On calculating **disconnected-type hadronic light-by-light scattering diagrams** from lattice QCD

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Background and motivation

- We focus on the anomalous magnetic dipole moment of the muon, a_μ.

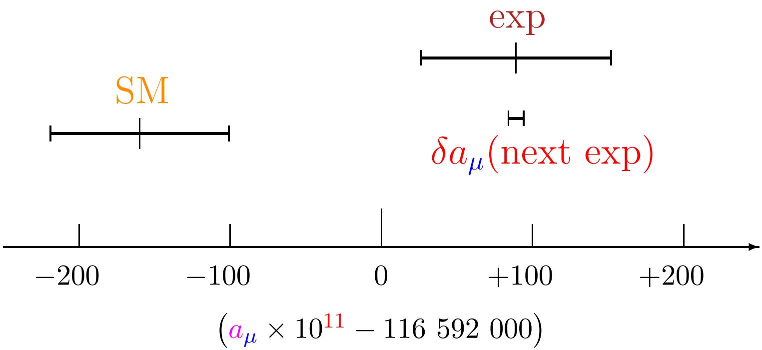
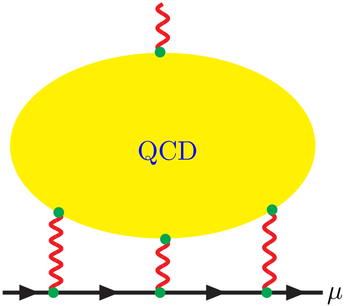


Figure: Theoretical prediction (SM) to a_μ and its measured value (exp).

Hadronic light-by-light scattering contribution

- There are *two* kinds of QCD contributions important to the most precise comparison between $a_\mu(\mathrm{SM})$ and $a_\mu(\mathrm{exp})$:
  - Hadronic vacuum polarization contribution (HVP) induced by

  - Hadronic light-by-light scattering contribution (HLBL), $a_\mu(\text{had-lbyl})$



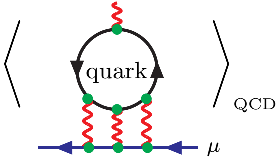
Significance of lattice study of HLBL

$10^{11} \times a_\mu(\text{had-lbyl}) =$	$116\ (40)$
$10^{11} \times \{a_\mu(\text{exp}) - a_\mu(\text{SM})\} =$	$249\ (87)$
$10^{11} \times \delta a_\mu(\text{next exp}) =$	$O(1)$

- Despite its importance, $a_\mu(\text{had-lbyl})$ has been *only estimated* by the low energy effective theory and the models with several hadrons as dynamical variables.
- Imprecise knowledge on chiral dynamics and igonarance on the other hadronic intermediate state contributions *hinder quantification of uncertainty*.
- Computation of $a_\mu(\text{had-lbyl})$ by means of the lattice QCD is indispensable to make a more reliable prediction of a_μ.

Connected-type diagrams in HLBL

- Check of feasibility of lattice QCD study on HLBL :
  - T. Blum, S. Chowdhury, M. H and T. Izubuchi, Phys. Rev. Lett. **114**, 012001 (2015) [arXiv:1407.2923 [hep-lat]]
  - Luchang's talk on new methods and extensive calculation
- For the hadronic light-by-light scattering amplitude, J. Green. O. Gryniuk, G. Hippel, H. B. Meyer and V. Pascalutsa, arXiv:1507.01577.
- Lattice QCD simulation has been carried out for the connected-type diagrams :

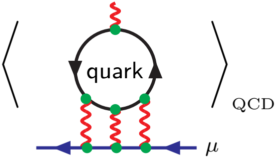


$$+\ (\text{permutations of QED vertices}(\bullet)\ \text{on muon side})\,,$$

where each line in the quark part denotes $D_q[U]^{-1}$ for a given gluon field U.

Disconnected-type diagrams in HLBL

- The connected-type diagrams are the ones in which four QED interaction vertices in the QCD part lie on a single quark loop.



- There are disconnected-type diagrams in which four QED vertices in the QCD part are distributed over more than one quark loop.

