The curvature of the chiral phase transition line for small values of the chemical potential

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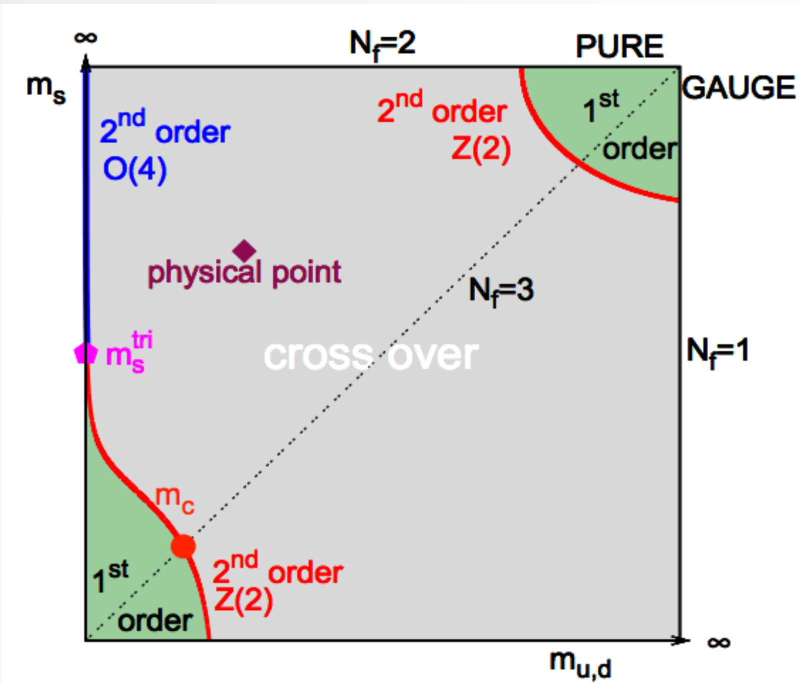
The chiral phase transition



Columbia plot

The phase transition between the broken and restored phases is believed to be second-order in the limit of the light quark mass $m_l \to 0$ and $m_s > m_s^{tric}$.

There is some evidence that the universality class is O(4), in accordance with the Pisarski-Wilczek picture [Pisarski and Wilczek, PRD29, 338 (1984)].

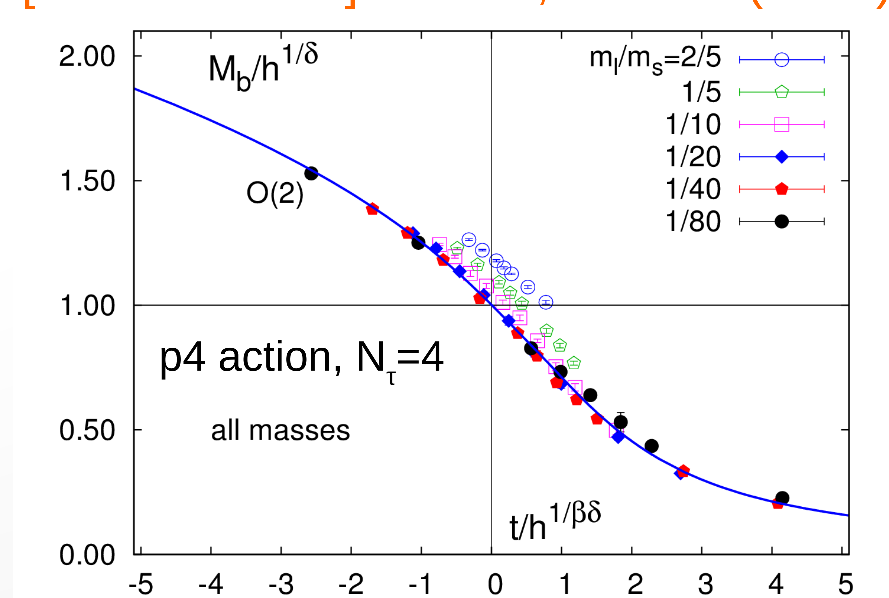
$$t = \frac{1}{t_0}\frac{T - T_c}{T_c}$$

