Renormalization of two-dimensional XQCD

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arXiv:1507.02392 [hep-th] “Renormalization of Extended QCD_2”

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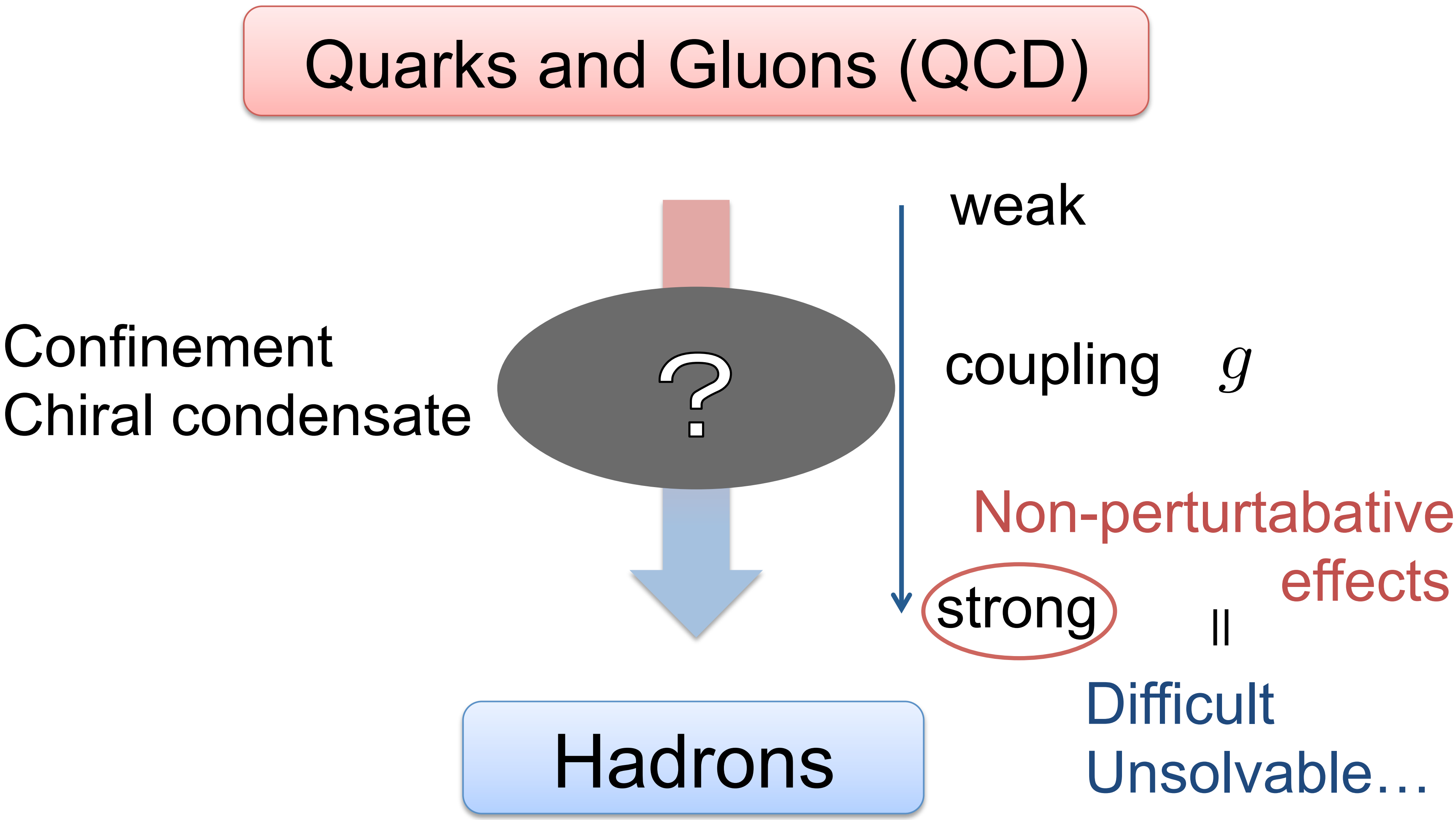
15/07/15

Lattice 2015 @Kobe

PARTICLE THEORY OSAKA

From QCD to Hadrons



New approach = XQCD

Reformulation of QCD

Kaplan(2013)

Extended QCD (XQCD) ＝ QCD + Auxiliary fields

XQCD is exactly equivalent to QCD

However,

XQCD contains low energy pictures more naturally !