An example of disconnected-type diagram; $(1_E,\ 1,\ 1,\ 1)$-type

- If four QED interaction vertices are distributed over $\textit{four separate quark loops}$,

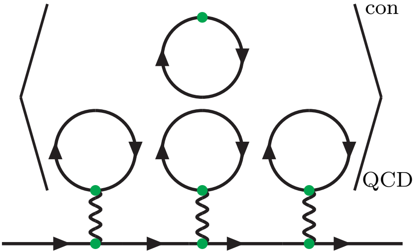


Figure: $(1_E,\ 1,\ 1,\ 1)$-type diagrams

Quark loops are connected with one another by the gluons to be supplied by the average over QCD configurations.

All disconnected-type diagrams

- Disconnected-type diagrams can be classified *according to how* four QED interaction vertices are distributed over more than one quark loop :

$$(3_E,\,1)$$
$$(2_E,\,2)$$
$$(1_E,\,3)$$
$$(2_E,\,1,\,1)$$
$$(1_E,\,1,\,2)$$
$$(1_E,\,1,\,1,\,1)$$

- Each number denotes the total number of QED vertices contained in a quark loop.
- In particular, the number with the subscript E denotes the total number of QED vertices contained in the quark loop with the external QED vertex.

Diagrams of $(3_E,\, 1)$-type

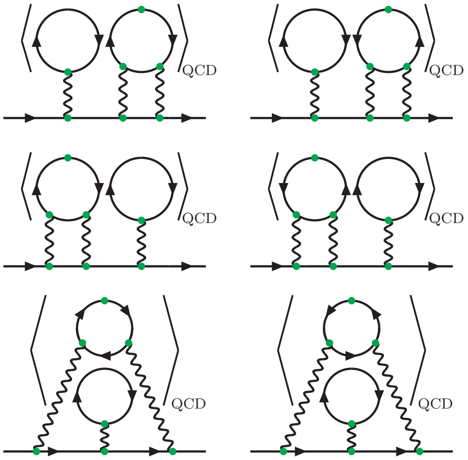


FIG. 1: $(3_E,\, 1)$-type diagrams. The diagrams with $O(a)$ local QED vertices are not shown.