$$h = \frac{H}{h_0}$$

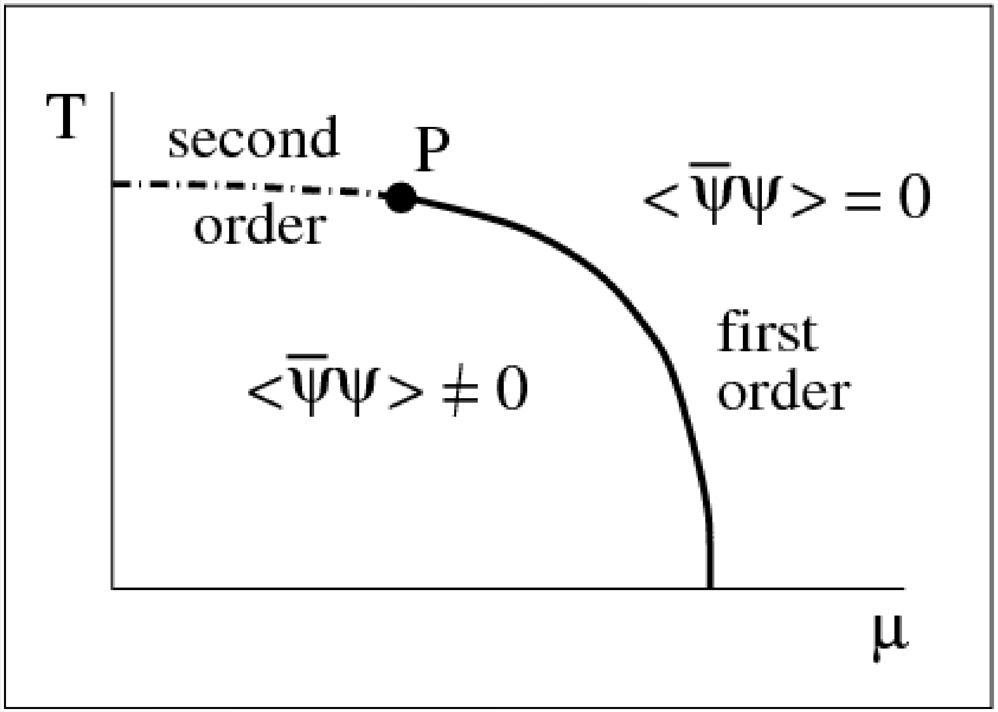
$$z = t/h^{1/\beta\delta}$$

$$M = h^{1/\delta} f_G(z)$$

[S.Ejiri et al. [BNL-Bielefeld] PRD80, 014504 (2009)]



What happens at μ>0?



The second-order transition meets a first-order line at the tricritical point P [Y.Hatta & T.Ikeda, PRD 67, 014028 (2003)].

The quark chemical potential μ does not break chiral symmetry.

Second-order transition persists; however the scaling variable now becomes μ-dependent.

$$t = \frac{1}{t_0}\left[\frac{T - T_c}{T_c} + \kappa\left(\frac{\mu}{T}\right)^2\right]$$

curvature

$$t \sim \frac{T - T_c(\mu)}{T_c(\mu)} = 0 \implies T_c(\mu) = T_c(0)\left[1 - \kappa\left(\frac{\mu}{T}\right)^2 + \ldots\right]$$

Calculating the curvature

Lattice simulations not possible at μ>0, however:

- Derivatives of observables w.r.t. μ defined at μ=0. Therefore,
- Extend the scaling functions to μ>0

