Gradient flow and IR fixed point in SU(2) with 8 flavors

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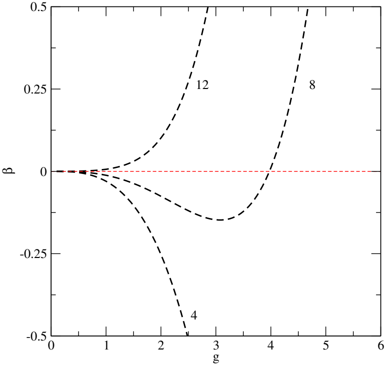
Intro

- A Class of gauge theories have an IRFP
- Nearly conformal theories can have walking behavior needed by technicolor
- β-function can be measured with gradient flow method
- SU(2) with 8 massless fundamental fermions [1] [2]
- IRFP expected at large coupling



[1] H. Ohki, et al., PoS LATTICE2010 (2010) 066 (hep-lat/1011.0373) ,

[2] J. Rantaharju, et al., PoS LATTICE2014 (2014) 258 (hep-lat/1411.4879)

The Model

$$S = (1 - c_g)S_G(U) + c_g S_G(V) + S_F(V)\,,$$

$$U_i(x) = 1\,,\ \text{when } x_0 = 0, L\,, \qquad U_\mu(x + L\hat{i}) = U_\mu(x)$$

$$\psi(x)\ = 0\,,\ \text{when } x_0 = 0, L\,, \qquad \psi(x + L\hat{i}) = \psi_\mu(x)$$

- HEX-smeared[1] gauge field V, unsmeared U
  - Three sequential stout smearing steps using only orthogonal directions
- Gauge action smearing tuned by $c_g = 0.5$

- S_G standard Wilson single plaquette gauge action
- S_F Clover improved Wilson fermion action $c_{SW} = 1$
- Schrödinger functional boundary conditions
- Trivial time boundaries

[1] S. Capitani, S. Durr and C. Hoelbling, JHEP 0611 (2006) 028

Gradient Flow

$$\partial_t B_{t,\mu} = D_{t,\mu} G_{t,\mu\nu} \, ,$$
$$B_{0,\mu} = A_\mu \, ,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \, .$$

- Smoothing operation of the initial gauge field
- Fix flow time t to L by setting scale: $c = \sqrt{8t}/L = 0.4$
- Measuring field strength and topological charge:

$$\langle E(t) \rangle = \frac{1}{4} \langle G_{t,\mu\nu} G_{t,\mu\nu} \rangle = \frac{3(N^2-1)g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4)$$
$$Q = \frac{1}{32\pi^2} \sum_x \sum_{\mu,\nu} \tilde{G}^a_{\mu\nu}(t,x) G^a_{\mu\nu}(t,x)$$

- Fixed boundary conditions: measure energy only using the middle time slice $x_0 = L/2$

Coupling from Gradient Flow

- Coupling defined as[1 2]:

$$g_{GF}^2 = \frac{1}{\mathcal{N}} t^2 \langle E(t) \rangle$$

- We improve the continuum limit by adding a correction to flow time[2]:

$$\begin{aligned} g_{GF}^2 &= \frac{t^2}{\mathcal{N}} \langle E(t + \tau_0 a^2) \rangle \\ &= \frac{t^2}{\mathcal{N}} \langle E(t) \rangle + \frac{t^2}{\mathcal{N}} \langle \frac{\partial E(t)}{\partial t} \rangle \tau_0 a^2 \end{aligned}$$

- τ_0 fixed by hand to remove a^2 effects

[1] M. Luscher and P. Weisz , JHEP 1102 (2011) 051 (hep-th/1101.0963) ,
[2] P. Fritzsch and A. Ramos , JHEP 1310 (2013) 008 (hep-lat/1301.4388) ,

[3] A. Cheng, A. Hasenfratz, Y. Liu, G. Petropoulos and D. Schaich. JHEP 1405 (2014) 137 (hep-lat/1404.0984)