（quark model, chiral perturbation, bag model…）

Lattice XQCD

Our final goal : Lattice XQCD

Serious sign problem… (even at $\mu_B = 0$)

$$\bar{\psi}\left(\partial\!\!\!/ + i\mathbf{A}\!\!\!/ + \underline{\Phi P_+ + \Phi^\dagger P_- + \mathbf{v}\!\!\!/ + i\mathbf{a}\!\!\!/\gamma_5}\right)\psi$$

Interactions with auxiliary fields

Today : 2d continuum (X)QCD in the large Nc limit

('t Hooft model)

Our work

||

Renormalization group (RG) analysis of (XQCD)

What is the role of auxiliary fields at low-energy ?

Similar works：From Quarks and Gluons to Hadrons
Braun, Jens et al. arXiv:1412.1045 [hep-ph]

Contents

Definition of XQCD

partition function: $Z_{\mathrm{QCD}} = \int e^{-S_{\mathrm{QCD}}}$ (4d Euclidean)

Multiply $1 = \int e^{-S_{\mathrm{aux}}}$

$$\underline{Z_{\mathrm{QCD}} = Z_{\mathrm{XQCD}}} = \int e^{-S_{\mathrm{QCD}} - S_{\mathrm{aux}}}$$

$$S_{\mathrm{XQCD}} \equiv S_{\mathrm{QCD}} + S_{\mathrm{aux}}$$

Cancellation by Fierz identity

$$1 = \int e^{-S_{\rm aux}}$$ ⬅ $S_{\rm aux}$ is Gaussian.

$$S_{\rm aux} \propto (\Phi^\dagger + \bar\psi P_+ \psi)(\Phi + \bar\psi P_- \psi)$$

Non-renormalizable

$$= \Phi^\dagger \Phi + \bar\psi(\Phi P_+ + \Phi^\dagger P_-)\psi + (\bar\psi P_+ \psi)(\bar\psi P_- \psi)$$

Fierz identity ⇒ Cancellation

$$(\bar\psi P_+ \psi)(\bar\psi P_- \psi) + \tfrac{1}{2}(\bar\psi \gamma_\mu \psi)(\bar\psi \gamma_\mu \psi) - \tfrac{1}{2}(\bar\psi \gamma_\mu \gamma_5 \psi)(\bar\psi \gamma_\mu \gamma_5 \psi) = 0$$

$$S_{\rm aux} \propto (\Phi^\dagger + \bar\psi P_+ \psi)(\Phi + \bar\psi P_- \psi) + \tfrac{1}{2}(\mathbf{v}_\mu + \bar\psi \gamma_\mu \psi)^2 + \tfrac{1}{2}(\mathbf{a}_\mu + i\bar\psi \gamma_\mu \gamma_5 \psi)^2$$

Quark model picture in XQCD

$$\bar{\psi}\left(i\not{\mathbf{A}} + \Phi P_+ + \Phi^\dagger P_- + \not{\mathbf{v}} + i\not{\mathbf{a}}\gamma_5\right)\psi$$

New interactions

① Repulsive interaction by $\mathbf{v}_\mu$ exchanges

⇒ weakening the attractive interaction

② Constituent quark mass by $\langle\Phi\rangle$

$$(\Phi P_+ + \Phi^\dagger P_-)\bar{\psi}\psi \to \langle\Phi\rangle\left(P_+ + P_-\right)\bar{\psi}\psi = M\bar{\psi}\psi$$

①+② = weakly interacting and massive quarks

Quark model picture !