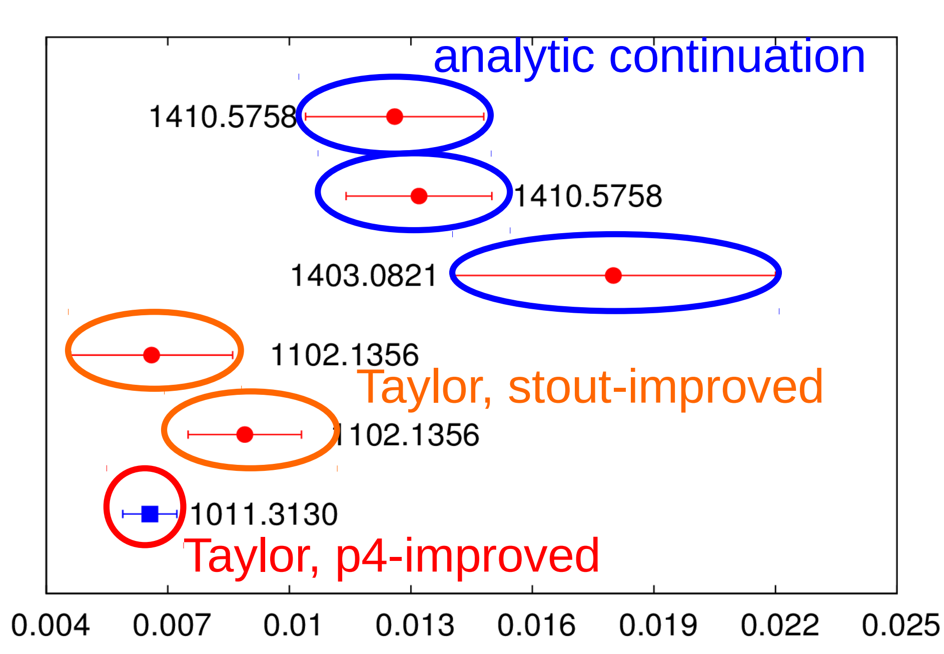
$$t = \frac{1}{t_0}\left[\frac{T-T_c}{T_c} + \kappa\left(\frac{\mu}{T}\right)^2\right] \qquad z = t/h^{1/\beta\delta}$$

- and calculate derivatives of chiral observables e.g.

$$\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial(\mu/T)^2} = \frac{\chi_{\mathrm{mixed}}}{T} = \frac{2\kappa T}{t_0 m_s} h^{-(1-\beta)/\beta\delta} f'_G(z)$$

Alternatively, work at imaginary μ and perform an analytic continuation to real μ [de Forcrand and Philipsen, NP B642, 270 (2002), B673, 190 (2003); M.D'Elia and M.P.Lombardo, PR D67 014505 (2003) D70 014709 (2004)].

Curvature calculation: Current status



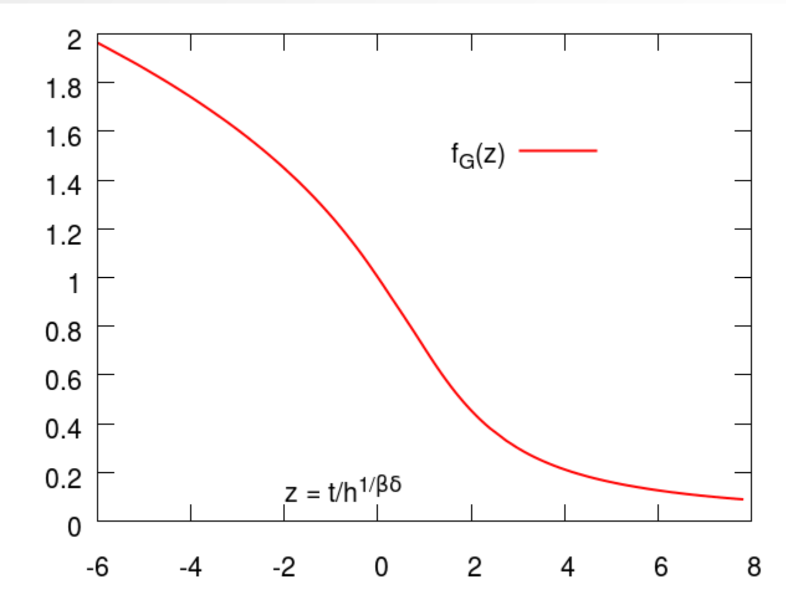
Difference of more than a factor of 2 between the various results.