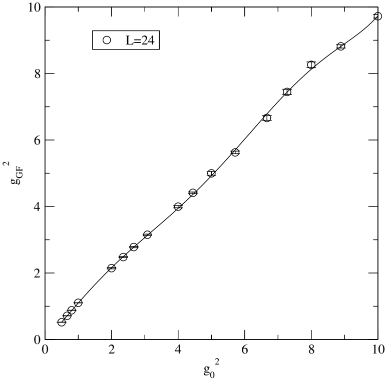
Step scaling function

$$\Sigma(u,s,a/L) = g^2_{GF}(g_0, s\frac{L}{a})\Big|_{g^2_{GF}(g_0,\frac{L}{a})=u},$$

$$\sigma(u,s) = \lim_{a/L\to 0}\Sigma(u,s,a/L)$$

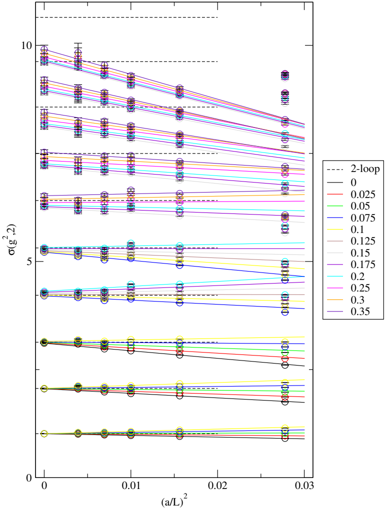
- s=2
- Can be used to measure running of the coupling
- Fix τ_0 in continuum limit: $\Sigma(u,a/L) = \sigma(u) + c((\frac{a}{L})^2)$
- Calculated using interpolated coupling:

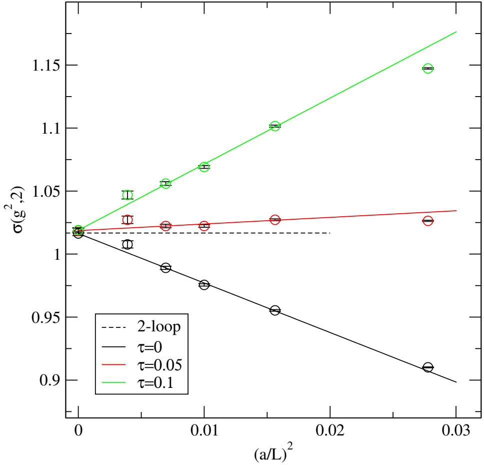
$$g^2_{GF}\left(g_0,\frac{a}{L}\right) = g_0^2\left[1+\sum_{i=1}^{6} a_i g_0^{2i}\right]$$