Next step of XQCD

XQCD = QCD + auxiliary fields

- Quark model picture
- Massless pion
- bag model

↑

low energy picture of QCD

⇒ naturally explained in XQCD

Auxiliary fields in the low energy region ?

⬇

RG analysis

3. RG analysis of 2d (X)QCD

RG analysis of 2d large Nc (X)QCD

↑

('t Hooft model)

't Hooft (1974)

Simple and Solvable

Chiral Symmetry Breaking in the large Nc limit

➡ Good for the test model

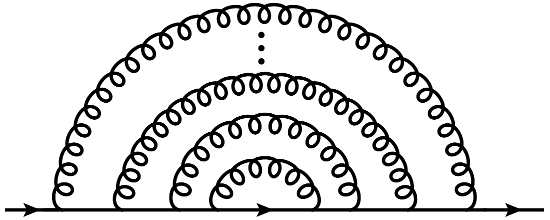
’t Hooft model

’t Hooft model = 2d QCD in the large Nc limit

No gluon self-interaction + planarity = solvable

’t Hooft coupling $g^2 \equiv g_0^2 N_c$ is fixed.

Ladder approximation is exact



➡ constituent mass : $M^2(m,g) = m^2 - g^2/\pi$

Set up

$$\int [d\phi_l] e^{iS_{\rm kin}[\phi_l]} \int \underline{[d\phi_h]\ e^{iS_{\rm kin}[\phi_h]+iS_{\rm int}[\phi_h+\phi_l]}}$$

$$= \int [d\phi_l] e^{iS_{\rm kin}[\phi_l]+iS_{{\rm int},\Lambda}[\phi_l]}$$

① soft cut-off at Λ
(reguralization)

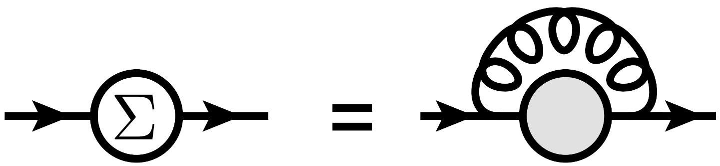
② Counter terms (e.g. preserve the symmetry)

① regularization + ② counter term = one scheme

③ Truncation

Neglect large Nc subleading and $O(\Lambda^{-4})$ terms.

RG flow of 2d large Nc QCD



Self-consistent equation ⇒ Non-perturbative result

$$Z_\psi^2(\Lambda) = 1 + \frac{\frac{g^2}{\pi\Lambda^2}\left(\log\left|\frac{\Lambda^2}{M^2}\right| - 1\right)}{1 - \frac{g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|},$$

$$m_R^2(\Lambda) = m^2\left(1 + \frac{\frac{2g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|}{1 - \frac{g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|}\right)$$

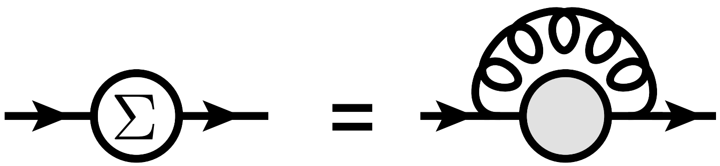
$$g_R^2(\Lambda) = \frac{Z_\psi^2(\Lambda)g^2}{1 - \frac{g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|}.$$