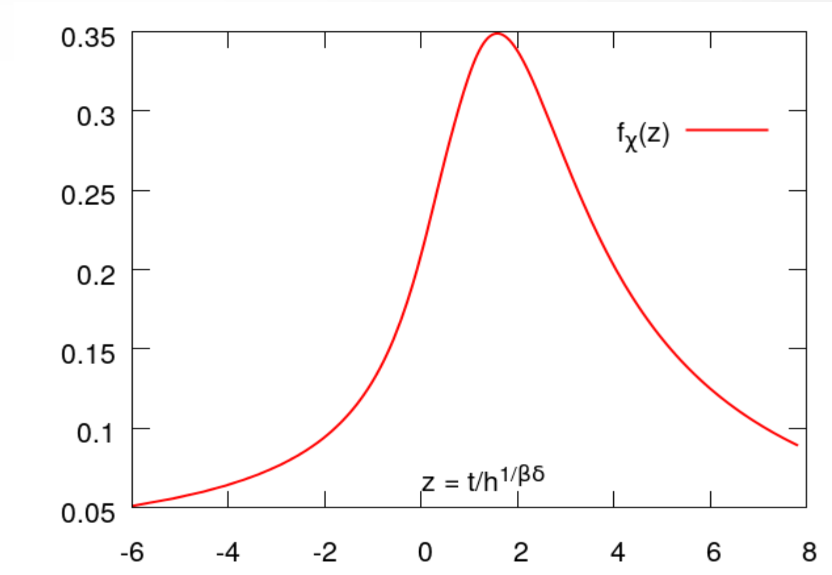
Our previous calculation was done using the p4 action on $N_\tau=4$ lattices. In this talk we will report on an improved calculation done on $N_\tau=6$ lattices with the HISQ action.

The strange quark was set to its physical value, and the light quark mass was varied so that 160 MeV ≥ m_π ≥ 80 MeV.

For each quark mass, we generated ~10,000 configurations for 4-5 temperatures in the transition region, and measured the chiral condensate and its μ-derivatives stochastically, using ~500 random sources.

Light quark chiral observables





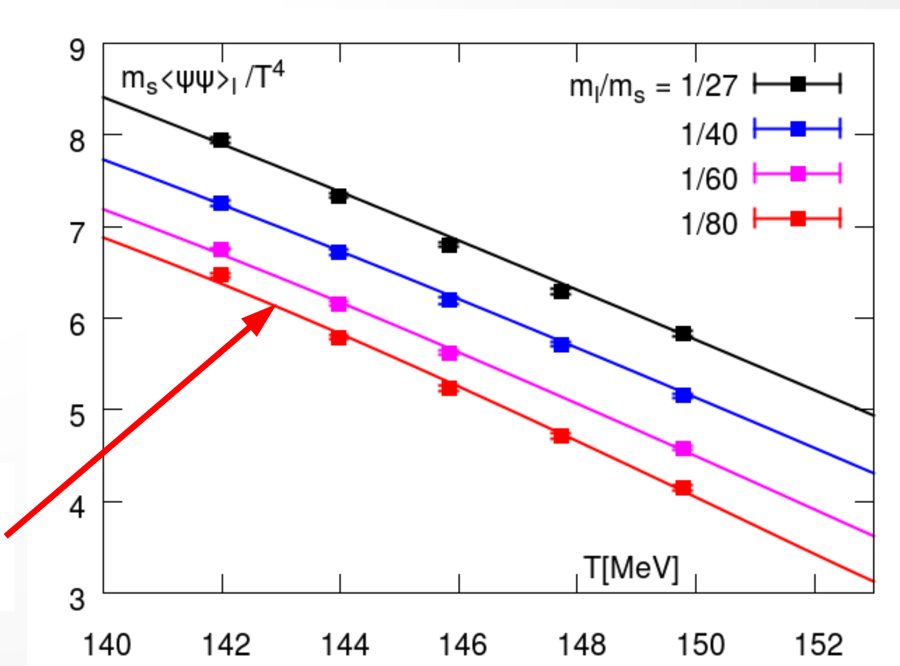
$$\langle\bar{\psi}\psi\rangle = \langle \mathrm{tr} M^{-1}\rangle$$