Diagrams of $(2_E,\, 2)$-type

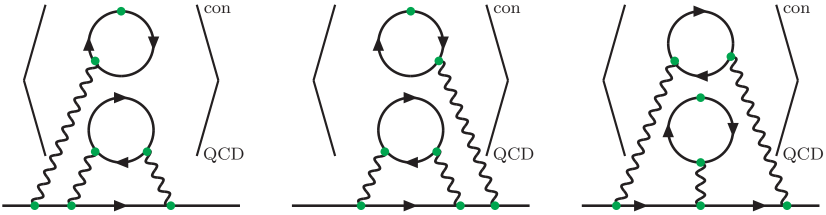


FIG. 1: $(2_E,\, 2)$-type diagrams

Diagrams of $(1_E, 3)$-type

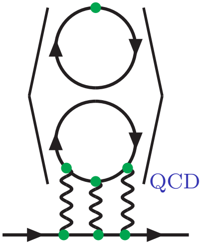


Figure: $(1_E, 3)$-type diagrams

Diagrams of $(2_E, 1, 1)$-type

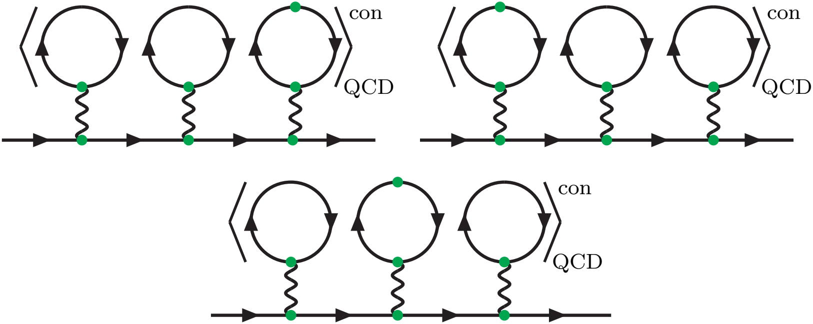


Figure: $(2_E, 1, 1)$-type diagrams

Diagrams of $(1_E, 1, 2)$-type

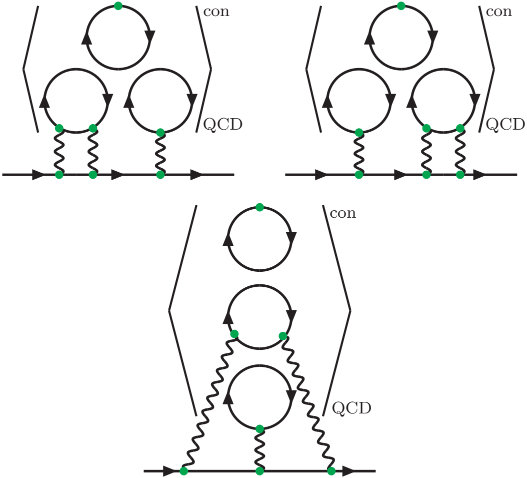


FIG. 1: $(1_E, 1, 2)$-type diagrams

Diagrams of $(1_E, 1, 1, 1)$-type

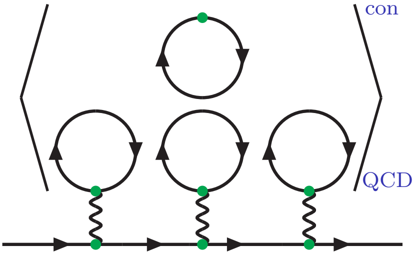


Figure: $(1_E, 1, 1, 1)$-type diagrams

Method to calculate disconnected-type diagrams

- One method to calculate all HLBL including disconnected-type diagrams was proposed in
  T. Blum, M. H. and T. Izubuchi, PoS LATTICE **2012**, 022 (2012) [arXiv:1301.2607 [hep-lat]]
  which will be reviewed later on.
- It needs modification because one point was overlooked.
- The succeeding content of this talk is as follows;
  - I explain the overlooked point, which is actually a general remark that must be taken into account in any method to calculate the disconnected-type HLBL diagrams.
  - I illustrate how to reflect such a remark in the calculation by modifying the nonperturbative QED method in PoS LATTICE **2012**, 022 (2012).

Field-theoretically disconnected components

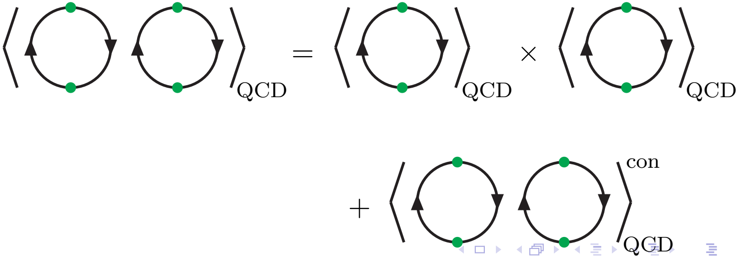
- Lattice QCD simulation may enable to compute the VEV of four hadronic electromagnetic (EM) currents $j_\mu(x)$

$$
\begin{aligned}
&\left\langle j_{\mu_{(1)}}(x_{(1)})\, j_{\mu_{(2)}}(x_{(2)})\, j_{\mu_{(3)}}(x_{(3)})\, j_{\mu_{(4)}}(x_{(4)}) \right\rangle_{\mathrm{QCD}} \\
&= \frac{1}{Z_{\mathrm{QCD}}} \int dU \int dq d\overline{q}\, e^{-S_{\mathrm{QCD}}[U, q, \overline{q}]} \\
&\qquad j_{\mu_{(1)}}(x_{(1)})\, j_{\mu_{(2)}}(x_{(2)})\, j_{\mu_{(3)}}(x_{(3)})\, j_{\mu_{(4)}}(x_{(4)}) \, .
\end{aligned}
$$

- It *inevitably contains* the contribution caused by *field-theoretically disconnected diagrams*.
- A field-theoretically disconnected diagram is a *Feynman diagram consisting of more than one connected component*.