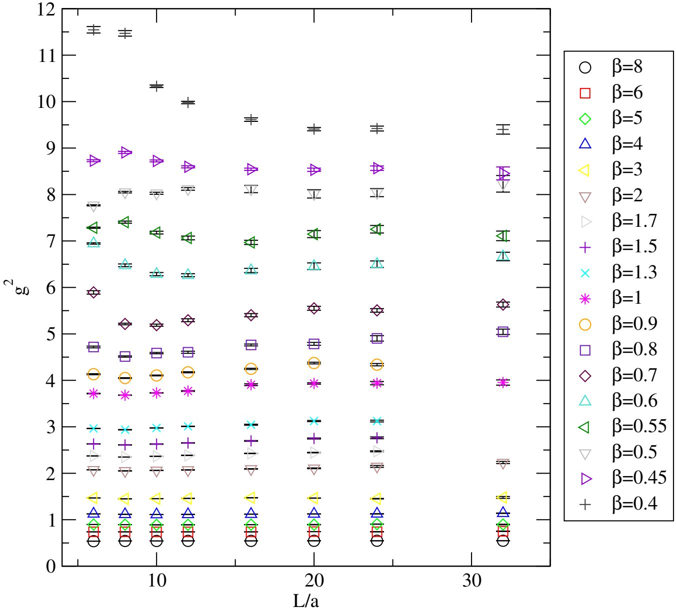
Fixing τ

- Choosing $\tau = 0.032665 g_0$ for $c = 0.4$

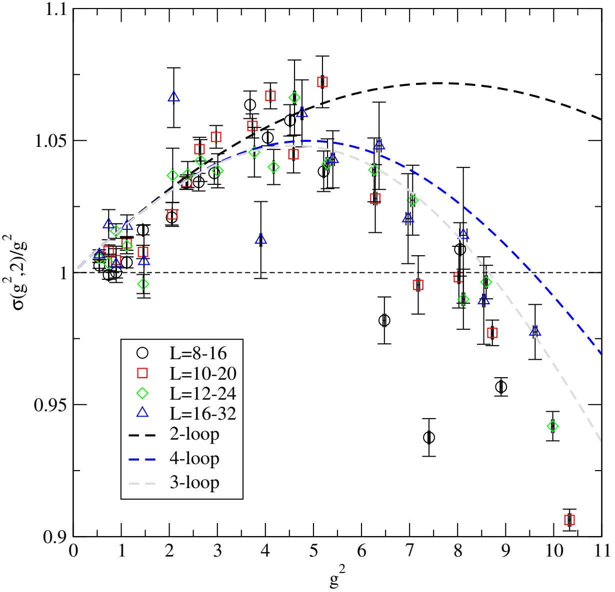




Measured couplings

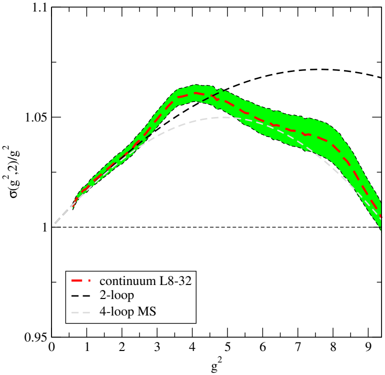


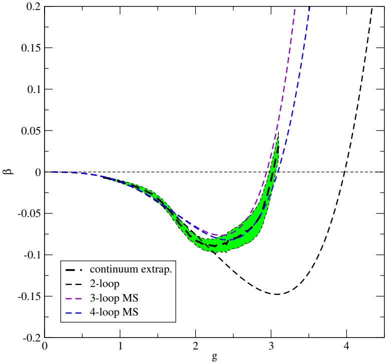
Step scaling on the lattice



Continuum limit and the β

- β-function approximated near IRFP as: $\beta(g) \approx \frac{g}{2\ln(s)}\left(1 - \frac{\sigma}{g^2}\right)$





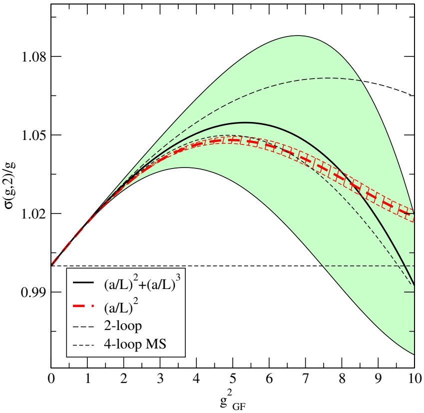
Step scaling function (Alternative continuum limit)

- Parametrize both the continuum step scaling function and discretization errors as polynomial functions of the coupling
- Constrain $c_{1,2}$ to perturbative values

$$\Sigma(u, \frac{a}{L}) = \sigma(u) + \sum_{k=2}^{n_0} f_k(u) \frac{a^k}{L^k}$$

$$\sigma(u) = 1 + \sum_{l=1}^{m} c_l u^l$$

$$f_k(u) = \sum_{l=0}^{n_k} a_{k,l} u^l$$



PRELIMINARY

Mass anomalous dimension

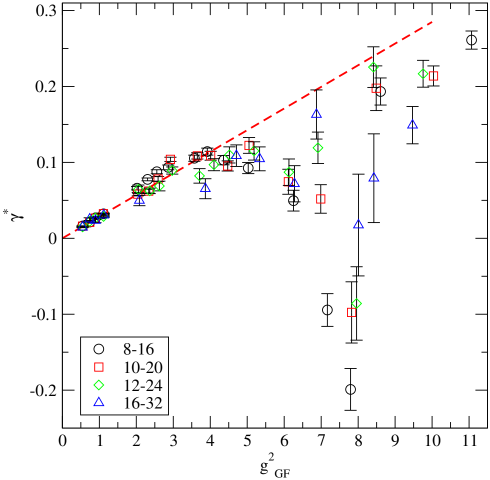
- Schrödinger functional pseudoscalar density renormalization constant allows calculation of γ_m[1]
- Near fixed point approximate as $\gamma^\star$

$$Z_P(g_0, \frac{L}{a}) = \frac{\sqrt{3f_1}}{f_p(L/2a)}$$

$$\Sigma_P(u, \frac{a}{L}) = \left.\frac{Z_P(g_0, \frac{2L}{a})}{Z_P(g_0, \frac{L}{a})}\right|_{g^2_{GF}=u}$$

$$\sigma_P(g^2) = \lim_{a\to 0} \Sigma_P(g^2, \frac{a}{L})$$

$$\gamma^\star = -\frac{\log \sigma_P(g^2)}{\log 2}$$



Note: Poster by Joni Suorsa about γ with spectral density model (21)

[1] S. Capitani, M. Luscher, R. Sommer and H. Witting *Nucl. Phys.* B 544 (1999) (hep-lat/9810063)

Conclusions

- SU(2) with 8 fundamental representation fermions
  - Running coupling
    - Gradient flow coupling
    - τ-correction
    - Step scaling function
  - Mass anomalous dimension from Schrödinger functional
- We can get up to relatively large couplings
- We see clear indication of IRFP in the step scaling function