Around $\Lambda^2 \simeq M^2$,

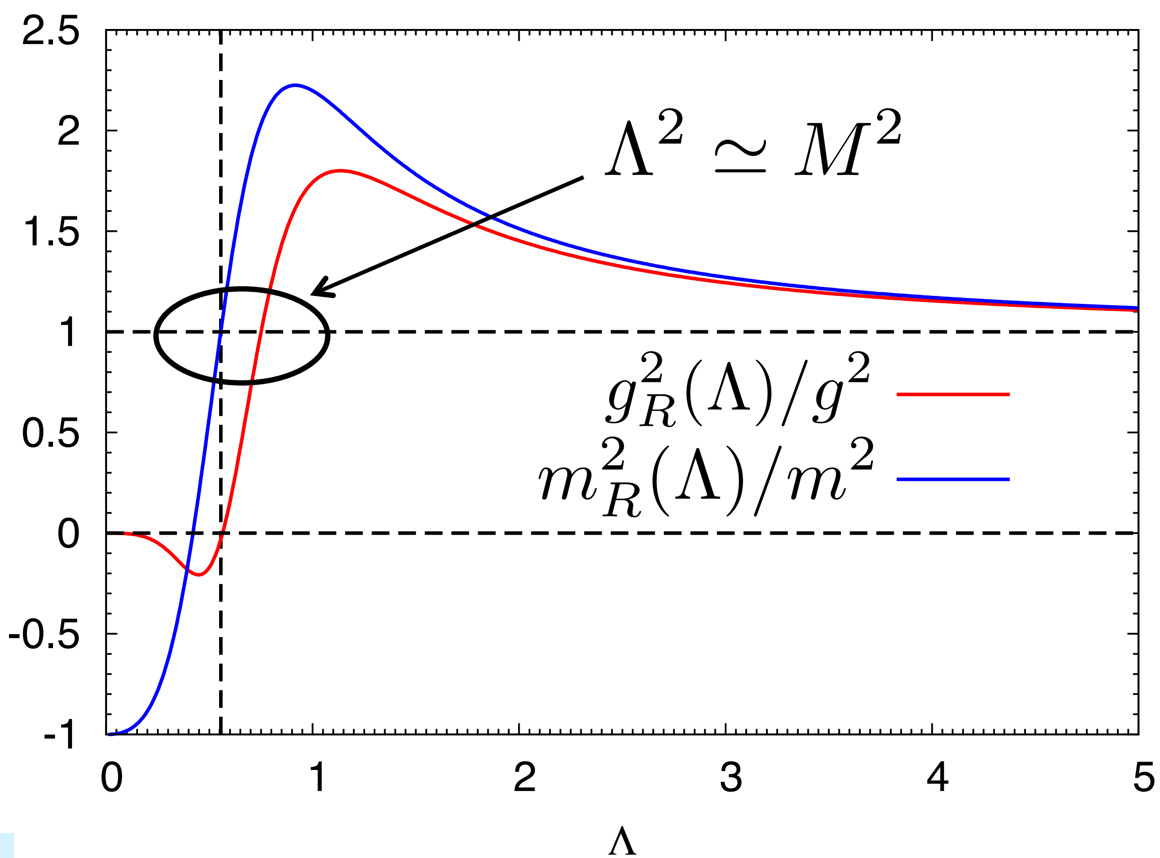
$(m_R, g_R) \sim (m, g)$

Reasonable results !