connected / disconnected

$$\chi^{\mathrm{total}} = \langle \mathrm{tr} M^{-2}\rangle + \langle (\mathrm{tr} M^{-1})^2\rangle - \langle \mathrm{tr} M^{-1}\rangle^2$$

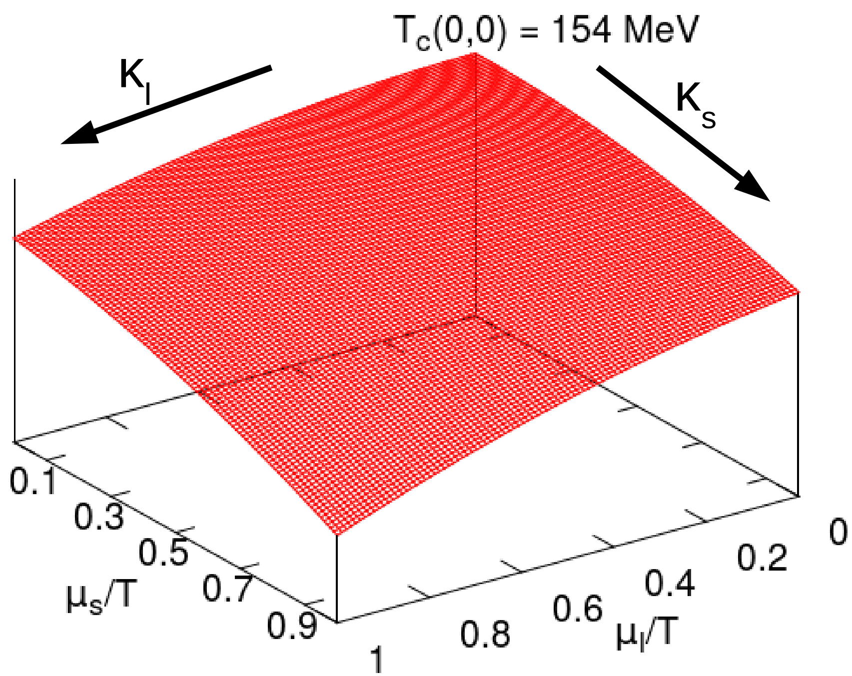
universal scaling function / regular contribution

$$h^{1/\delta} f_G(z) + \frac{m_l}{m_s}\left(a_0 + a_1\left(\frac{T-T_c}{T_c}\right)\right)$$

