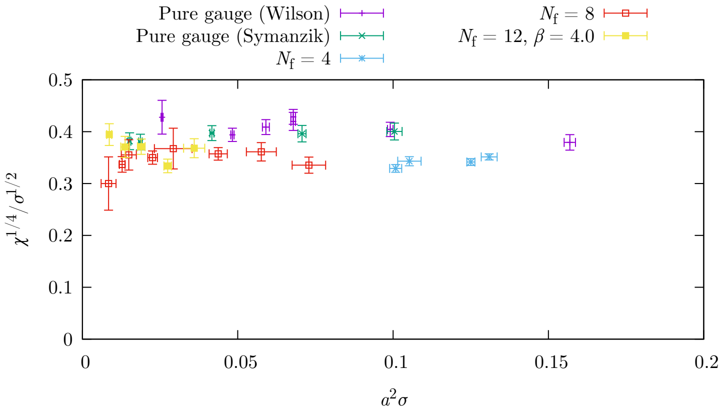
$N_f = 12$



$\beta = 3.7$ $\beta = 4.0$

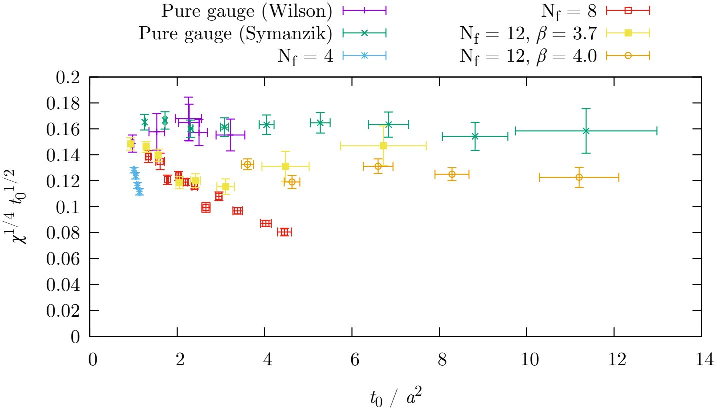
- Obvious freezing, becoming more severe at low m
- Subvolume method used here

Scaling with $a\sqrt{\sigma}$



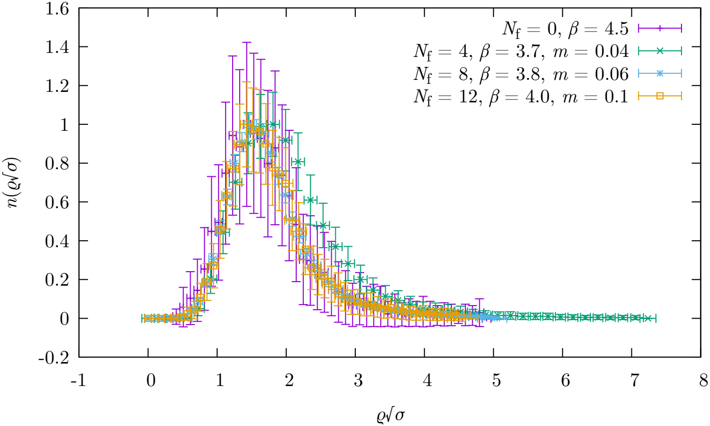
- $N_f = 0$ is roughly flat
- $N_f = 12$ is near-flat; matches $N_f = 0$ at low $a\sqrt{\sigma}$
- $N_f = 4$ has positive gradient
- $N_f = 8$ matches $N_f = 12$ at high $a\sqrt{\sigma}$, but turns over moving towards the chiral limit

Scaling with t_0



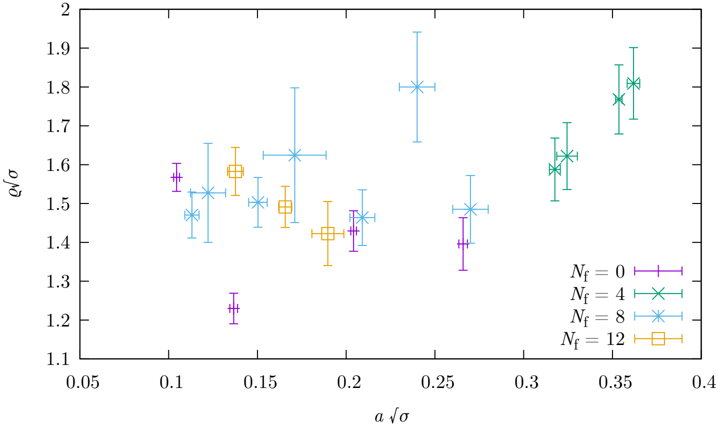
- Dimensionless product $\chi^{1/4} t_0^{1/2}$ is flat for pure gauge
- Theories roughly match in quenched (small t_0) limit
- $N_f = 12$ rapidly flattens off; $N_f = 8, 4$ have increasingly steep gradients

Instanton size distribution



- Instanton size distributions for $N_\mathrm{f} = 8, 12$ match $N_\mathrm{f} = 0$ in physical units
- $N_\mathrm{f} = 4$ diverges slightly
- Can we use this to view $\langle \rho \rangle$ as a function of m_f?

Scaling of ρ



- Meaning here unclear.

Conclusions

- LatKMI's $N_\mathsf{f} = 4$ and 8 QCD simulations are topologically ergodic; $N_\mathsf{f} = 12$ is borderline but shows ergodicity in the topological charge density
- Scaling of χ consistent with $N_\mathsf{f} = 12$ being (near-)conformal, $N_\mathsf{f} = 8$ walking, $N_\mathsf{f} = 4$ confining and chirally broken
- Instanton size somewhat supports these results, but better understanding needed

Next steps:

- A look at $N_\mathsf{f} > 12$ without constricted volume would be interesting
- C.f. $\mathrm{SU}(2)$ with fundamental matter