RG flow of 2d large Nc QCD



Self-consistent equation ⇒ Non-perturbative result

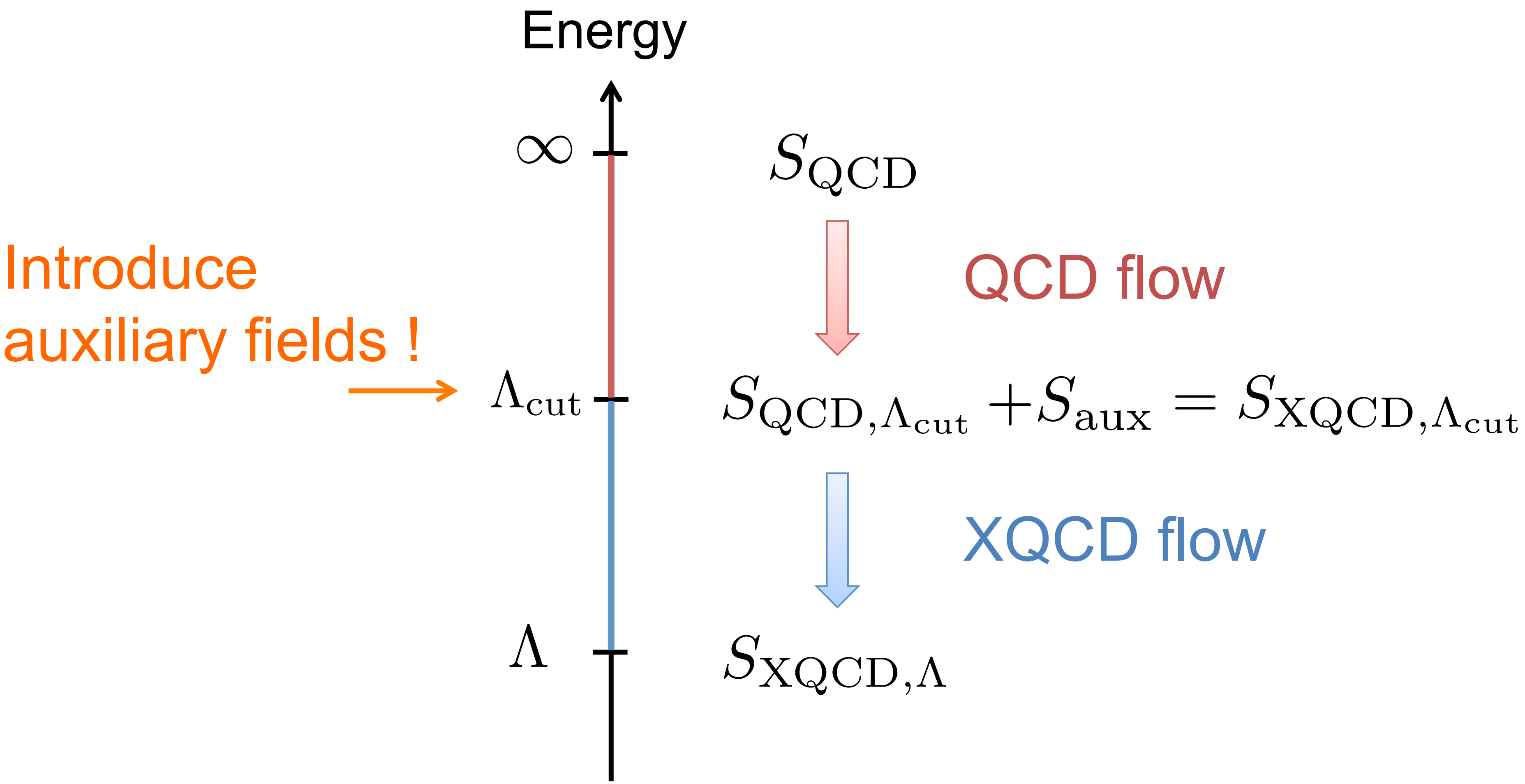


Around $\Lambda^2 \simeq M^2$,

$(m_R, g_R) \sim (m, g)$

Reasonable results !

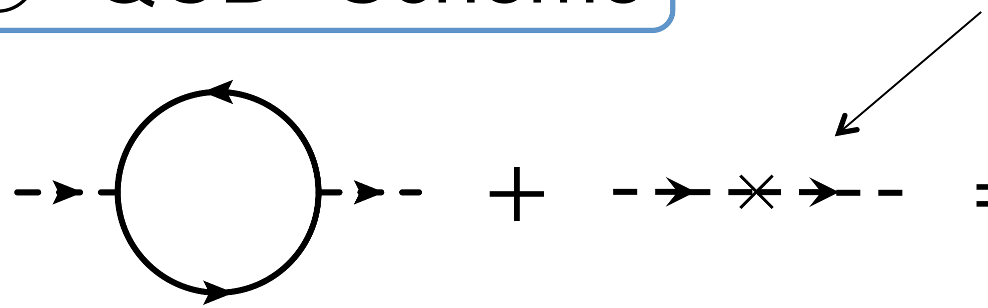
Set up in the RG study of XQCD



How does the flow change ?