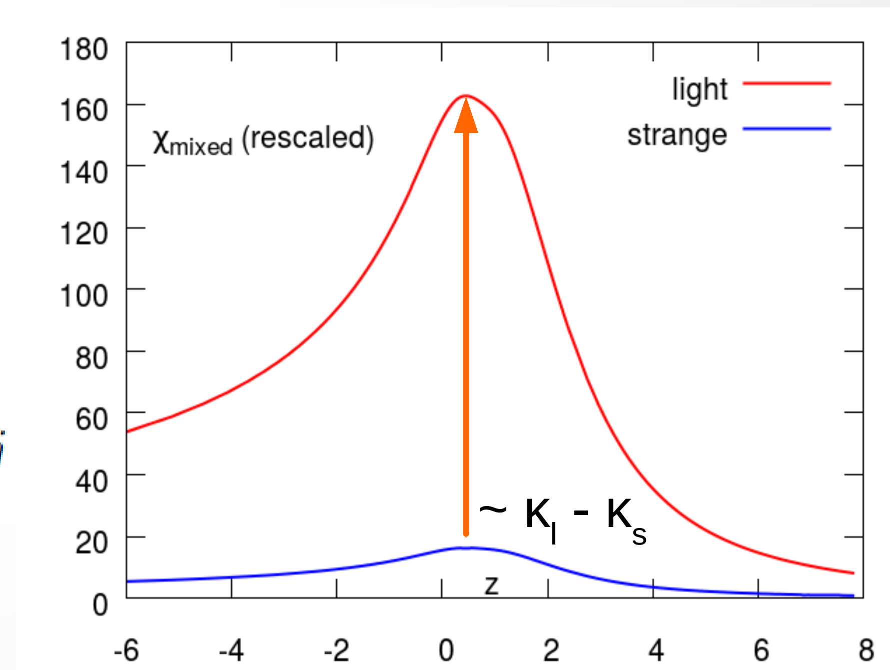


See the talk by H.-T.Ding on Wednesday

The curvature matrix



Once t_0, h_0 and T_c are determined from a scaling analysis, the only unknown in the formula below is κ_{ij}.



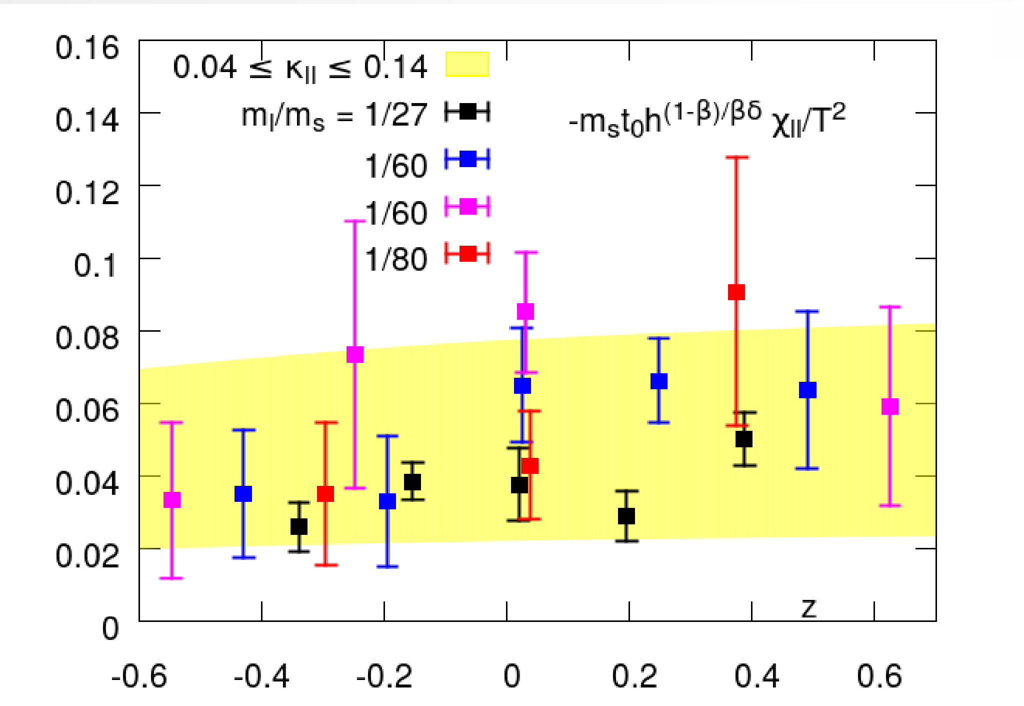
$$\frac{1}{T}\frac{\partial\langle\bar{\psi}\psi\rangle_l}{\partial\mu_i\partial\mu_j} = \kappa_{ij}\frac{2T}{t_0 m_s}\, h^{-(1-\beta)/\beta\delta} f'_G(z) \qquad i,j$$