Field-theoretically disconnected components

- The contribution from all field-theoretically connected diagrams to $\mathcal{A}$ is denoted by $\langle \mathcal{A} \rangle_{\mathrm{QCD}}^{\mathrm{con}}$.
- The quantity necessary for HLBL is
$$\left\langle j_{\mu_{(1)}}(x_{(1)})\, j_{\mu_{(2)}}(x_{(2)})\, j_{\mu_{(3)}}(x_{(3)})\, j_{\mu_{(4)}}(x_{(4)}) \right\rangle_{\mathrm{QCD}}^{\mathrm{con}}.$$
- For instance, (In momentum space, disconnected components appear only for special kinematics.)



Field-theoretically disconnected components

- In lattice QCD simulation, it is difficult to isolate $\langle\mathcal{A}\rangle^{\mathrm{con}}_{\mathrm{QCD}}$ $directly$.
- Every method to calculate the HLBL diagrams with more than one quark loop with QED vertices (,$i.e.$ disconnected-type ones in terms of lattice field theory,) inevitably picks up $O(\alpha^3)$ HVP, called $unwanted$ contribution hereafter.
- If we adopt the value for HVP derived using experimental data, we must subtract such an unwanted contribution in the Green function to avoid overcounting.
- The feasibility of the method is affected by the possibility of subtraction of the unwanted contribution, and by the efficiency of subtraction scheme even if possible.

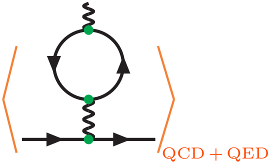
Nonperturbative QED method

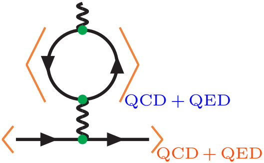
- To see how subtraction of disconnected components is realized, we try to modify the method proposed in
  T. Blum, M. H. and T. Izubuchi, PoS LATTICE **2012**, 022 (2012) [arXiv:1301.2607 [hep-lat]]
- The term $(-\mathcal{K}_D)$ is introduced to remove $O(\alpha^3)$ HVP contribution

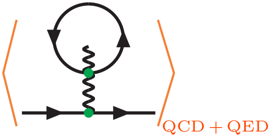
$$\frac{1}{3}\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\}$$

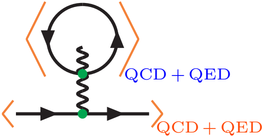
Nonperturbative QED method

$$\frac{1}{3}\left\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\right\}$$

$\mathcal{M}_C =$ 

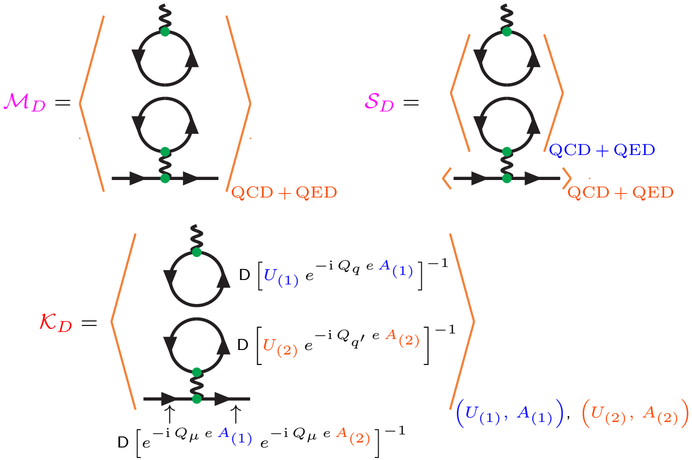
$\mathcal{S}_C =$ 

$\mathcal{M}_{C'} =$ 

$\mathcal{S}_{C'} =$ 

Nonperturbative QED method

$$\frac{1}{3}\left\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\right\}$$



Nonperturbative QED method

Every HLBL diagram is generated with *triplicate redundancy* :

Table: The multipication factor of each diagram provided by the individual terms in nonperturbative QED method. C, say, denotes $\mathcal{M}_C - \mathcal{S}_C$.