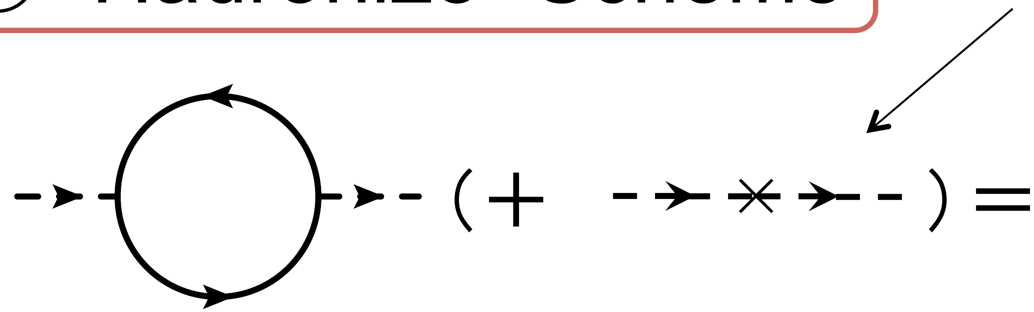
RG schemes

① “QCD” Scheme



⇒ Same as the QCD flow

② “Hadronize” Scheme



⇒ auxiliary field becomes dynamical

Φ becomes dynamical

New operators

$$Z_\Phi(\Lambda)\partial_\mu\Phi^\dagger\partial^\mu\Phi \qquad m_\Phi^2(\Lambda)\Phi^\dagger\Phi \qquad y(\Lambda)\bar{\psi}\Phi P_+\psi + \mathrm{h.c.}$$

$$\cancel{Z_v(\Lambda)(\partial_\mu \mathbf{v}_\nu)^2} \qquad m_v^2\xcancel{(\Lambda)}\mathbf{v}_\mu^2 \qquad \alpha\xcancel{(\Lambda)}\bar{\psi}\not{\mathbf{v}}\psi$$

(Others are at the 2-loop level)

Note : Φ acquires the kinetic term

$\mathbf{v}_\mu$ remains to be an auxiliary field.

1-loop RG flow of XQCD

$$Z_\Phi(\Lambda) = \frac{y^2(\Lambda)}{\pi}\left(\frac{1}{\Lambda^2} - \frac{1}{\Lambda_{\mathrm{cut}}^2}\right) + O(\Lambda^{-4}),$$

$$m_\Phi^2(\Lambda) = \lambda^2 - \frac{y^2(\Lambda)}{\pi}\log\left(\frac{\Lambda_{\mathrm{cut}}}{\Lambda}\right) + O(\Lambda^{-2}),$$

$$y(\Lambda) = \frac{\alpha\lambda}{1 + \frac{\alpha_R^2(\Lambda)}{\pi}\log\left(\frac{\Lambda_{\mathrm{cut}}}{\Lambda}\right)} + O(\Lambda^{-2}),$$

$$m_R^2(\Lambda) = m^2\left(1 + \frac{\frac{2g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|}{1 - \frac{g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|}\right),$$

$$g_R^2(\Lambda) = \frac{Z_\psi^2(\Lambda)g^2}{1 - \frac{g^2}{\pi\Lambda^2}\log\left|\frac{\Lambda^2}{M^2}\right|}.$$

Flows are not changed…

What is interesting in XQCD ?

$$Z_\Phi(\Lambda)\partial_\mu\Phi^\dagger\partial^\mu\Phi + m_\Phi(\Lambda)\Phi^\dagger\Phi + y(\Lambda)\bar{\psi}\Phi P_+\psi + \mathrm{h.c.}$$

When normalizing $Z_\Phi(\Lambda) = 1$,

$$Z^{-1/2}(\Lambda)y(\Lambda) \sim \sqrt{\pi}\Lambda$$

$$Z^{-1}(\Lambda)m_\Phi^2(\Lambda) \sim \frac{\pi\Lambda^2}{y^2(\Lambda)}\left[\lambda^2 - \frac{y^2(\Lambda)}{2\pi}\log\left(\frac{\Lambda_{\mathrm{cut}}}{\Lambda}\right)\right]$$

$(\Lambda^2 \ll \Lambda_{\mathrm{cut}}^2)$

Looks like quadratic divergence

$\Rightarrow$ Auxiliary fields decouple ?
No effect on the low energy physics ?