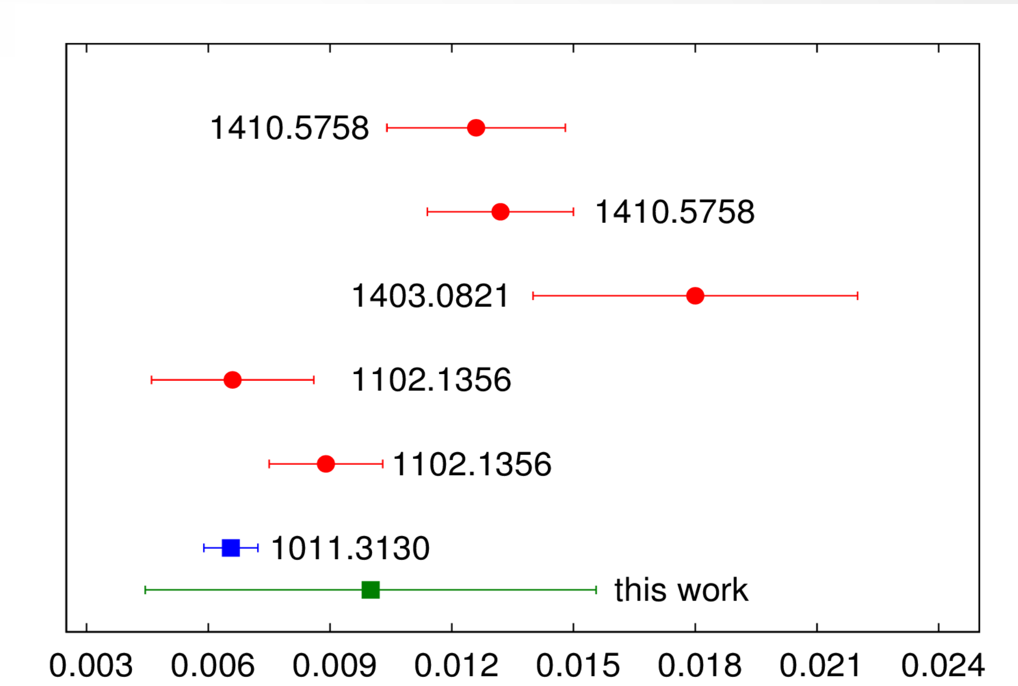
Curvature coefficients $\quad i,j = l,s$

Mixed susceptibility

The same scaling function controls the behavior along both μ_l- and μ_s-directions.