	$C + C'$	D
4_E	3	0
$(1_E, 3)$	0	3
$(2_E, 2)$	1	2
$(3_E, 1)$	2	1
$(1_E, 1, 2)$	0	3
$(2_E, 1, 1)$	1	2
$(1_E, 1, 1, 1)$	0	3

Nonperturbative QED method

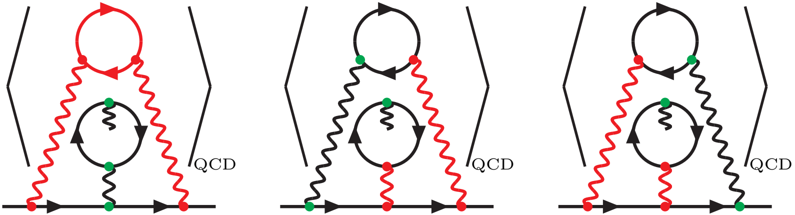


Figure: An identical diagram of $(2_E, 2)$-type is generated in three ways from $\mathcal{M}_C$ (left) and $\mathcal{M}_D$ (middle, right). The red stuffs are generated by the ensemble average of $(\mathrm{QCD} + \mathrm{QED})$.

Structure of unwanted HVP

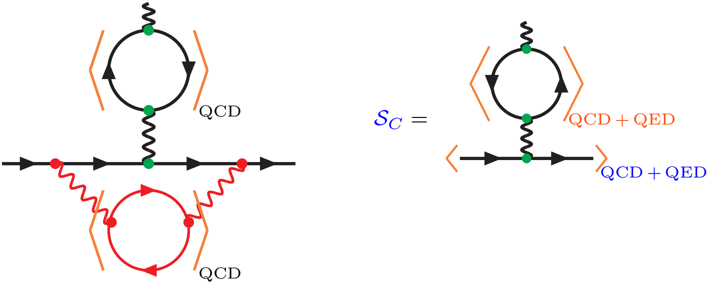


Figure: The left diagram is the disconnected component involved in a diagram of type $(2_E, 2)$ induced from $\mathcal{M}_C$. It is canceled by $(-\mathcal{S}_C)$.

- The disconnected contribution with a HVP function generated *entirely* by ensemble average is canceled by $(-\mathcal{S}_C)$, $(-\mathcal{S}_{C'})$ or $(-\mathcal{S}_D)$. This cancellation takes place with use of the photon field A generated by $(\mathrm{QCD} + \mathrm{QED})$ for $D[A]^{-1}$ in the muon part.

Structure of unwanted HVP

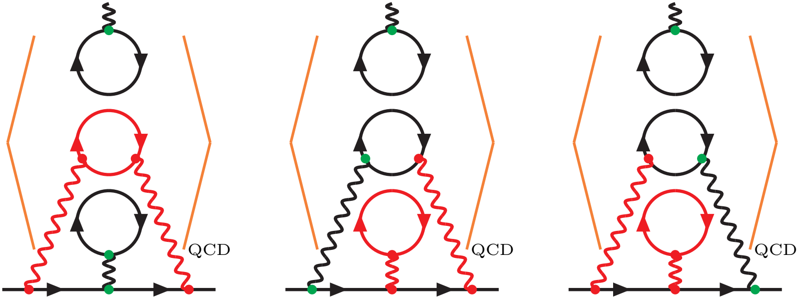


Figure: An identical diagram of $(1_E, 1, 2)$-type is generated from $\mathcal{M}_D$ in three ways. The disconnected component of the left diagram is canceled by $(-\mathcal{S}_D)$. However, *the other two disconnected components survive without being subtracted*.

Structure of unwanted HPV

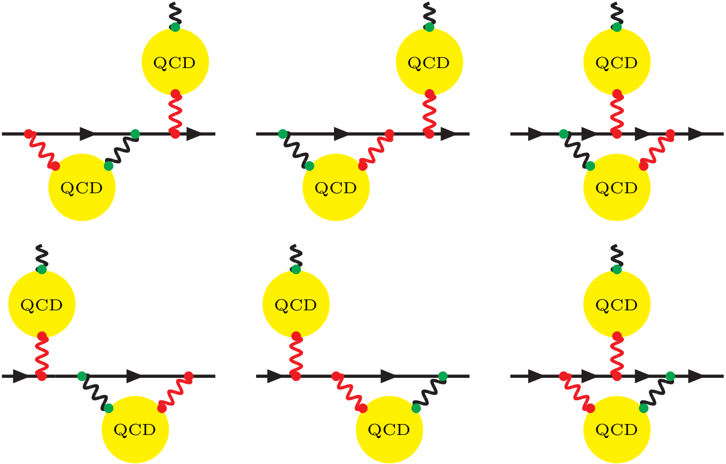


Figure: Summary of unwanted diagrams. Every diagram appears with *duplicate redundancy*.