π becomes dominant

$$m_\Phi^2 \Phi^\dagger \Phi,\ m(\Phi + \Phi^\dagger) \longrightarrow m_\pi^2 = m \langle \bar{\psi}\psi \rangle + O(m)$$

$$\Phi = \langle \Phi \rangle e^{\sigma + i\pi}\ , \langle \Phi \rangle \propto \langle \bar{\psi}\psi \rangle$$

When $m \to 0$, $m_\pi \to 0$.

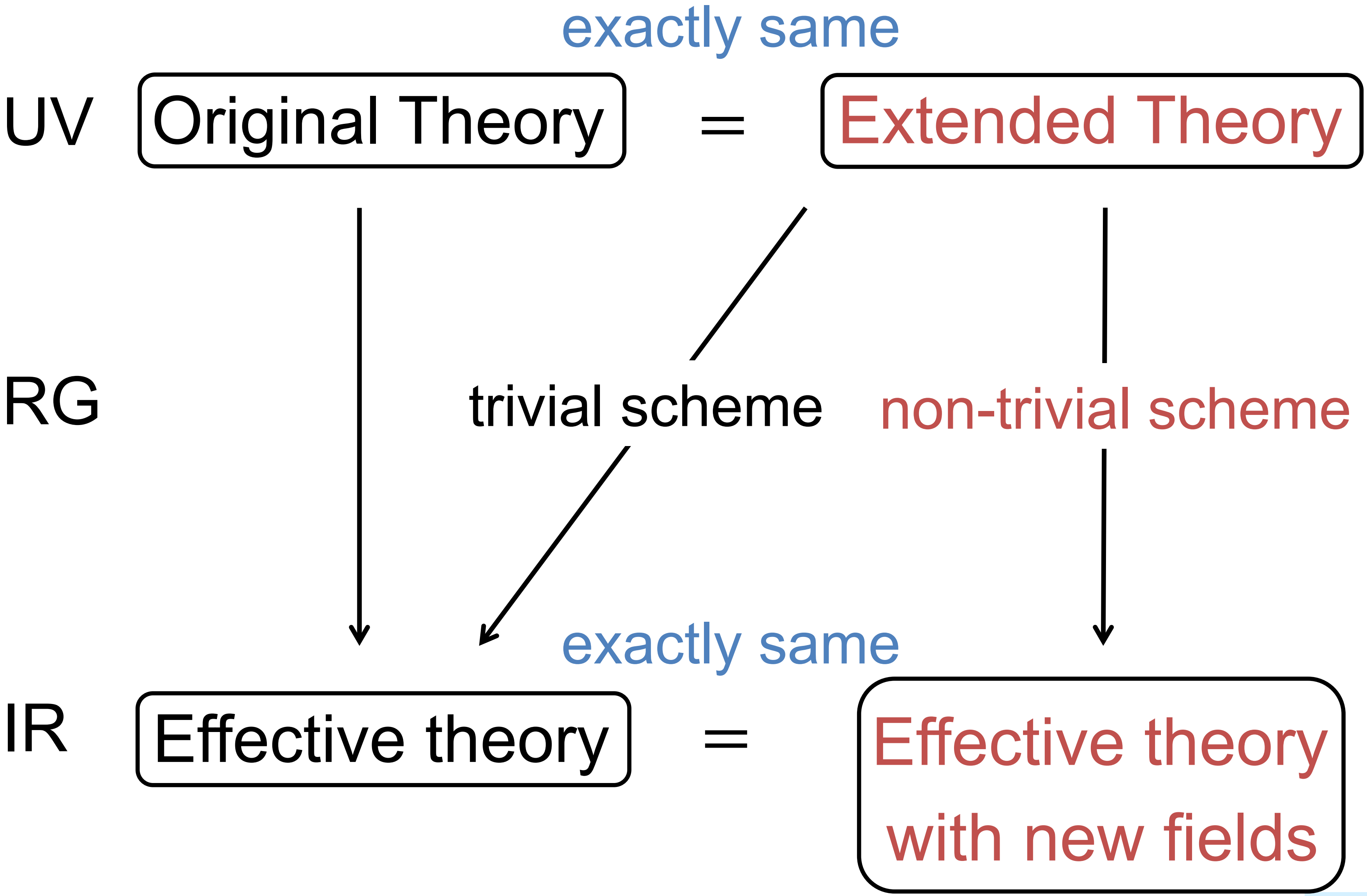
This fact always holds along the RG flow.