The curvature along the μ_l direction



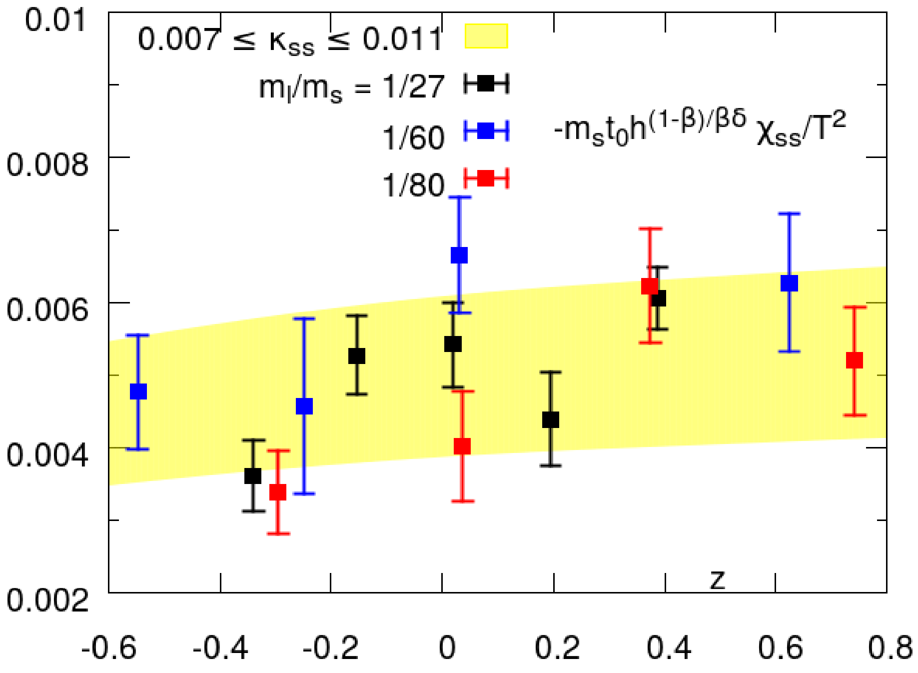


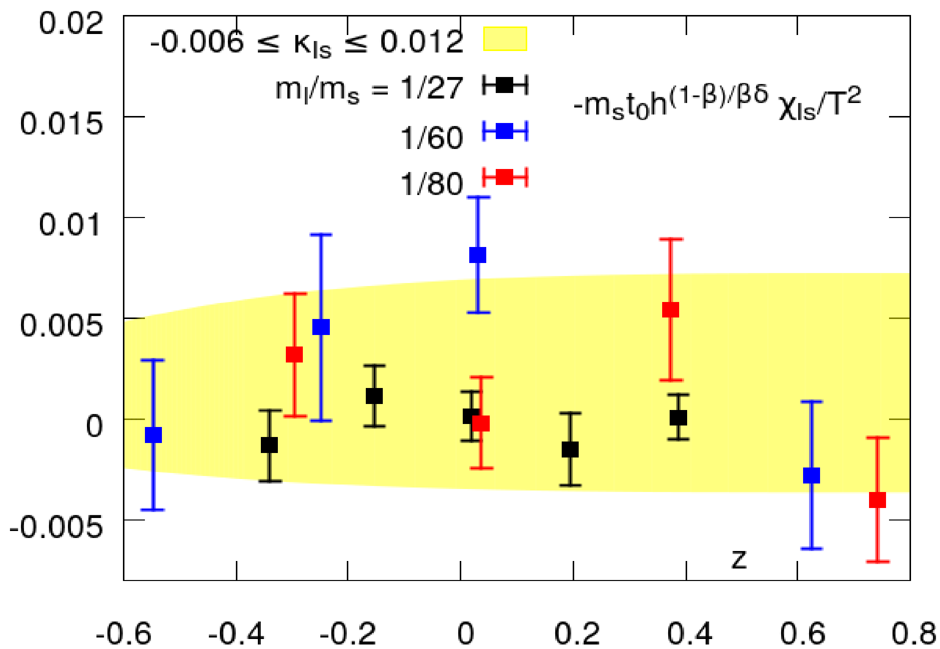
Rough estimate: Vary κ_{ll} by hand to obtain upper and lower bounds.

Rigorous approach: Do a global one-parameter (i.e. κ_{ll}) fit to all quark masses. Should also lead to smaller errors on κ_{ll}.

Our new results are consistent with our previous result [arXiv:1011.3130] that was done using the p4 action and on N_τ=4 lattices.

The μ_s and μ_l-μ_s curvatures





The new observables that we have compared to [arXiv:1011.3130] are κ_{ss} and κ_{ls}.

κ_{ss} is an order of magnitude smaller than κ_{ll}. This tells us that $\kappa_B \approx \kappa_{ll}/9$ to a very good approximation.

Currently, the off-diagonal curvature κ_{ls} is less constrained than both κ_{ll} and κ_{ss} and could have either sign. However, it is very unlikely to be bigger than κ_{ss}.

Summary

The chiral transition temperature as a function of μ_B can be determined from the universal properties of QCD.

For small values of μ_B and in 2+1-flavor QCD, the behavior is characterized by a curvature matrix.

We are currently in the process of calculating the elements of this matrix using HISQ fermions.

We found that the largest matrix element is κ_{ll}. This implies that $\kappa_B=\kappa_{ll}/9$ to a very good approximation.

Of the other two elements, both κ_{ls} and κ_{ss} are an order of magnitude smaller. κ_{ls} could even be zero or negative.