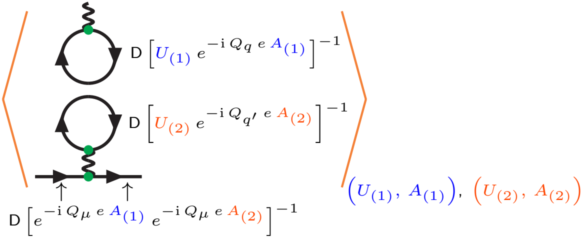
Structure of unwanted HPV

Here, HVP function consists of connected-type contribution and disconnected-type contribution in terms of lattice field theory (diagrams with $O(a)$ QED vertices are not shown here)

where the red stuffs are generated by the ensemble average of $(\mathrm{QCD} + \mathrm{QED})$.

On $\mathcal{K}_D$

$\mathcal{K}_D =$



The leading-order diagrams from $\mathcal{K}_D$ correspond exactly to the full set of the *unwanted* diagrams, including multiplicity :

- Let's focus on the quark part which involves two independent QCD averages.
- $\langle j_\mu \rangle_{\mathrm{QCD}} = 0.$ For the QCD average to be nontrivial at the leading order of α, one additional photon must be supplied by each of the QED averages.

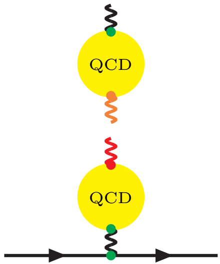
On $\mathcal{K}_D$

- That photon can emerge either from the valence quark loop (1st term on RHS) or from the sea quark loop (2nd term), giving the whole hadronic vacuum polarization function (diagrams with $O(a)$ QED vertices are not shown here)

where the red stuffs are generated by the ensemble average.

On $\mathcal{K}_D$

- The situation reached so far for $\mathcal{K}_D$ at the leading order is



- These two photons are the quanta of different $\mathrm{U}(1)$ gauge fields so that they cannot be identical with each other.
- Hence, both photon lines must be stuck somewhere on the muon lines.

On $\mathcal{K}_D$

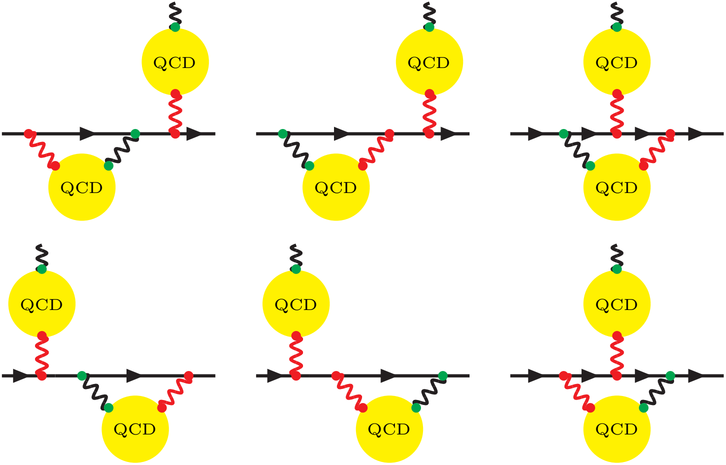


Figure: $\mathcal{K}_D$ at the leading-order, which coincides with the set of *unwanted* diagrams

Summary

- The VEV of four EM currents calculated by lattice QCD simulation inevitably contains the field-theoretically disconnected components.
- To invent the method to calculate the hadronic light-by-light scattering diagrams, it is necessary to specify the way to subtract $O(\alpha^3)$ hadronic vacuum polarization contribution (HVP) caused by these disconnected components.
- In the context of nonperturbative QED method, it is possible to add a term $(-\mathcal{K}_D)$ to subtract the unwanted $O(\alpha^3)$ HVP contribution

$$\frac{1}{3}\left\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\right\} .$$