(∵ Chiral symmetry)

π is dominant in the low energy region !

↑ We can never see this in the QCD flow.

Extended RG scheme



XQCD flow is interesting !

1. 2d QCD flow is obtained non-perturbatively.
2. 2d XQCD flow at the 1-loop level is studied

The auxiliary field Φ acquires the kinetic term.

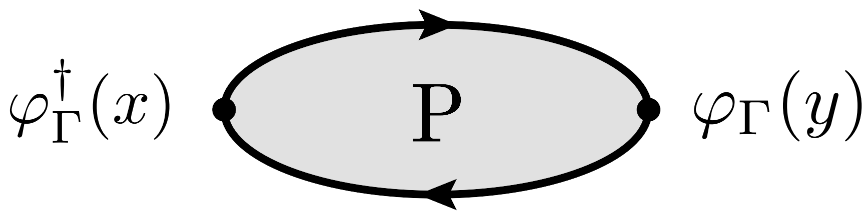
π becomes dominant in the low energy region !

↑ We can never see this in the QCD flow.

Back up

Massless pion

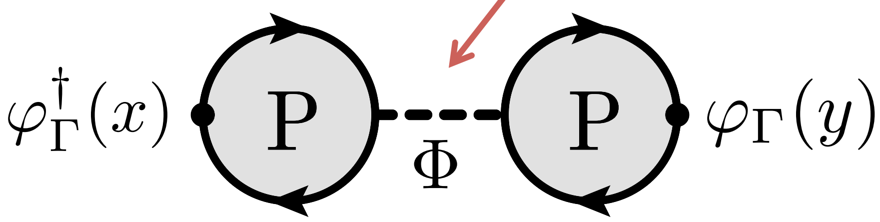
Meson correlator : $\langle \varphi^\dagger_\Gamma(x)\varphi_\Gamma(y)\rangle \quad \varphi_\Gamma(x)=\bar{\psi}(x)\Gamma\psi(y)$



Common for all mesons

↓ XQCD Only

massless pole



same quantum number

$$\Gamma \sim \Phi$$

Quark model and massless pion are compatible !

Symmetries in RG flow

Assume the gauge-invariant ansatz :

$$S_\Lambda = \int d^2x \left[-\frac{1}{2}\mathrm{Tr}\ (\mathbf{A}_+)_R \partial_-^2 (\mathbf{A}_+)_R + \bar{\psi}_R(i\partial\!\!\!/ - m_R(\Lambda))\psi_R \right.$$
$$\left. - \frac{g_R(\Lambda)}{\sqrt{N_c}}\bar{\psi}_R \mathbf{A}_+ \gamma^+ \psi_R + \cdots \right]$$

RG condition :

$$\langle \psi(x)\bar{\psi}(y)\rangle_{\mathrm{exact}} = Z_\psi(\Lambda)\,\langle \psi_R(x)\bar{\psi}_R(y)\rangle$$

⇓

$g_R(\Lambda)$ and $m_R(\Lambda)$ are obtained.